Review of Complex variables and functions

\[ s = \sigma + j \omega \quad \text{Re}(s) = \sigma \quad \text{Im}(s) = \omega \]

\[ |s| = \sqrt{\sigma^2 + \omega^2} \quad \angle s = \tan^{-1}\left(\frac{\omega}{\sigma}\right) \]

\[ s = |s| \left(\cos(\angle s) + j \sin(\angle s)\right) \]

\[ \bar{s} = \sigma - j \omega \quad |s|^2 = s\bar{s} \]
Laplace transform of a derivative

Lower case $f$ indicates function of time.
Upper case $F$ indicates function of $s$.

$$L\left\{ \frac{d}{dt} f(t) \right\} = sF(s) - f(0)$$

Multiplication by $s$ is differentiation w.r.t. time

Primes and dots are often used as alternative notations for the derivative.

Dots are almost always used to denote time derivatives.

Primes might denote either time or space derivatives.

In problems with both time and space derivatives, primes are space derivatives and dots are time derivatives.
Solving ODEs

Laplace transforms are the primary tool used to solve ODEs in control engineering.

When initial conditions are zero:
\[
\frac{d}{dt} \leftrightarrow L \rightarrow s \\
\frac{d^n}{dt^n} \leftrightarrow L \rightarrow s^n
\]

For non zero initial conditions:
\[
L \left[ \frac{d}{dt} y(t) \right] = sY(s) - y(0)
\]

\[
L \left[ \frac{d^2}{dt^2} y(t) \right] = ? \\
L \left[ \frac{d^3}{dt^3} y(t) \right] = ? \\
L \left[ \frac{d^n}{dt^n} y(t) \right] = ?
\]

\[
L \left\{ \int_{\tau=0}^{\tau=t} f(\tau) d\tau \right\} = ?
\]
Solving ODEs

\[ L \left[ \frac{d^2}{dt^2} y(t) \right] = s^2 Y(s) - sy(0) - \dot{y}(0) \]

\[ L \left[ \frac{d^3}{dt^3} y(t) \right] = s^3 Y(s) - s^2 y(0) - s\dot{y}(0) - \ddot{y}(0) \]

\[ L \left[ \frac{d^n}{dt^n} y(t) \right] = s^n Y(s) - s^{n-1} y(0) - s^{n-2} \dot{y}(0) - s^{n-3} \ddot{y}(0) - \cdots - y^{(n-1)}(0) \]

\[ L \left\{ \int_{\tau=0}^{\tau=t} f(\tau) d\tau \right\} = \frac{F(s)}{s} \]