OPTIMAL FILTERING TECHNIQUES FOR ANALYTICAL STREAMFLOW FORECASTING

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Overview

- Motivation
- Hydrology Model
- Problem formulation
- H-infinity estimator design
- Experimental results
- Conclusion
- Future work

Motivation

- Hydrology
- Hydrology Modeling
  - Conceptual Hydrologic Model
  - Used by NWS (National Weather Service)

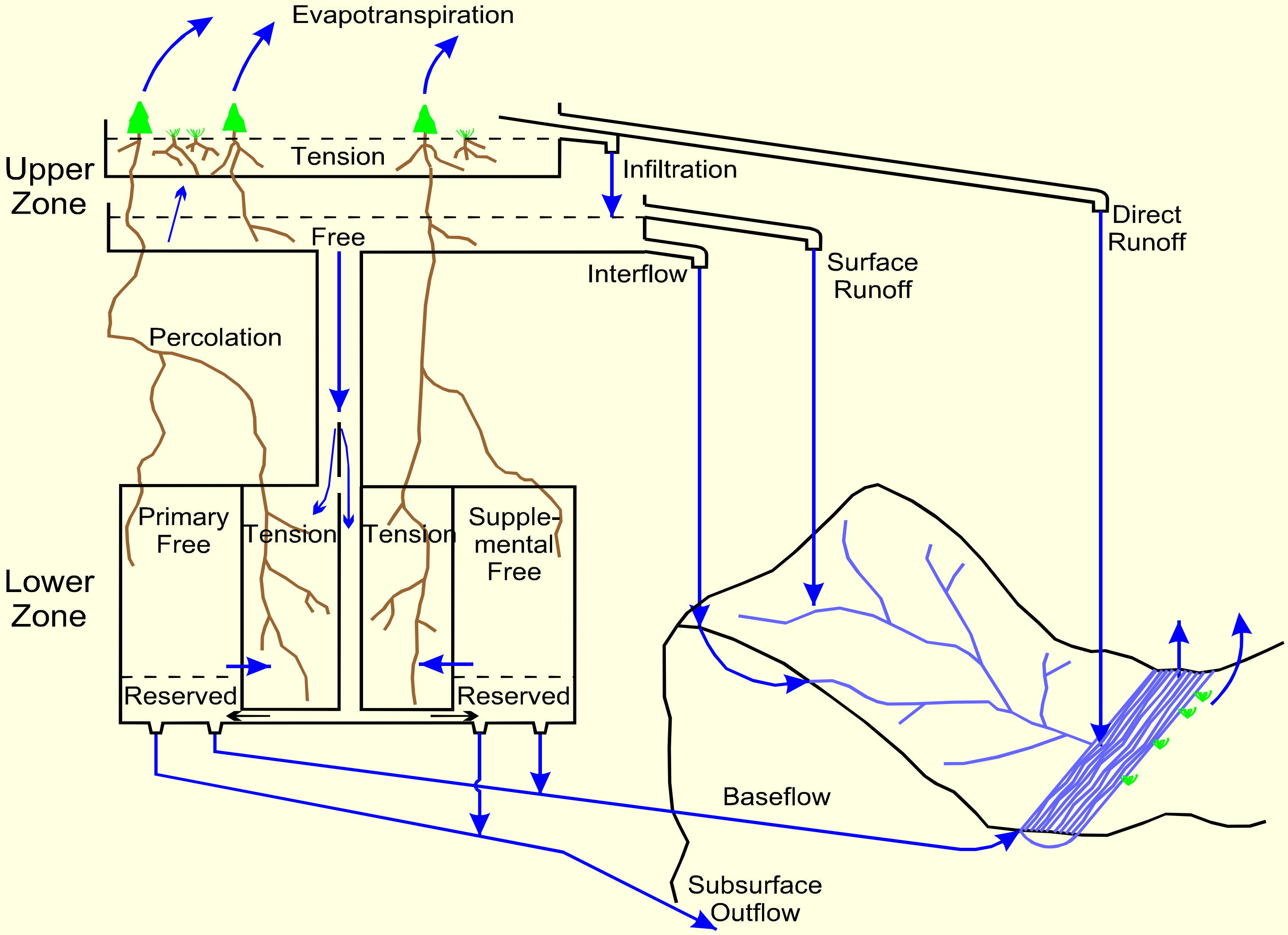
- Errors related to model
  - Measurement, Input, Model Errors

