Chaebols and Big Deals: Cross-Market Cost-Interdependencies and the Excess Diversification Dilemma in Multi-Market Business Groups*

Myong-Hun Chang  
Department of Economics  
Cleveland State University  
Cleveland, OH 44115  
216-687-4523, -9206 (Fax)  
m.chang@csuohio.edu  
www.csuohio.edu/changm

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Abstract

This paper investigates the multi-market business groups’ tendency to excessively diversify as an equilibrium phenomenon. The question of diversification is viewed as that of choosing between the focused strategy (where the firm concentrates on only one market or product) and the broad strategy (where the firm commits to more than one market). The main objective is to investigate how cross-market cost-interdependencies influence the firms’ equilibrium strategies. It is found that cross-market cost-complementarities do indeed result in mutual pursuit of the broad strategy in equilibrium. However, if the cost advantage from multi-market operation is small, this equilibrium turns out to be less profitable than an alternative market configuration which entails specialization of each firm to a single (non-overlapping) market through the adoption of focused strategies. In that the firms fail to support this mutually beneficial configuration as an equilibrium, they are viewed as being confronted with the excess diversification dilemma. It is also found that the pursuit of excess diversification can exist even when diversification generates competitive disadvantages. The zone of excess diversification dilemma is identified and characterized for the case of duopoly. It is further shown that such a zone exists for a more general oligopoly.
1. Introduction

1.1. Chaebols and Big Deals

In January of 1997, Hanbo Steel, a leading South Korean steel maker, defaulted on loans, starting a series of major corporate failures in 1997. The eventual financial crisis caused by the mounting non-performing corporate debts, coupled with the currency melt-downs in Thailand, Malaysia, and Indonesia, finally hit South Korea in November of 1997, leading to a rescue loan package led by the International Monetary Funds. The IMF-led rescue package for South Korea was a record $57 billion.

The root cause of the crisis is widely believed to be the reckless business expansions in recent years by the Korean conglomerates (also known as chaebols). Since the debt crisis, the Korean government has been pushing ahead with a major corporate reform program for large conglomerates. The central component of the program is what is dubbed as “big deals,” the swapping of the businesses among multi-market business groups. The targets of the big deal program have been the largest five chaebols: Hyundai, Samsung, Daewoo, LG (Lucky-Goldstar), and SK (Sunkyong) groups. The justification for this program has been that the chaebols are over-extended into too many overlapping industrial sectors. Excess capacity is rampant in almost all strategic industries, which has increased the level of corporate borrowings to a previously unprecedented high. The proposed solution to this excess capacity is to exchange each other’s businesses so as to allow for specialization by a single firm in each individual sector. The following quotes from the newspaper articles capture the essence of the South Korean government’s policy regarding big deal:

“Big Deals are a cornerstone of President Kim Dae Jung’s plan to build a more efficient and competitive Korea...... Korea’s chaebol dominate its economy – the top five account for 10% of the nation’s gross domestic product. But in their single-minded drive to expand, they sank too many fingers in too many pies, leading to overlapping and often unprofitable investments. Of the top five, three have major interests in car manufactur-
ing, three in shipbuilding, four in oil and petrochemicals, four in electronics, and all five in telecommunications equipment. Many investments seem to be based more on pride than strategy. In Kim’s vision, Big Deals would see chaebol swap businesses among themselves so that just two or three remain in each sector, eliminating redundancies and cut-throat internecine competition while raising efficiency, profitability and global competitiveness.” – *Asia week* (September 18, 1998)

“Samsung Group has agreed to swap its auto-making business for Daewoo Group’s electronics operations in a major realignment of South Korean industry that could have important consequences for the world car and electronics markets....The asset swap completes a radical reorganization of the Korean automobile industry. A year ago, Korea had five independent car makers; now only two remain....News of the swap helped spark a 4.9% rally in the Korea Stock Exchange, to its highest point since March....Questions remain about the economic benefits of the swap......Chaebol executives complain that they are now being pressured into merging and swapping affiliates, and the government has repeatedly threatened that banks will cut off finance to chaebols that don’t toe the reform line.” – *The Wall Street Journal* (December 8, 1998)\(^1\)

While the big deal reform of the chaebols in South Korea is a slow on-going process with uncertain future, the events observed thus far raise the following questions. One, why did these conglomerates drive themselves into the state of excessive diversification, from which the government must step in to “rescue” them? Two, there has been substantial resistance to the program from the chaebols themselves. If the move toward specialization is indeed profitable (as the government contends), why are these conglomerates not doing it voluntarily? Three, will the active

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\(^1\)The proposed swap has since fallen through. The Samsung Group backed off from its earlier plan of selling off Samsung Motors, Inc. to Daewoo and instead chose to seek court receivership for its debt-laden auto-making affiliate. Daewoo, under a severe liquidity crunch, is currently going through a debt restructuring programme by its creditors. The Korean government has pledged to provide sufficient liquidity to financial institutions that are exposed to Daewoo.
industrial policy such as the big deal reform be effective in the long run? The purpose of this paper is to gain some preliminary insights into these issues.

