Test 1 Differential Equations 2/15/02

Write your name on the back of your test. Show all work. Where possible test your solutions.

1.(10) Use the guess and test method to solve the second order differential equation initial value problem:

\[
\frac{d^2 y}{dt^2} + 6 \frac{dy}{dt} + 8y = 0, \quad y(0) = -2, \quad y'(0) = 1.
\]
2. (10) Solve the given initial value problem.

\[ \frac{dy}{dt} = \frac{t^2}{y + t^3y}, \quad y(0) = -2. \]
3.(10) Suppose we know that the graph below is the graph of the right-hand side \( f(y) \) of the differential equation \( \frac{dy}{dt} = f(y) \).

a) Give a rough sketch of the slope field that corresponds to this differential equation.

b) Sketch the phase line for this differential equation. Identify the equilibrium points as sinks, sources or nodes.
4. (15) Solve the partially decoupled system

\[
\begin{align*}
\frac{dx}{dt} &= -x + 2y, \\
\frac{dy}{dt} &= y.
\end{align*}
\]
5.(45) Consider the pair of systems:

\[(A) \quad \frac{dx}{dt} = -3x + 2xy \quad \frac{dy}{dt} = -5y + 3xy\]
\[(B) \quad \frac{dx}{dt} = 5x - x^2 - 3xy \quad \frac{dy}{dt} = 8y - 3xy - 3y^2\]

\(x(t)\) and \(y(t)\) represent populations of two different species. One system describes a competitive system in which both species are harmed by interaction, for example, cars and pedestrians. The other system describes a cooperative system in which both species benefit from interaction, for example, bees and flowers. A direction field/phase portrait is supplied for each of the systems. \(x(t)\) and \(y(t)\) graphs are supplied for one pair of initial conditions for each of the systems.

a)(5) Identify which system each direction field/phase portrait matches.

b)(5) Sketch at least 3 additional solutions on each of the phase portraits.

c)(15) Identify the equilibrium points of each system. Describe how the systems evolve over time from different initial points.

d)(20) Briefly discuss each of the terms in each system. For example, what does the coefficient on \(xy\) in the equation for \(dx/dt\) represent in model A? As part of this discussion, indicate which system represents a competitive system, and which represents a cooperative system and how you know.