Hydrology Model

Soil Division

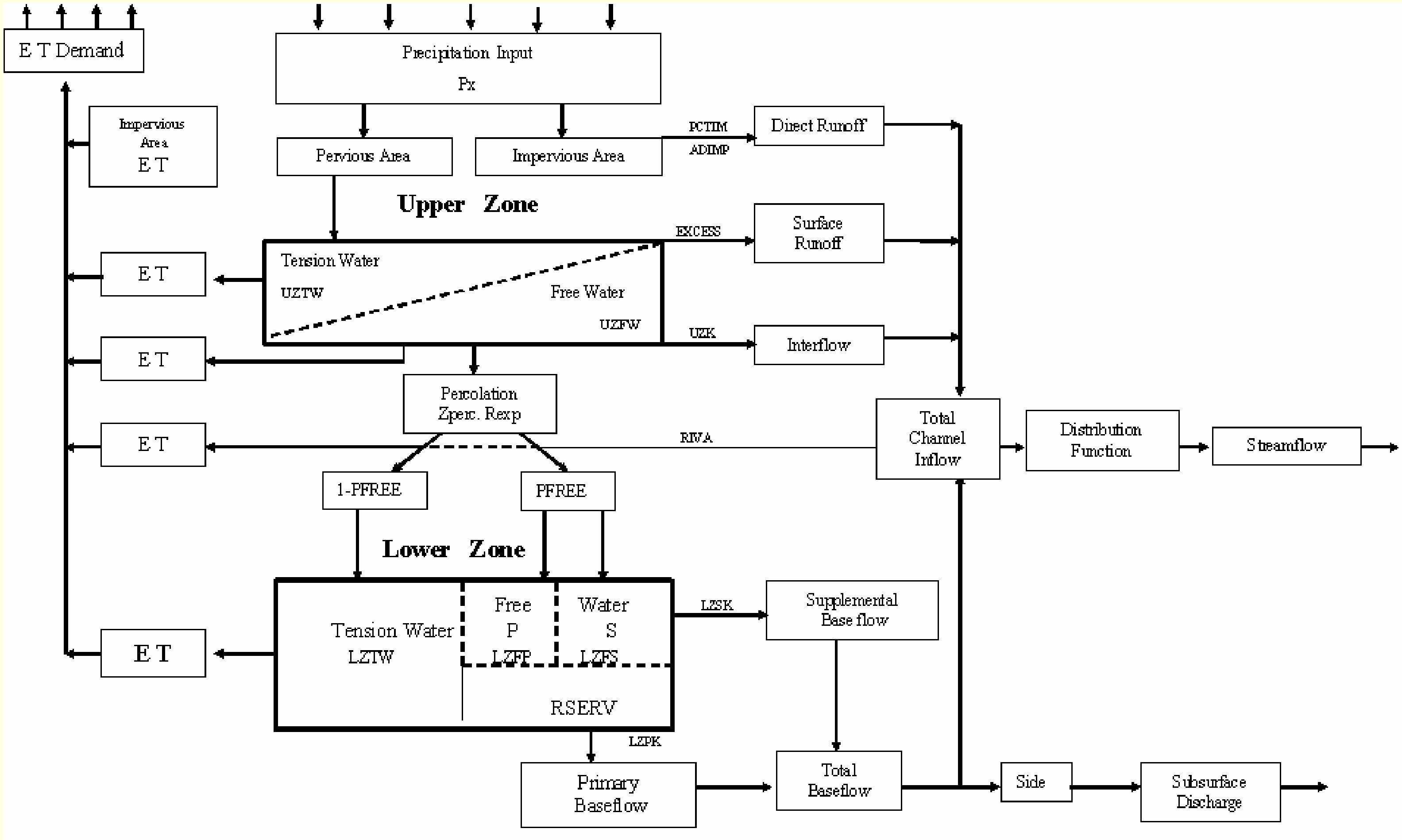
- Upper zone
  - Upper zone tension water
  - Upper zone free water
- Lower zone
  - Lower zone tension water
  - Lower zone free water
    - Primary free water storage
    - Secondary free water storage

Hydrology Model



Hydrology Model

- Sacramento Catchment model



Problem formulation

States

- x_1 = upper zone tension water content
- x_2 = upper zone free water content
- x_3 = lower zone tension water content
- x_4 = lower zone primary free water content
- x_5 = lower zone secondary free water content
- x_6 = additional impervious