1.2. Corporate Strategy and Scope of Firm

The horizontal scope of or the extent of diversification in conglomerate business firms is one of the central issues in corporate strategy. The following have been suggested as three major rationales – economic, financial, and managerial, respectively – for conglomerate diversification\(^2\): 1) when the technology exhibits cross-product cost-interdependencies, there may be incentives to exploit economies of scope across various products; 2) diversification may reduce the variation of a firm’s return, thereby securing the cash-flow stability; 3) diversification may be motivated by the manager’s self-interest that diverges from the firm’s objective (shareholder profit-maximization).

The validity of the second and third rationales have been questioned on theoretical grounds. Cash-flow stability can be much more easily accomplished by shareholders themselves through a diversified portfolio than by a manager acting on behalf of the firm. The divergence between the manager’s objective and the shareholder interests, if it exists, tends to create opportunities for the shareholders to increase their monitoring as well as to seek external candidates to replace the poorly performing managers.\(^3\) The role of market for corporate control as a disciplining device then substantially weakens the relevance of the third rationale. Furthermore, in the context of diversification by the South Korean chaebols the relevance of the third rationale appears rather weak, as those chaebols are all owned and run by the founding families. It seems unlikely that there would be any significant divergence between the interests of managers and the owners in these companies.

More compelling is the first rationale, which is based on a well-established theory of scope economies. This too, however, is not without difficulty. Analyzing the business activities of over

\(^2\)See Oster (1994) and Besanko, Dranove, and Shanley (1996) for general surveys.

\(^3\)Much of the hostile takeover activities which occurred in U.S. during the 1980’s provide ample evidence that there are indeed external threats providing effective checks on such managerial behavior.
180 large multi-product firms, Nathanson and Cassano (1982) found that a large number of these diversified firms are engaged in products and markets that appear rather poorly positioned to generate much scope economies, if at all. Compounding the difficulty is the evidence collected by Porter (1987): upon studying the diversification records of thirty-three large U.S. companies over the 1950-1986 period, he found that most companies have dissipated rather than created shareholder value in the course of pursuing the broad expansion strategy. In short, the excessive tendency to diversify is not a phenomenon unique to South Korean chaebols. Even if we accept the existence of scope economies as the main driver of diversification, it still falls short of explaining why the firms in general may have an excessive tendency to engage in multi-market operations.

This paper advances an alternative theory that combines cross-market production linkages with strategic interactions among conglomerates. It explains excessive diversification as an equilibrium phenomenon without resorting to the divergence between managerial and shareholder objectives. The question of conglomerate diversification is viewed as that of choosing between the focused strategy (where the firm concentrates on only one market or product) and the broad strategy (where the firm commits to more than one market). The markets are assumed to be independent on the demand side. However, there are cross-market interdependencies on the production side such that a firm’s competitiveness in one market is affected by whether or not it is also present in another market. These cross-market linkages may have both positive and negative impact on the competitiveness of a firm. On the positive side, there are scale economies that can be realized on common shared inputs between productions in multiple markets. Knowledge sharing and transfer of skills may facilitate production efficiency. In some cases, there are political advantages that arise

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1See Montgomery (1994) for a survey of papers reaching similar conclusions.

5One potential source of this benefit is the product design modularity. Modular design allows for many shared components (common parts), which can be produced under economies of scale. See Pine (1993) and Baldwin and Clark (1999).

6To the extent that the knowledge and skills can be considered production inputs, the effectiveness of these intra-organizational transfers will depend on the degree of similarity in production processes. See Chang and Harrington (1999a, 1999b) for discussions of how uniformity in business operations across units can facilitate mutual learning in
from the simple presence in multiple markets. This is especially important in those export-oriented economies in which the government’s industrial policy actively promotes large-sized conglomerates capable of competing globally. The political advantages from the size and scope manifest themselves in the form of favorable loan rates from the government-coerced lending institutions as well as selectively lowered regulatory barriers.\(^7\) On the negative side, stretching managerial capacity over multiple markets may lead to deterioration in the quality of administrative decision-making. The sheer cost of coordination at the central headquarters will increase as well.\(^8\) Just as significant is the excessive influence cost incurred by the units competing for a bigger share of the internally available resources [Milgrom (1988), Milgrom and Roberts (1990, 1992)].\(^9\) How these competing factors net out determines the ultimate impact of cross-market linkages on the firm’s competitiveness. My purpose in this paper is not to identify the exact sources of these linkages, but to investigate how the net impact of such linkages influences the equilibrium choice of diversification strategies.

