Fuzzy Membership Function Optimization for System Identification Using an Extended Kalman Filter

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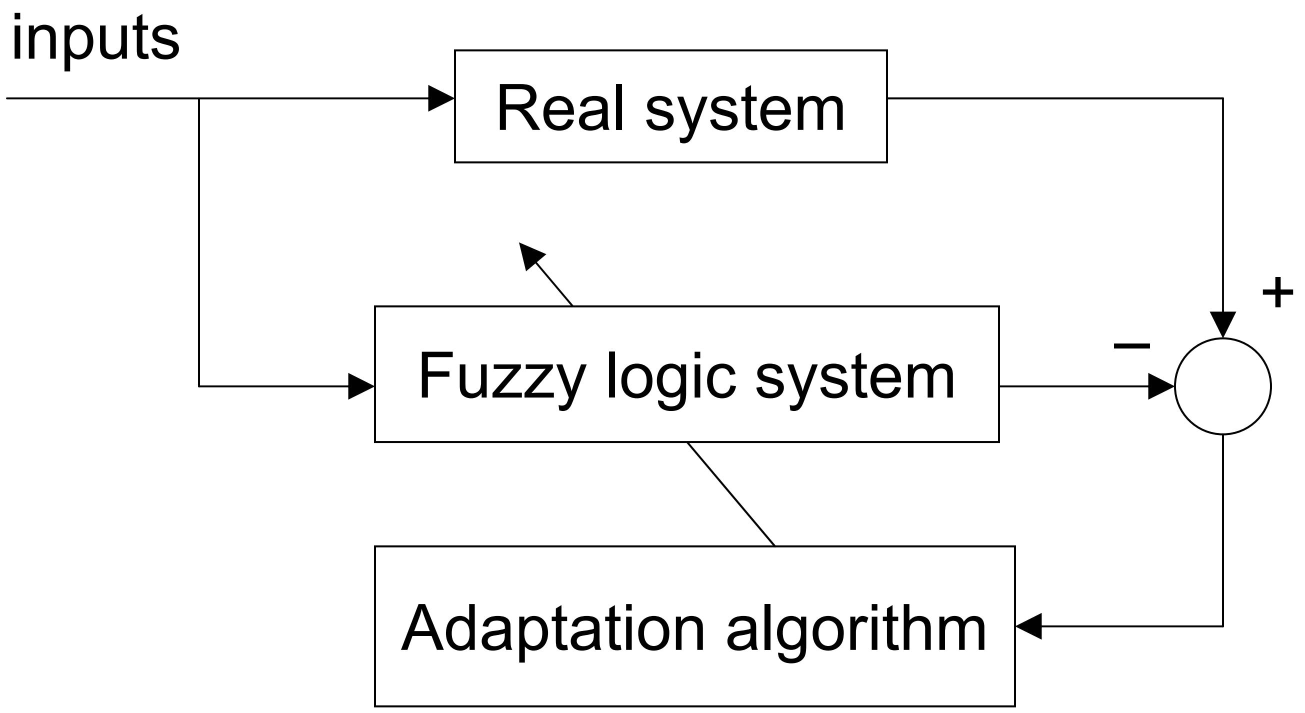
Overview


inputs
Real system
Fuzzy logic system
Adaptation algorithm
+
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Overview

1. Introduction
2. Membership optimization
3. Extended Kalman filter
4. Fuzzy model identification
5. Experimental results
6. Conclusion and future work

Introduction

- Fuzzy system performance depends on
  – Rule base
  – Membership functions
- Given a rule base, is membership function identification trivial?
  – In most real world problems it is not
  – Membership function tuning improves performance

Introduction

- Fuzzy membership function identification
  - Derivative-based methods
    - Simplex
    - Least squares
    - Backpropagation
    - Optimal filtering
  - Derivative-free methods
    - Genetic algorithms
    - Swarm optimization
    - Other methods of computer intelligence

Introduction

- Advantage of Kalman filtering
  - Use of optimal state estimation theory
    - Intuitively attractive in that we are identifying unknown parameters using available theory
  - Should work better than first-order methods (e.g., gradient descent)
  - Opens up new research in the area of membership function optimization