Inputs

- U_p = mean areal precipitation
- U_e = mean areal evapotranspiration demand

Output

- U_c = channel inflow per unit time

Problem formulation

Mathematical Equations Involved in Modeling

- Model developed By Dr. Georgakakos

$$\frac{dx_1}{dt} = [1 - \left(\frac{x_1}{x^o_1}\right)^{m1}] u_p - u_e \left(\frac{x_1}{x^o_1}\right)$$

$$\frac{dx_2}{dt} = \left(\frac{x_1}{x^o_1}\right)^{m1}] u_p [1 - \left(\frac{x_2}{x^o_2}\right)^{m2}] - d_u x_2 - C_1 (1 + \varepsilon y^{\theta}) \left(\frac{x_2}{x^o_2}\right)$$

$$\frac{dx_3}{dt} = C_1 (1 + \varepsilon y^{\theta}) \left(\frac{x_2}{x^o_2}\right) (1 - p_f) [1 - \left(\frac{x_3}{x^o_3}\right)^{m3}] - u_e (1 - \frac{x_1}{x^o_1}) \left(\frac{x_3}{x^o_1 + x^o_3}\right)$$

$$\frac{dx_4}{dt} = -d'_l x_4 + C_1 (1 + \varepsilon y^{\theta}) \left(\frac{x_2}{x^o_2}\right) [1 - (1 - p_f) [1 - \left(\frac{x_2}{x^o_2}\right)^{m3}]] [(C_2 \frac{x_5}{x^o_5} - 1) \left(\frac{x_2}{x^o_2}\right) + 1]$$

$$\frac{dx_5}{dt} = -d''_l x_5 + C_1 (1 + \varepsilon y^{\theta}) \left(\frac{x_2}{x^o_2}\right) [1 - (1 - p_f) [1 - \left(\frac{x_3}{x^o_4}\right)^{m3}]] [(1 - C_2 \frac{x_5}{x^o_5}) \left(\frac{x_4}{x^o_4}\right)]$$

$$\frac{dx_6}{dt} = [1 - \left(\frac{x_6}{x^o_3}\right)^2 [1 - \left(\frac{x_2}{x^o_2}\right)^{m2}] \left(\frac{x_1}{x^o_1}\right)^{m1} u_p - u_e (1 - \frac{x_1}{x^o_1}) \left(\frac{x_6}{x^o_3 + x^o_1}\right)$$

Problem formulation

- Output Equation

$$u_c = (d_u x_2 + \frac{d''_l x_4 + d_l'' x_5}{1+\mu})(1-\beta_1-\beta_2) + u_p\beta_2 + \left(\frac{x_6}{x^o{}_3}\right)^2 u_p \left(\frac{x_1}{x^o{}_1}\right)^{m1} \beta_1 +$$

$$u_p \left(\frac{x_1}{x^o{}_1}\right)^{m1} \left(\frac{x_2}{x^o{}_2}\right)^{m2} (1-\beta_1-\beta_2) + [1-\left(\frac{x_6}{x^o{}_3}\right)^2] \left(\frac{x_2}{x^o{}_2}\right)^{m2} \left(\frac{x_1}{x^o{}_1}\right)^{m1} u_p \beta_1$$

- Routing model

H-infinity estimator design

- The Linearized model

$$\Delta \dot{x} = A \Delta x + B \Delta U$$

$$\Delta x = \Delta x + \Delta \dot{x} \, dt$$

$$x = x_{nom} + \Delta x$$

H-infinity estimator design

- Errors to the system enter via B_w and D_w

$$\Delta \dot{x} = A\Delta x + B\Delta U + B_w \Delta W$$
$$\Delta m = C_m \Delta x + D_m \Delta U + D_w \Delta W$$
$$\Delta y = C_y \Delta x$$

- ΔW is 6 X 3 matrix, with errors induced in states and measurement
- m is the measured signal
- y is the signal desired to estimate

H-infinity estimator design

- Conditions

$$D_m B_w^T = 0$$

$$D_m D_m^T = I$$

$$\Delta \dot{\hat{x}} = A\Delta\hat{x} + B\Delta U + G(\Delta m - C_m \Delta\hat{x} - D_m \Delta U)$$

$$G = QC_m^T$$

$$Q = \text{Riccatti} \quad \text{equation} \quad \text{solution}$$

$$\Delta\hat{x} = \Delta\hat{x} + \Delta \dot{\hat{x}}\, dt$$

$$\hat{x} = x_{nom} + \Delta\hat{x}$$

H-infinity estimator design

- γ is set to 80

$$\gamma = \sup_{w \neq 0} \frac{\left\| y - \hat{y} \right\|_{2,|0,tf|}}{\left\| w \right\|_{2,|0,tf|}}$$