I find that cost-complementarities that give rise to competitive advantage do induce the firms to pursue broad strategies in equilibrium. However, if the cost advantage is small (though still positive), this equilibrium turns out to be less profitable to the firms than an alternative market configuration which entails specialization of each firm to a single (non-overlapping) market through the adoption of focused strategies. In that the firms fail to support this Pareto-superior configuration as an equilibrium, the firms are viewed as being confronted with the excess diversification

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\(^7\)It is well-known that the South Korean chaebols historically had much more relaxed borrowing constraints than other smaller and more focused firms.

\(^8\)Of course, the cost of coordination is affected by the choice of organizational structure and, hence, the crucial relationship between strategy and structure [Chandler (1969)]. In this paper, however, I assume the organizational structure to be exogenously given so as to concentrate on the issue of strategy only.

\(^9\)A somewhat related literature on the “diversification discount” finds that the market values of diversified firms are lower on average than those of single-segment firms in the same industries. [Lang and Stulz (1994), Berger and Ofek (1995) and Servaes (1996)] Many of the recent works in this line of research attribute its cause to the misallocation of internal funds in diversified firms. See Scharfstein and Stein (1997) and Rajan, Servaes and Zingales (1999).
dilemma - excessive investment into overlapping markets to the detriment of their mutual benefits. It is further found that the pursuit of excess diversification can exist even when diversification generates competitive disadvantages: even when a firm’s cost in a market is adversely affected by its engagement in another market, the firm may still have the equilibrium incentive to pursue broad strategy. In this case, the excess diversification dilemma can be a major concern for the conglomerates as diversification creates no economic value in the process.

2. The Model

Consider two separate markets, A and B. These markets are completely separated from one another on the demand-side in that the consumption demand in A is in no way affected by the demand situation in market B. There is, however, a possibility that there may be interdependence among these two markets on the supply-side such that the cost of supplying the product in market A may be influenced by the firm’s engagement in market B.

There exist two business groups which consider entering into these two markets. We shall call them firm 1 and firm 2. Let \( q^A \) and \( q^B \) be vectors of outputs in respective markets A and B such that \( q^A \equiv (q_1^A, q_2^A) \) and \( q^B \equiv (q_1^B, q_2^B) \), where \( q_i^I \) is firm i’s output in market I, \( i = 1, 2 \) and \( I = A, B \). The market demand conditions are characterized by the following inverse demand functions:

\[
P^I(Q^I) = a - bQ^I,
\]

where \( Q^I = q_1^I + q_2^I \) and \( I = A, B \). For simplicity, the market demand functions are assumed to be identical for both markets.

The production in market I ( \( I = A, B \) ) entails constant returns to scale:

\[
C^I_i(q_i^I) = \begin{cases} 
  c \cdot q_i^I, & \text{if firm } i \text{ produces only in } I \in \{A, B\}, \\
  \tilde{c} \cdot q_i^I, & \text{if firm } i \text{ produces for both } A \text{ and } B.
\end{cases}
\]

(2.2)

Diversification leads to competitive advantage if \( c > \tilde{c} \), while the opposite is true for \( c \leq \tilde{c} \). I assume that \( a - c > 0 \) and \( a - \tilde{c} > 0 \) to ensure the feasibility of these markets.
In that the selection of markets to serve precedes the output decisions in the chosen markets, it is assumed that the game proceeds in two stages. In stage 1, both firms simultaneously choose which market(s) to serve. Their choices are then revealed at the end of stage 1. Subsequently in stage 2, both firms simultaneously choose the production quantities in the markets chosen in stage 1.\textsuperscript{10} A pure strategy for firm $i$ is then a pair $(m_i, q_i)$, where $m_i \in \{A, B, AB\}$ and $q_i = (q_i^A, q_i^B) : \{A, B, AB\}^2 \rightarrow \mathbb{R}_+^2$ is a function mapping from the firms’ chosen markets to the space of production quantities. $m_i = A(B)$ refers to the focused strategy of entering market $A(B)$ only, while $m_i = AB$ corresponds to the broad strategy of entering both markets simultaneously. This also implies that if $m_i = A(B)$, then $q_i^A > 0$ and $q_i^B = 0$ ($q_i^A = 0$ and $q_i^B > 0$), while if $m_i = AB$, then $q_i^A > 0$ and $q_i^B > 0$. The cost of entry is assumed to be zero.