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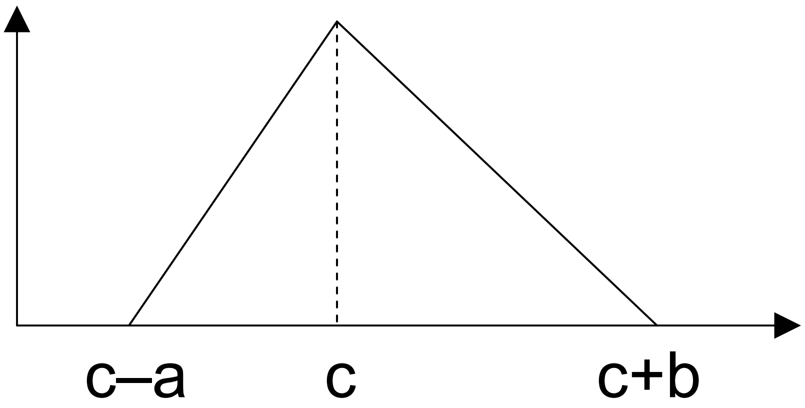
Membership Optimization

- Consider triangular membership functions for simplicity and proof of concept
- Each membership function is characterized by its centroid, lower half-width and upper half-width (c, a, and b, respectively)



Membership Optimization

- Degree of membership for the *i* th membership function of the *j* th input is

$$f_{ij}(x)=\begin{cases} 1+(x-c_{ij})/a_{ij} & \text{if } -a_{ij}\le(x-c_{ij})\le 0 \\ 1-(x-c_{ij})/b_{ij} & \text{if } 0\le(x-c_{ij})\le b_{ij} \\ 0 & \text{otherwise} \end{cases}$$

- Similar equation for output membership functions

Membership Optimization

- The mapping from the fuzzy output to a crisp value is given by

$$\text{crisp output} = \frac{\sum_{j=1}^{n} m(\gamma_{\mathrm{j}})\gamma_{\mathrm{j}} J_j}{\sum_{j=1}^{n} m(\gamma_{\mathrm{j}}) J_j}$$

where *m( )* is the output membership function, γ_{j} and J_j are the centroid and area of the *j* th output fuzzy membership function, and *n* is the number of fuzzy output sets

Membership Optimization

- Derivative-based training
- Consider

$$E = \frac{1}{2N}\sum_{q=1}^{N} g_q (y_q - \hat{y}_q)^2$$

  - *g* is the user defined weighting function
  - *y* is the target value and *y*-hat is the output of the fuzzy system
- *E* can be optimized w/r to centroids and half widths

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Extended Kalman filter

- Proposed by Rudolf Kalman
- Highly celebrated for the purpose of optimal state estimation
- Developed in late 1950's and early 1960's

Extended Kalman filter

- Consider

$$x_{n+1} = f(x_n) + w_n$$

$$d_n = h(x_n) + v_n$$

where *x* is the state, *w* is the process noise, *d* is the measurement, and *v* is the measurement noise

Extended Kalman filter

- Consider

$$E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T] = P_0$$

$$E(w_n w_l^T) = Q\delta_{nl}$$

$$E(v_n v_l^T) = R\delta_{nl}$$

Extended Kalman filter

- The problem addressed by the KF is to

find $\hat{x}_{n+1}$ of x_{n+1} given $d_j \ (j = 0, \ldots, n)$

- If the nonlinearities are smooth, the system can be simplified to

$$x_{n+1} = F_n x_n + w_n + f(\hat{x}_n) - F_n \hat{x}_n$$
$$d_n = H_n x_n + v_n + h(\hat{x}_n) - H_n \hat{x}_n$$

Extended Kalman filter

- Knowing the system dynamics, the desired estimate can be computed as

$$K_n = P_n H_n (R_n + H_n^T P_n H_n)^{-1}$$

$$\hat{x}_n = f(\hat{x}_{n-1}) + K_n[d_n - h(\hat{x}_{n-1})]$$

$$P_{n+1} = F_n(P_n - K_n H_n^T P_n) F_n^T + Q_n$$

Extended Kalman filter

- If the computational effort is too high, and the derivative of *h( )* w/r to *x* does not change too much during optimization, the following assumption can be made:

$$H_n \approx H_0 = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_0}$$

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Fuzzy model identification

- Consider a permanent magnet synchronous motor (a nonlinear system)
- The fuzzy system model can be used to predict the motor output, which can then be used for closed-loop control

Fuzzy model identification

- Consider a two-input, one-output fuzzy logic system
- Consider a fuzzy system which has μ fuzzy sets for the first input, ν fuzzy sets for second input, and κ fuzzy sets for the output

Fuzzy model identification

Rule base for fuzzy filter

		Input 2						
		NL	NM	NS	Z	PS	PM	PL
Input 1	NL	NL	NL	NM	NM	NS	NS	Z
	NM	NL	NM	NM	NS	NS	Z	PS
	NS	NM	NM	NS	NS	Z	PS	PS
	Z	NM	NS	NS	Z	PS	PS	PM
	PS	NS	NS	Z	PS	PS	PM	PM
	PM	NS	Z	PS	PS	PM	PM	PL
	PL	Z	PS	PS	PM	PM	PL	PL

NL = negative large, NM = negative medium, NS = negative small, Z = zero, PS = positive small, PM = positive medium, PL = positive large.