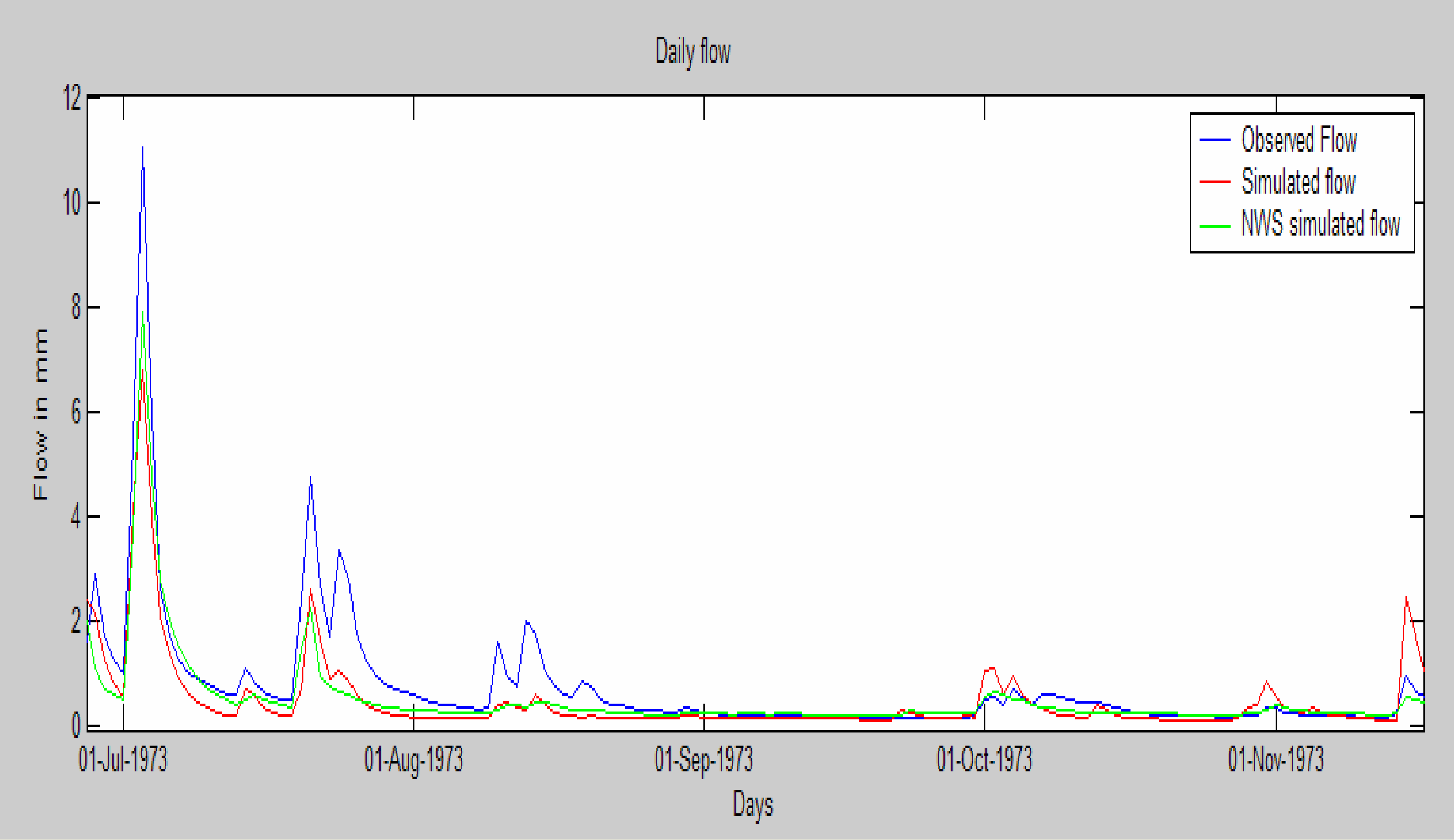
$$Riccatti \quad equation$$

$$\dot{Q} = QA^T + AQ + BB_w^T - Q(C_m C_m^T - I/\gamma^2)Q$$

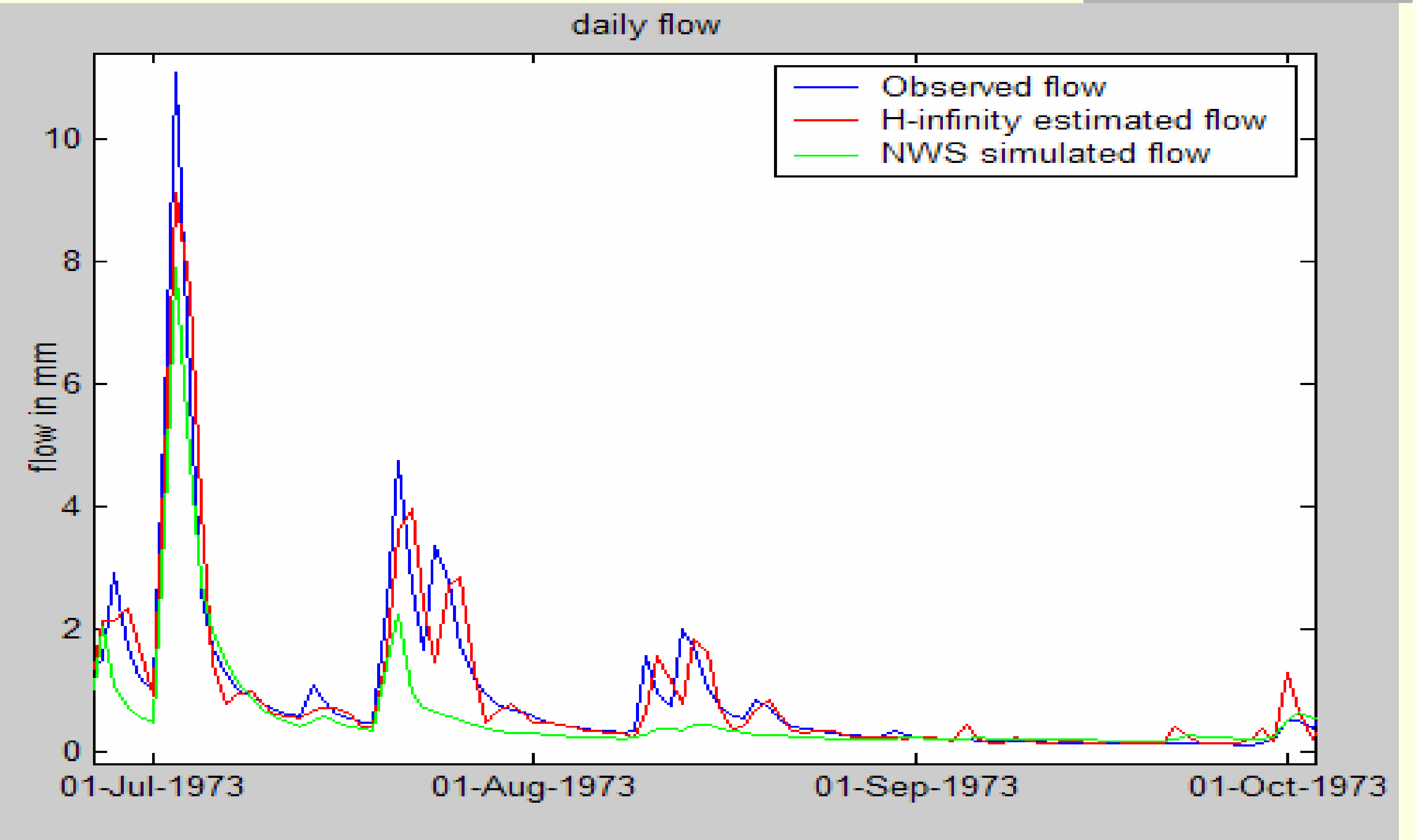
Catchment Description