Given the demand and cost conditions, it is easy to define the profit functions for these firms. For firm $i$, its total profit, $\hat{\Pi}_i ((m_1, q_1), (m_2, q_2))$ is simply the sum of its profits in market $A$ and market $B$ such that:

$$\hat{\Pi}_i ((m_1, q_1), (m_2, q_2)) = \pi_i^A ((m_1, q_1^A), (m_2, q_2^A)) + \pi_i^B ((m_1, q_1^B), (m_2, q_2^B)),$$

(2.3)

and

$$\pi_i^l ((m_1, q_1^l), (m_2, q_2^l)) = \begin{cases} [a - b(q_1^l + q_2^l) - c]q_1^l, & \text{if } m_i \neq AB, \\ [a - b(q_1^l + q_2^l) - c]q_1^l, & \text{if } m_i = AB. \end{cases}$$

(2.4)

The firms are restricted to using pure strategies. The relevant solution concept is subgame perfect equilibrium. Given the sequential nature of the game, backward induction is used in solving for the subgame perfect equilibrium. As such, I first derive the Cournot-Nash equilibrium quantities in stage 2 for all market configurations. The equilibrium selection of markets in stage 1 is then derived, given that the firms rationally expect the Cournot-Nash equilibrium to be played in stage 2.

\textsuperscript{10}The stage-1 strategy may be viewed as corporate strategy, while the stage-2 strategy may be viewed as business unit (or competitive) strategy. See Porter (1987) for a discussion of the essential linkage between these two levels of strategy formulation.
3. Duopoly Equilibrium

3.1. Equilibrium Quantities in Stage 2

The strategies in stage 2 must be contingent on the actual selection of markets by the firms in stage 1. While there are nine possible configurations of firms’ stage 1 choices, these can be divided into the following four cases: 1) both firms may pursue the focused strategy targeting a single \( \text{identical} \) market; 2) both firms may pursue the focused strategy, but target \( \text{different} \) markets; 3) one firm may pursue the focused strategy targeting only one of the markets, while the other firm may pursue the \( \text{broad strategy} \) targeting both markets; 4) both firms may pursue broad strategies entering both markets. The first case is when there is duopoly competition in either \( A \) or \( B \), while the second market is left untapped: \( (m_1, m_2) \in \{(A, A), (B, B)\} \). The second case corresponds to the perfect market segmentation, in which each firm avoids competition by specializing in a niche market as a local monopolist: \( (m_1, m_2) \in \{(A, B), (B, A)\} \). In the third case, there is duopoly competition in one of the markets, but the second market is monopolized by the firm with the broad strategy: \( (m_1, m_2) \in \{(A, AB), (B, AB), (AB, A), (AB, B)\} \). The final case captures the full duopoly competition, in which both firms engage in head-to-head competition in all markets: \( (m_1, m_2) = (AB, AB) \). Given the symmetry, it is only necessary for us to consider one configuration out of each of these four cases. Without any loss of generality, I assume that it is market \( A \) in which duopoly competition occurs if there is indeed only one market where \( \text{both} \) firms are engaged in. As such, I will consider only \( (A, A) \), \( (A, B) \), \( (A, AB) \), and \( (AB, AB) \).

**Proposition 3.1.** The Cournot-Nash equilibrium in stage 2 is contingent on the markets selected by the firms in stage 1: 1) When firm 1 is in \( A \) only and firm 2 is in \( A \) only, the Nash equilibrium entails \( (q_1^A, q_2^A) = \left( \frac{a-c}{2b}, \frac{a-c}{2b} \right) \) and \( (q_1^B, q_2^B) = (0, 0) \); 2) when firm 1 is in \( A \) only and firm 2 is in \( B \) only, the Nash equilibrium entails \( (q_1^A, q_2^A) = \left( \frac{a-c}{2b}, 0 \right) \) and \( (q_1^B, q_2^B) = (0, \frac{a-c}{2b}) \); 3) when firm 1 is in \( A \) only and firm 2 is in both \( A \) and \( B \), the Nash equilibrium entails \( (q_1^A, q_2^A) = \left( \frac{a-2c+\hat{e}}{3b}, \frac{a-c-2\hat{e}}{3b} \right) \) and \( (q_1^B, q_2^B) = \left( 0, \frac{a-c}{2b} \right) \); 4) when firm 1 is in both \( A \) and \( B \) and firm 2 is in both \( A \) and \( B \), the
Nash equilibrium entails \((q_1^A, q_2^A) = \left( \frac{a-c}{3b}, \frac{a-c}{3b} \right)\) and \((q_1^B, q_2^B) = \left( \frac{a-c}{3b}, \frac{a-c}{3b} \right)\).