Fuzzy model identification

- If both the half-widths and centroids for the inputs and outputs are considered as parameters defining the fuzzy system, the state vector of the dynamic system for the KF can be given as

$$\left.\begin{aligned} x = [&a_{11}\ b_{11}\ \mathrm{c}_{11} \ldots \mathrm{a}_{\mu 1}\ b_{\mu 1}\ \mathrm{c}_{\mu 1} \\ &a_{12}\ b_{12}\ \mathrm{c}_{12} \ldots a_{\nu 2}\ b_{\nu 2}\ \mathrm{c}_{\nu 2} \\ &p_1\ q_1\ r_1 \ldots p_\kappa\ q_\kappa\ r_\kappa]^T \end{aligned}\right\} \text{3×3×5 = 45 elements}$$

Fuzzy model identification

- System description

$$x_{n+1} = x_n$$
$$d_n = h(x_n)$$

- For implementing the Kalman filter, we add artificial noise to the model

$$x_{n+1} = x_n + w_n$$
$$d_n = h(x_n) + v_n$$

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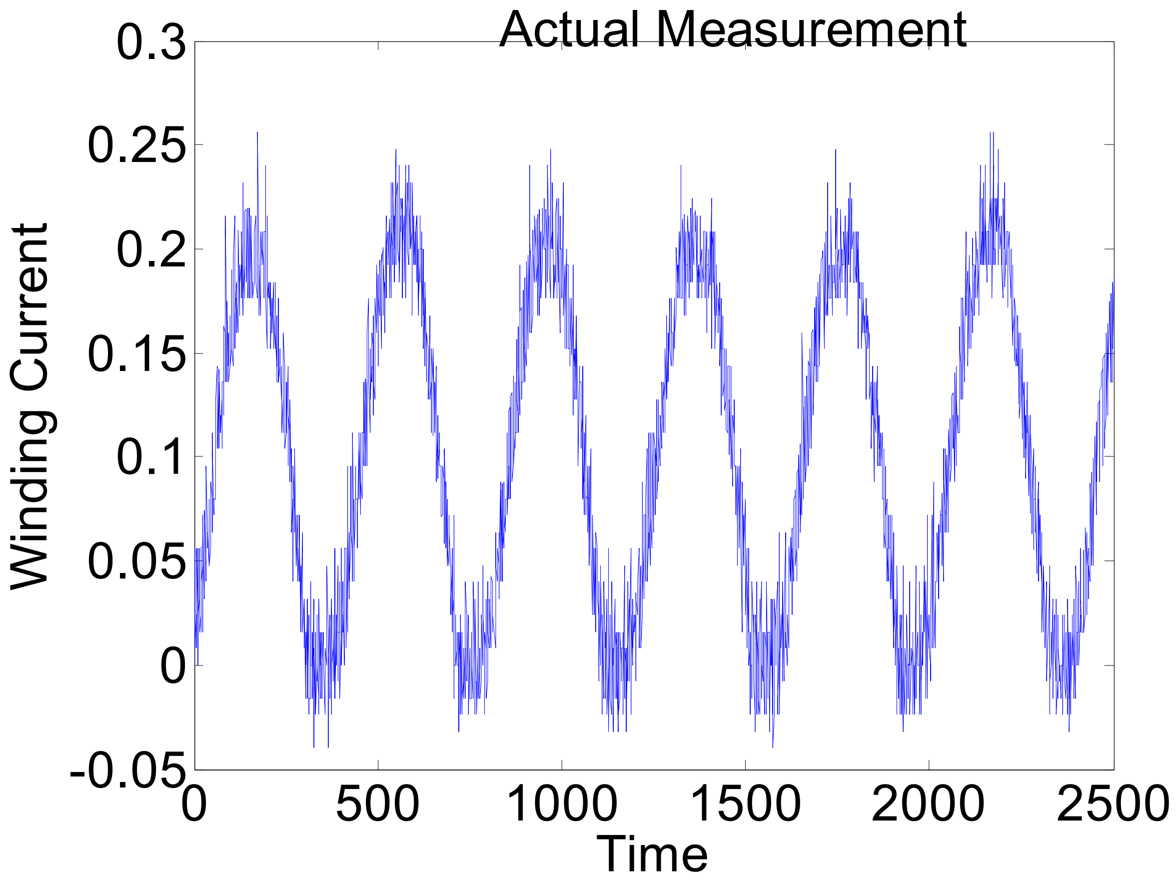
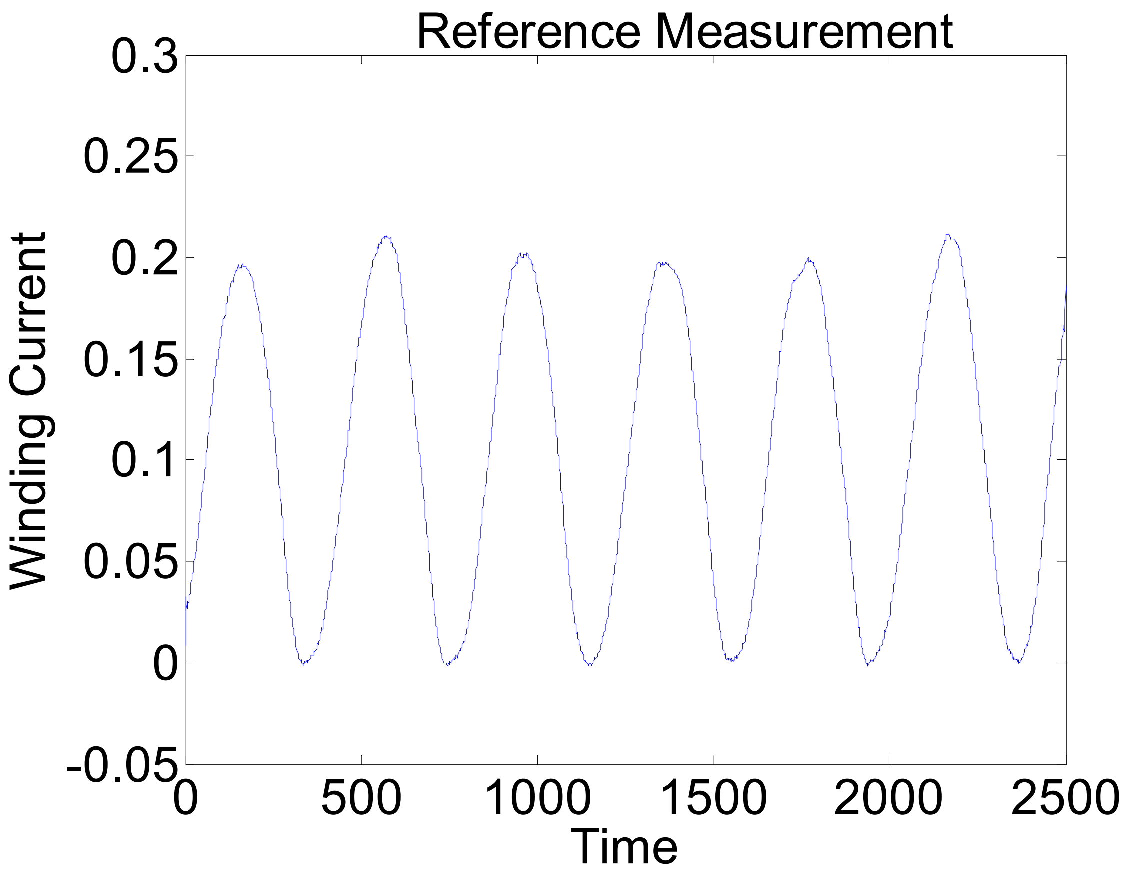
Experimental results