- Scioto River Basin
- Deer Creek Watershed in Ohio
- 228 square miles

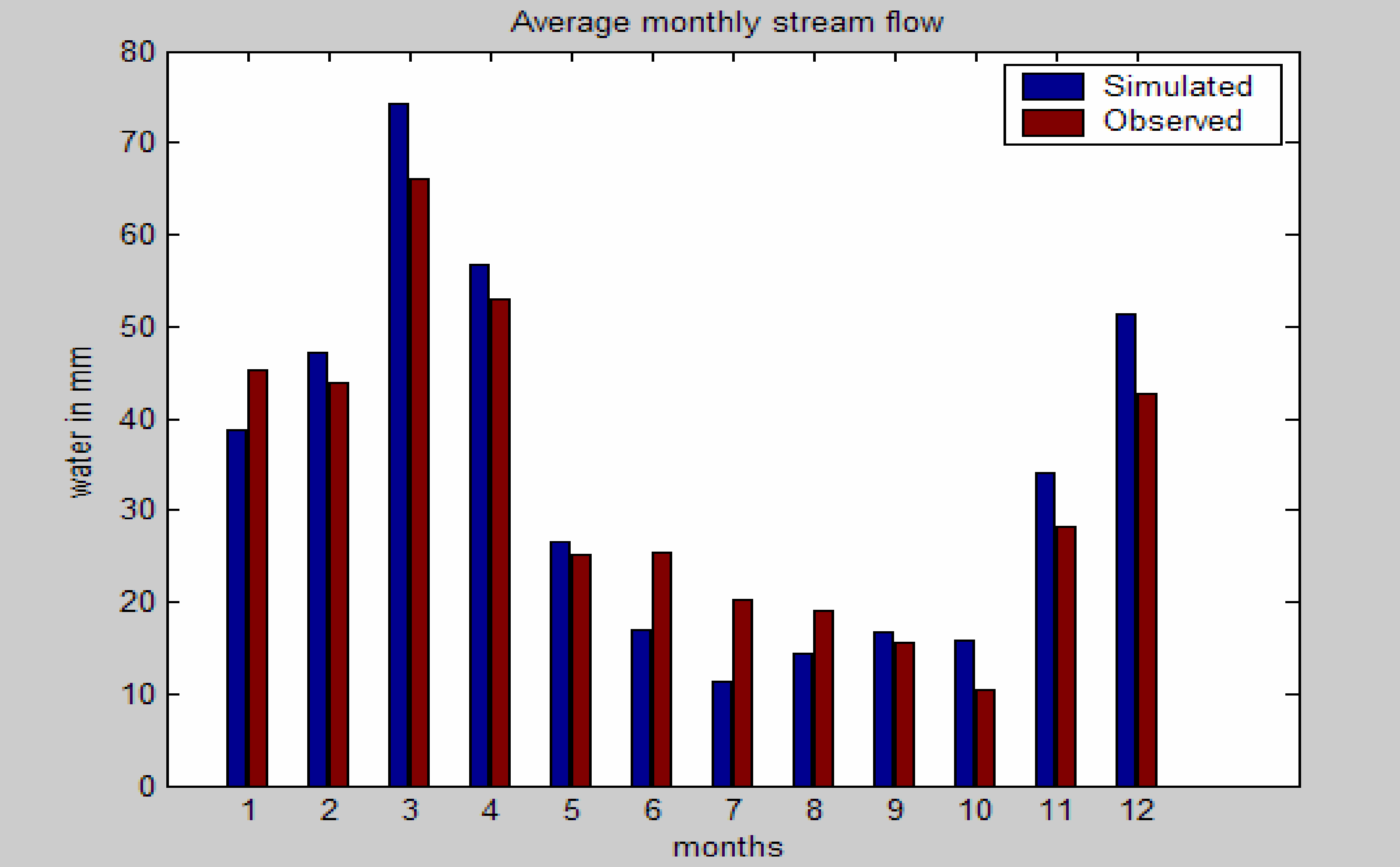
Experimental Results



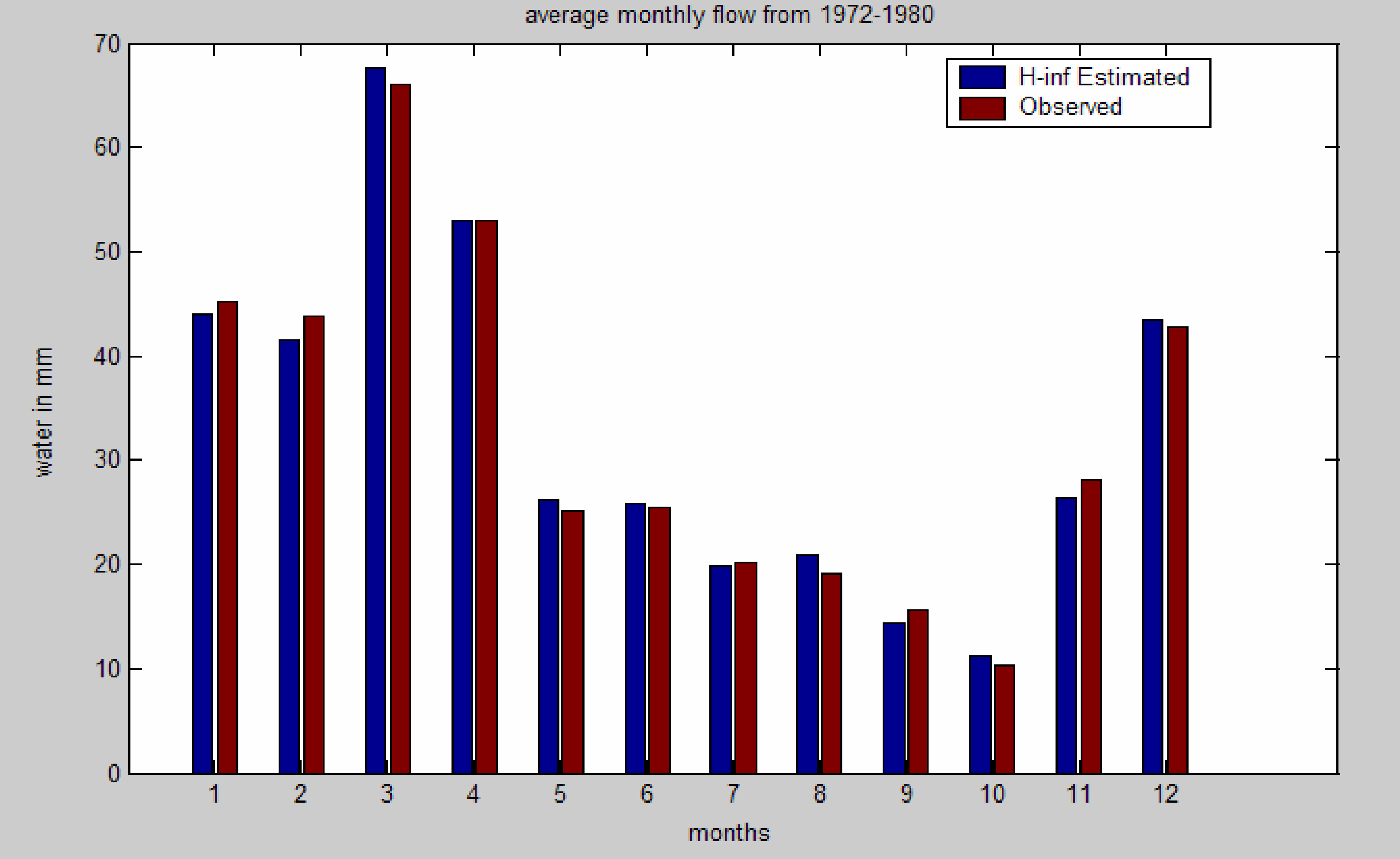