**Proof.** See Appendix. ■

The corresponding equilibrium market prices as well as firm profits for these four contingencies are provided in Table 1.

### 3.2. Equilibrium Selection of Markets in Stage 1

Given the Cournot-Nash equilibrium in stage 2, the firms choose markets to enter in stage 1. The market selection decision in stage-1 affects the marginal cost of production in stage 2: if the firm enters only one market, the marginal cost is \(c\), while if the firm enters both markets, the marginal cost is \(\hat{c}\). Let the impact of cross-market cost-interdependency be labelled \(\Delta\) such that \(\hat{c} = c + \Delta\), where \(\Delta \in (-(a-c), a-c)\).\(^{11}\) Diversification leads to competitive advantage (disadvantage) in individual markets, if \(\Delta < 0 \ (> 0)\). When \(\Delta = 0\), the business units are self-contained in that the engagement in one market does not affect the cost of operating in the other market.

The following three lemmas form the basis for a full characterization of the subgame perfect equilibrium in firms’ selection of the markets. Note that \(\hat{\Pi}_i(m_i, m_j)\) is firm \(i\)'s total profit from choosing \(m_i\) in stage 1, given that the rival firm \(j\) chooses \(m_j\) and the Nash equilibrium strategies are played in the stage 2 subgames as characterized in Proposition 3.1.

**Lemma 3.2.** Define \(L(\Delta) \equiv 4(a-c-2\Delta) - 18\Delta(a-c) + 9\Delta^2\). There exists \(\Delta_L(a,c) \in (0, a-c)\) such that

\[
\hat{\Pi}_i(AB, A) > \hat{\Pi}_i(B, A) \quad \text{for} \ \Delta \in (-(a-c), \Delta_L(a,c)),
\]

\[
\hat{\Pi}_i(AB, A) \leq \hat{\Pi}_i(B, A) \quad \text{for} \ \Delta \in [\Delta_L(a,c), a-c),
\]

where \(i = 1, 2\) and \(\Delta_L(a,c)\) satisfies \(L(\Delta_L(a,c)) = 0\).

**Proof.** Note that \(\hat{\Pi}_i(AB, A) \geq \hat{\Pi}_i(B, A)\) implies

\[
\hat{\Pi}_i(AB, A) \equiv \frac{(a + c - 2\hat{c})^2}{9b} + \frac{(a - \hat{c})^2}{4b} \geq \frac{(a - c)^2}{4b} \equiv \hat{\Pi}_i(B, A).
\]

\(^{11}\)The upper and lower bounds on \(\Delta\) are necessary to insure \(a - \hat{c} > 0\) and, hence, the feasibility of the markets for the firm pursuing the broad strategy.
Rearranging, one gets
\[4(a - \hat{c} + c - \hat{c}) + 9(a - \hat{c})^2 - 9(a - c)^2 \geq 0. \tag{3.3}\]
Substituting \(c + \Delta\) for \(\hat{c}\),
\[L(\Delta) \equiv 4(a - c - 2\Delta) - 18\Delta(a - c) + 9\Delta^2 \geq 0. \tag{3.4}\]
It is straightforward to show the following properties about \(L(\Delta)\):
\[
\begin{align*}
L(0) &= 4(a - c) > 0, \\
L'(\Delta) &= -8 - 18(a - c) + 18\Delta < 0 \text{ for all } \Delta \in (-(a - c), a - c), \\
L''(\Delta) &= 18 > 0, \\
L(a - c) &= -4(a - c) - 9(a - c)^2 < 0.
\end{align*}
\tag{3.5}\]
These properties together imply that there exists a unique \(\Delta_L(a, c) \in (0, a - c)\) such that \(L(\Delta) > 0\) for \(\Delta \in (-(a - c), \Delta_L(a, c))\) and \(L(\Delta) \leq 0\) for \(\Delta \in [\Delta_L(a, c), a - c)\), and, hence, prove the lemma.

Solving for the quadratic root of \(L(\Delta) = 0\), one finds the closed form expression for \(\Delta_L(a, c)\):
\[\Delta_L(a, c) = \frac{9(a - c) + 4 - \sqrt{81(a - c)^2 + 36(a - c) + 16}}{9}. \tag{3.6}\]

**Lemma 3.3.** Given \(\Delta_L(a, c)\) defined above,
\[
\begin{align*}
\hat{\Pi}_i(AB, B) &> \hat{\Pi}_i(A, B) \quad \text{for } \Delta \in (-(a - c), \Delta_L(a, c)), \\
\hat{\Pi}_i(AB, B) &\leq \hat{\Pi}_i(A, B) \quad \text{for } \Delta \in [\Delta_L(a, c), a - c). 
\end{align*}
\tag{3.7}\]

**Proof.** Given the identical market demand functions for \(A\) and \(B\), Lemma 3.3 follows immediately from Lemma 3.2.