Actual measurement



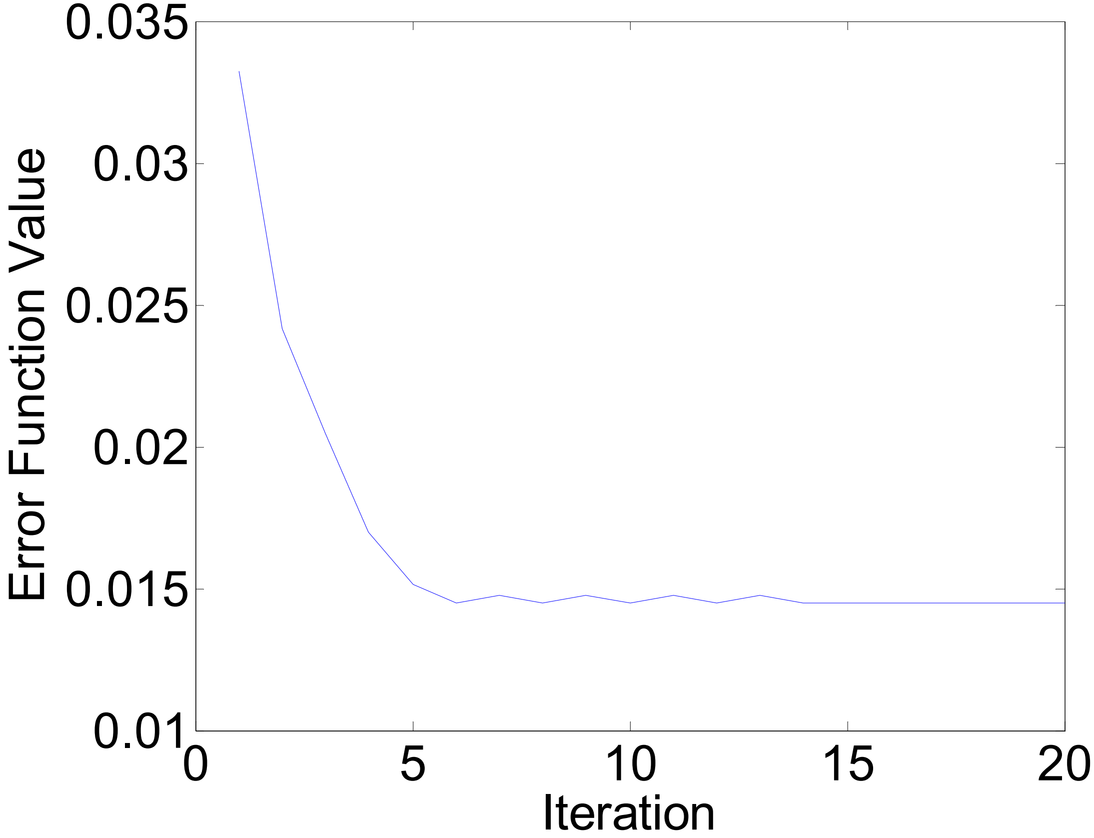
Experimental results

Reference measurement



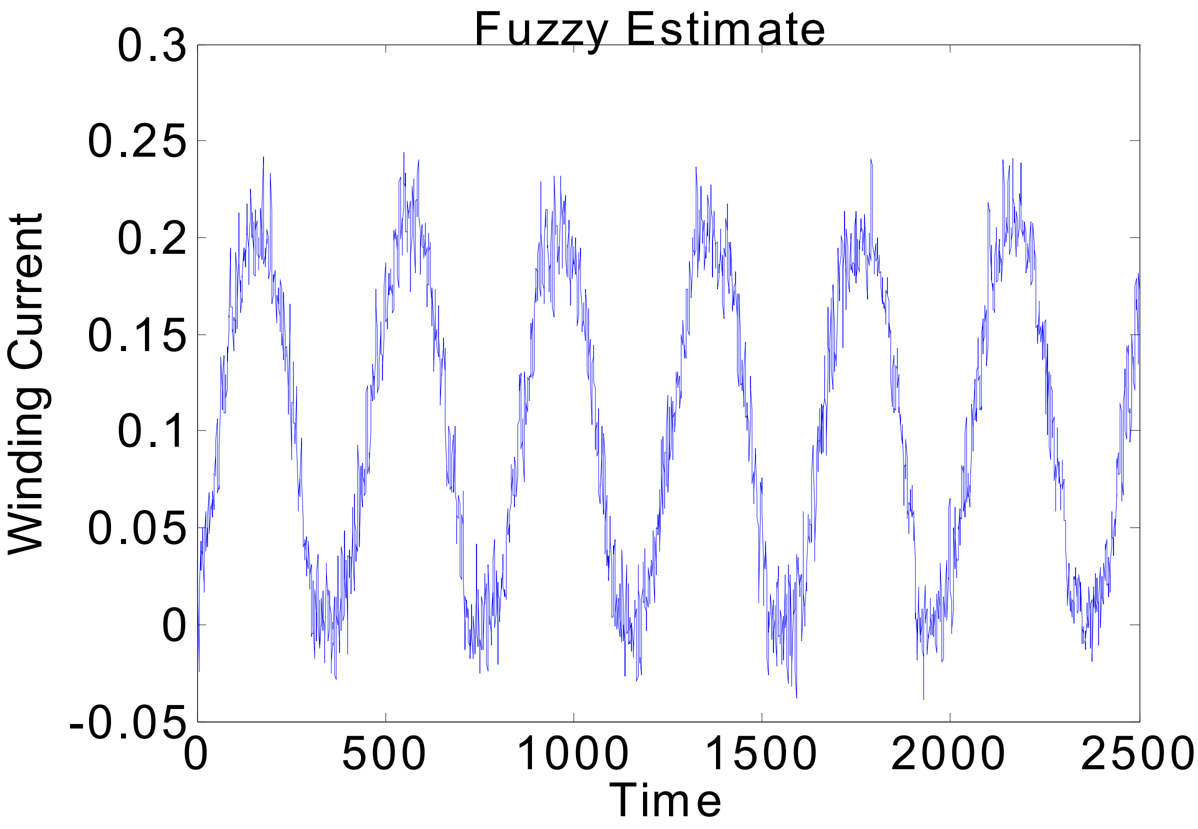
Experimental results

KF training progress



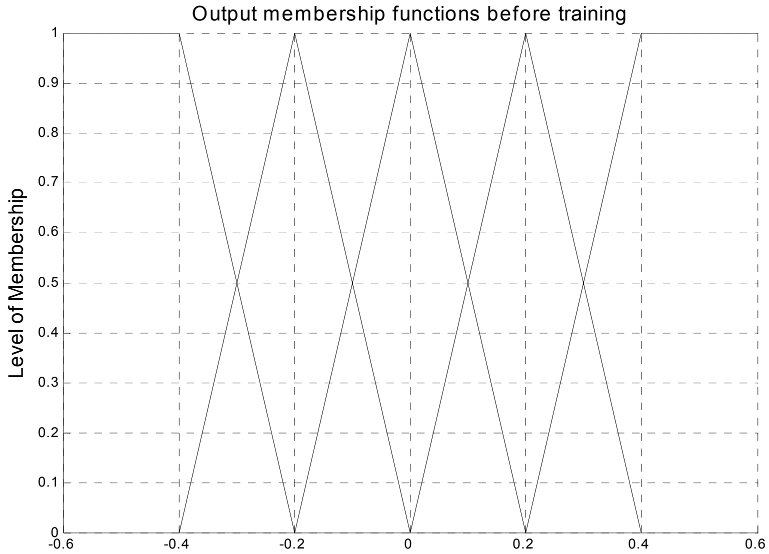
Experimental results

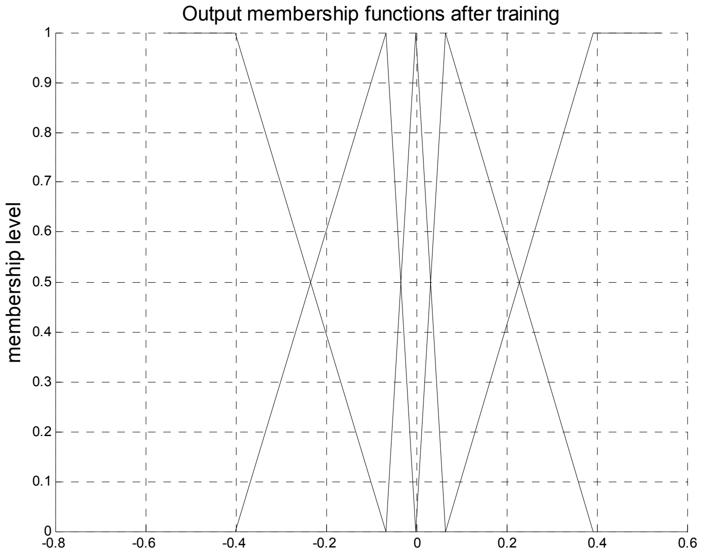
Fuzzy output



Experimental results

Output membership functions before and after training





Conclusion and future work

- Optimization of fuzzy parameters can be posed as an optimal filtering problem
- Fuzzy systems can be used as black-box models for general dynamic systems
- Kalman filter parameters can be further tuned
- Other filtering algorithms could be used (e.g., H-infinity and unscented Kalman filters)
- Establish convergence/stability results of the Kalman filter for this application