Experimental Results



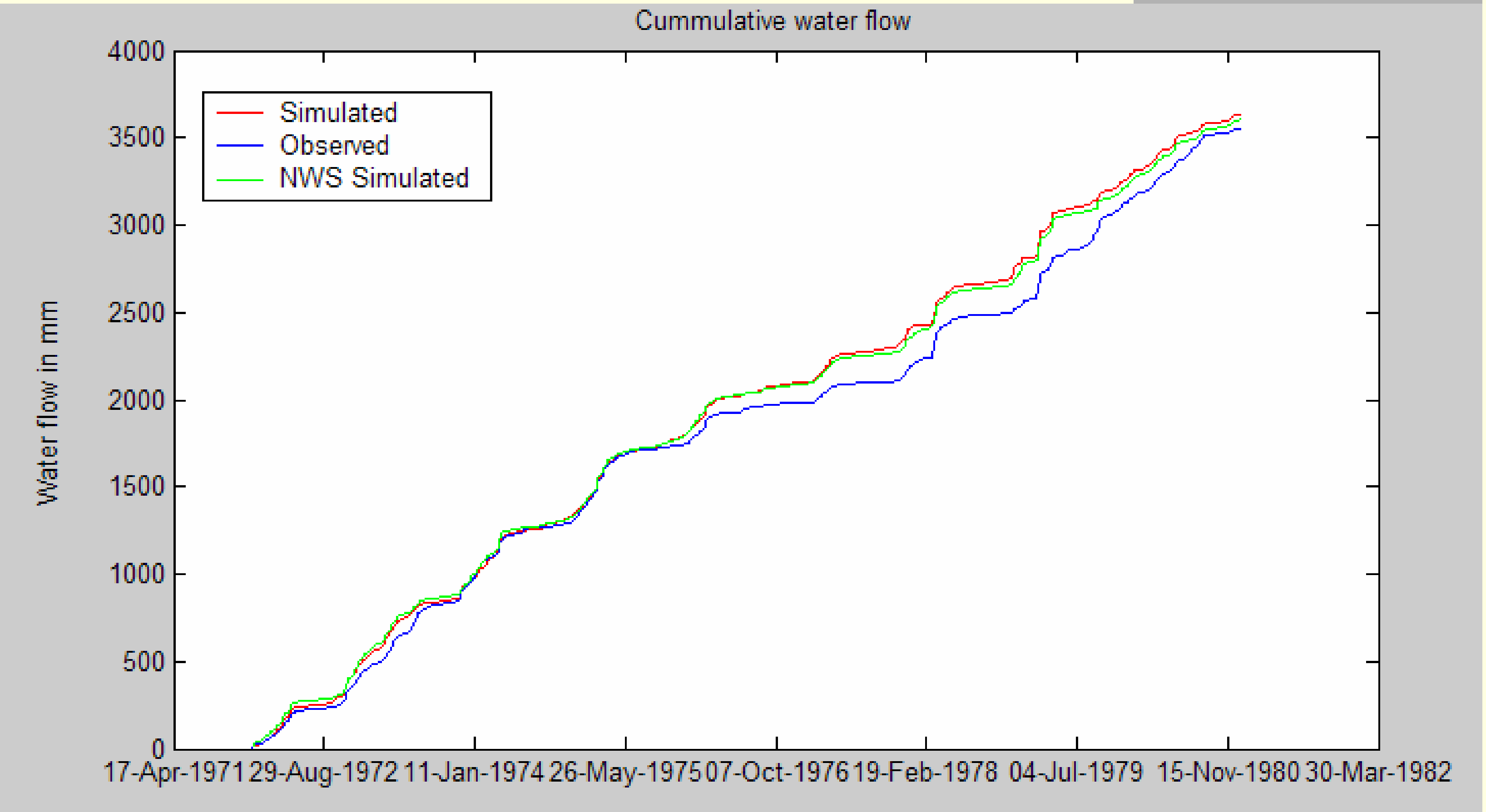
Experimental results



Experimental results



Experimental results



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