**Lemma 3.4.** Define \(R(\Delta) \equiv (a - c - \Delta)^2 - 4(a - c)\Delta\). There exists \(\Delta_R(a, c) \in (0, a - c)\) such that
\[
\begin{align*}
\hat{\Pi}_i(AB, AB) &> \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB) \quad \text{for } \Delta \in (-(a - c), \Delta_R(a, c)), \\
\hat{\Pi}_i(AB, AB) &\leq \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB) \quad \text{for } \Delta \in [\Delta_R(a, c), a - c),
\end{align*}
\tag{3.8}\]
where \(i = 1, 2\) and \(\Delta_R(a, c)\) satisfies \(R(\Delta_R(a, c)) = 0\).

**Proof.** Note that \(\hat{\Pi}_i(AB, AB) \geq \hat{\Pi}_i(A, AB)\) implies
\[\hat{\Pi}_i(AB, AB) \equiv \frac{2(a - \hat{c})^2}{9b} \geq \frac{(a - 2c + \hat{c})^2}{9b} \equiv \hat{\Pi}_i(A, AB). \tag{3.9}\]
Rearranging, one gets
\[(a - \tilde{c})^2 + 4(a - c)(c - \tilde{c}) \geq 0.\] (3.10)

Substituting \(c + \Delta\) for \(\tilde{c}\),
\[R(\Delta) \equiv (a - c - \Delta)^2 - 4(a - c)\Delta \geq 0.\] (3.11)

Note the following properties:
\[
\begin{align*}
R(0) &= (a - c)^2 > 0, \\
R'(\Delta) &= -2(a - c - \Delta) - 4(a - c) < 0 \text{ for all } \Delta \in (-a - c, a - c), \\
R''(\Delta) &= 2 > 0, \\
R(a - c) &= -4(a - c)^2 < 0.
\end{align*}
\] (3.12)

These properties imply that there exists a unique \(\Delta_R(a, c) \in (0, a - c)\) such that \(R(\Delta) > 0\) for all \(\Delta \in (-a - c, \Delta_R(a, c))\) and \(R(\Delta) \leq 0\) for all \(\Delta \in [\Delta_R(a, c), a - c)\), which proves the lemma. \(\blacksquare\)

Lemmas 3.2, 3.3, and 3.4 imply that \(m_i = AB\) is the dominant strategy for both firms if \(\Delta < 0\): the firms have profit incentives to pursue broad strategy regardless of the rival’s strategy if there exists cost complementarity. More interesting is the finding that \(\Delta_L(a, c) > 0\) and \(\Delta_R(a, c) > 0\), as this implies that the firms prefer the broad strategy over the focused strategy even when \(\Delta > 0\) as long as it is not excessively large. Finally, the next lemma characterizes \(\Delta_L(a, c)\) and \(\Delta_R(a, c)\) in terms of the market size, \(a - c\).

**Lemma 3.5.** \(\Delta_L(a, c) \leq \Delta_R(a, c)\) \(\text{ for } a - c \geq \frac{4(4\sqrt{2} - 5)}{9(8\sqrt{2} - 11)} \approx 0.930595.\)

**Proof.** First, note that \(R(\Delta_R(a, c)) \equiv (a - c - \Delta_R(a, c))^2 - 4(a - c)\Delta_R(a, c) = 0.\) Solving for the quadratic root of this equation, one finds
\[\Delta_R(a, c) = \frac{6(a - c) - \sqrt{36(a - c)^2 - 4(a - c)^2}}{2} = \left(3 - 2\sqrt{2}\right) (a - c).\] (3.13)

Given the closed-form expression for \(\Delta_R(a, c)\), it is straightforward to show that
\[\Delta_L(a, c) \leq \Delta_R(a, c), \text{ if and only if } L(\Delta_R(a, c)) \leq 0,\] (3.14)

where \(L(\Delta_R(a, c)) = 4\left(4\sqrt{2} - 5\right) (a - c) - 9\left(8\sqrt{2} - 11\right) (a - c)^2.\) It follows that \(\Delta_L(a, c) \leq \Delta_R(a, c)\) for \(a - c \geq \frac{4(4\sqrt{2} - 5)}{9(8\sqrt{2} - 11)} \approx 0.930595.\) \(\blacksquare\)
Let \((m_1^*, m_2^*)\) denote the subgame perfect equilibrium in stage-1 market-selection strategies. Obviously, a full characterization of the subgame perfect equilibrium must also include the Cournot-Nash equilibrium quantities in stage 2 identified in the previous section. For expository simplicity, however, I omit the description of the stage-2 equilibrium and refer the readers to Proposition 3.1.