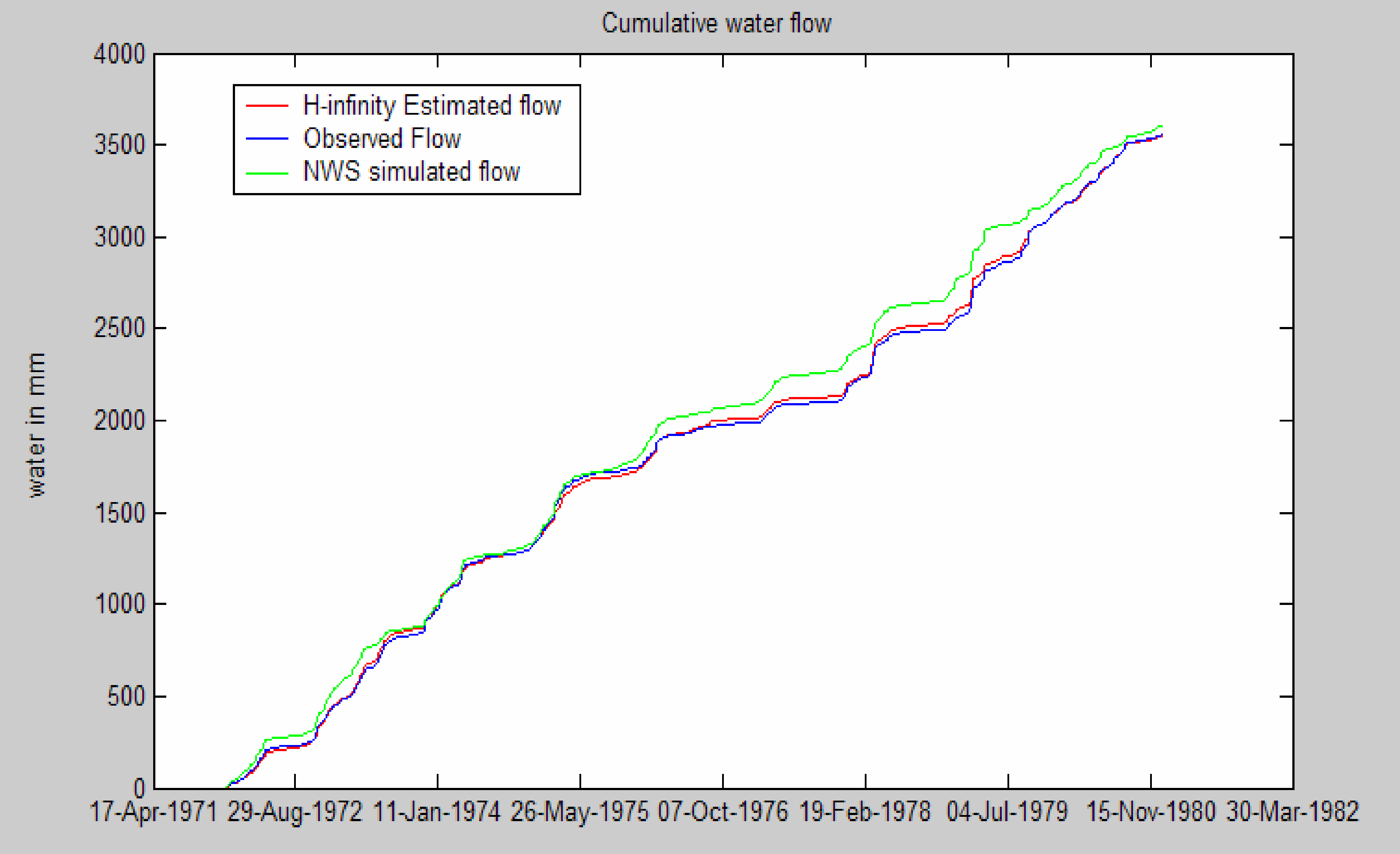

Conclusion

- H-infinity estimator works better than the model without state estimation.
- H-infinity estimator minimized the maximum estimation errors

Future work

- Tune H-infinity estimator so that it will more effectively estimate the states and better take the snow melt into account.
- Implement Kalman Filter and try to reduce the error.