**Theorem 3.6.** When the markets are sufficiently large that \(a - c > \frac{4(4\sqrt{2}-5)}{9(8\sqrt{2}-11)}\),

\[
(m_1^*, m_2^*) = \begin{cases}
(AB, AB) & \text{for all } \Delta \in \left( -(a-c), \Delta_L(a,c) \right); \\
(AB, AB), (A, B), (B, A) & \text{for all } \Delta \in \left[ \Delta_L(a,c), \Delta_R(a,c) \right]; \\
(A, B), (B, A) & \text{for all } \Delta \in \left( \Delta_R(a,c), a-c \right);
\end{cases}
\]

When markets are small that \(a - c \leq \frac{4(4\sqrt{2}-5)}{9(8\sqrt{2}-11)}\),

\[
(m_1^*, m_2^*) = \begin{cases}
(AB, AB) & \text{for all } \Delta \in \left( -(a-c), \Delta_R(a,c) \right); \\
(A, AB), (B, AB), (AB, A), (AB, B) & \text{for all } \Delta \in \left[ \Delta_R(a,c), \Delta_L(a,c) \right]; \\
(A, B), (B, A) & \text{for all } \Delta \in \left( \Delta_L(a,c), a-c \right);
\end{cases}
\]

**Proof.** 1) First, recall that \(\Delta_L(a,c) < \Delta_R(a,c)\) for \(a - c > \frac{4(4\sqrt{2}-5)}{9(8\sqrt{2}-11)}\). We know from Lemmas 3.2, 3.3, and 3.4, that for all \(\Delta \in \left( -(a-c), \Delta_L(a,c) \right)\),

\[
\begin{align*}
\hat{\Pi}_i(AB, A) & > \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) & > \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) & > \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB).
\end{align*}
\]

Hence, \(m_i = AB\) is the dominant strategy for both \(i = 1\) and \(2\) and \((m_1^*, m_2^*) = (AB, AB)\) is the unique subgame perfect equilibrium. On the other hand, for \(\Delta \in \left[ \Delta_L(a,c), \Delta_R(a,c) \right]\), one observes

\[
\begin{align*}
\hat{\Pi}_i(AB, A) & \leq \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) & \leq \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) & > \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB),
\end{align*}
\]

which implies that all three of \((A, B), (B, A), \) and \((AB, AB)\) are sustainable as the subgame perfect equilibrium. Finally, for \(\Delta \in \left( \Delta_R(a,c), a-c \right)\) we have

\[
\begin{align*}
\hat{\Pi}_i(AB, A) & \leq \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) & \leq \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) & \leq \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB).
\end{align*}
\]

In this case, \((AB, AB)\) is no longer sustainable. \((A, B)\) and \((B, A)\) are the only equilibria for this range of \(\Delta\) values.
2) Next, recall that $\Delta_L(a, c) \geq \Delta_R(a, c)$ for $a - c \leq \frac{4(\sqrt{2} - 5)}{9(8\sqrt{2} - 11)}$. Then, for $\Delta \in (-a - c, \Delta_R(a, c))$,

\[
\begin{align*}
\hat{\Pi}_i(AB, A) > \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) > \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) > \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB).
\end{align*}
\]

(AB, AB) is the unique subgame perfect equilibrium for this range of $\Delta$’s. Alternatively, for \( \Delta \in [\Delta_R(a, c), \Delta_L(a, c)] \), we observe the following:

\[
\begin{align*}
\hat{\Pi}_i(AB, A) > \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) > \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) \leq \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB).
\end{align*}
\]

It is easy to see that a set of asymmetric equilibria can be supported in this case: (A, AB), (B, AB), (AB, A), (AB, B). Finally, for $\Delta \in (\Delta_L(a, c), a - c)$ we have

\[
\begin{align*}
\hat{\Pi}_i(AB, A) \leq \hat{\Pi}_i(B, A) > \hat{\Pi}_i(A, A), \\
\hat{\Pi}_i(AB, B) \leq \hat{\Pi}_i(A, B) > \hat{\Pi}_i(B, B), \text{ and} \\
\hat{\Pi}_i(AB, AB) \leq \hat{\Pi}_i(A, AB) = \hat{\Pi}_i(B, AB),
\end{align*}
\]

which implies that (A, B) and (B, A) are the only subgame perfect equilibria. □

Using the closed-form expressions for $\Delta_L(a, c)$ and $\Delta_R(a, c)$ in equations (3.6) and (3.13), Figure 1 captures the intuition behind Theorem 3.6. The relevant area for my analysis is that between the two thick lines, $\Delta = a - c$ and $\Delta = -(a - c)$. This area can then be divided into four non-overlapping regions, depending on the subgame perfect equilibrium attainable in stage-1 market-selection game. Two things should be noted. Pursuing the broad strategy can be a subgame perfect equilibrium even if diversification leads to competitive disadvantage, i.e., $0 \leq \Delta \leq \Delta_R(a, c)$. If the market size is sufficiently large that $a - c > 0.93$, then for $\Delta_L(a, c) < \Delta < \Delta_R(a, c)$ there exist multiple equilibria such that both the mutual pursuit of diversification as well as the market segmentation with focused strategy are self-enforcing; (AB, AB), (A, B), (B, A). Alternatively, when the market size is small such that $0 < a - c \leq 0.93$, a set of asymmetric equilibria exists for $\Delta_R(a, c) < \Delta < \Delta_L(a, c)$ such that a firm with focused strategy coexists with a firm with broad strategy; (A, AB), (B, AB), (AB, A), (AB, B).
4. The Excess Diversification Dilemma

Given the subgame perfect equilibrium identified in the previous section, I now compare the profit of a firm when both firms are diversified into the two markets (i.e., $m_1 = m_2 = AB$) with that when each firm pursues a focused strategy in its own niche market with no local competition from the rival (i.e., $m_1 = A$ and $m_2 = B$). The next theorem identifies the range of $\Delta$ values for which the broad strategy chosen in subgame-perfect equilibrium is payoff-dominated by the focused strategy.

**Theorem 4.1.** There exists $\Delta_\Gamma(a, c) \equiv \left(1 - \frac{3\sqrt{2}}{4}\right)(a - c) < 0$ such that

\[
\begin{align*}
\hat{\Pi}_i(A, B) &\leq \hat{\Pi}_i(AB, AB) & \text{for all } \Delta \in (- (a - c), \Delta_\Gamma(a, c)], \\
\hat{\Pi}_i(A, B) &> \hat{\Pi}_i(AB, AB) & \text{for all } \Delta \in (\Delta_\Gamma(a, c), a - c), \ i = 1, 2. \tag{4.1}
\end{align*}
\]

**Proof.** Note that

\[
\hat{\Pi}_i(A, B) \equiv \frac{(a - c)^2}{4b} \leq \frac{2(a - \hat{c})^2}{9b} \equiv \hat{\Pi}_i(AB, AB). \tag{4.2}
\]

Substituting $\hat{c} = c + \Delta$ and simplifying, one obtains

\[
\Gamma(\Delta) \equiv 8\Delta^2 - 16(a - c)\Delta - (a - c)^2 \geq 0, \tag{4.3}
\]

where

\[
\begin{align*}
\Gamma(- (a - c)) &= 23(a - c)^2 > 0; \\
\Gamma(0) &= -(a - c)^2 < 0; \\
\Gamma(a - c) &= -9(a - c)^2; \\
\Gamma'(\Delta) &= 16\Delta - 16(a - c) < 0; \\
\Gamma''(\Delta) &= 16 > 0;
\end{align*} \tag{4.4}
\]

Hence, there exists a unique $\Delta_\Gamma(a, c) \in (- (a - c), 0)$, for which $\Gamma(\Delta_\Gamma(a, c)) = 0$ and

\[
\Gamma(\Delta) \geq 0 \quad \text{for } \Delta \leq \Delta_\Gamma(a, c). \tag{4.5}
\]

Solving for the quadratic root of $\Gamma(\Delta) = 0$, it is straightforward to show that

\[
\Delta_\Gamma(a, c) \equiv \left(1 - \frac{3\sqrt{2}}{4}\right)(a - c) < 0. \tag{4.6}
\]

Combined with Theorem 3.6, the result in Theorem 4.1 implies that the firms could be stuck in the excess diversification dilemma unless the cost-savings from multi-market operations are
sufficiently large. While they would strictly prefer to segment the market and behave as local monopolists, this is not sustainable in the long run as it is not subgame perfect. This situation is reminiscent of the traditional prisoner’s dilemma game in which the players are unable to support mutually beneficial strategies in equilibrium.

Figure 2 captures the zone of excess diversification dilemma, where the firms pursue broad strategies in equilibrium even if they are mutually better off pursuing the focused strategies. There are two separate roles that the government can play in this zone. Firstly, when there exist multiple equilibria in this zone \((a - c > 0.93 \text{ and } \Delta_L(a, c) \leq \Delta \leq \Delta_R(a, c))\), the government performs the role of coordinating which equilibrium should be chosen by the firms. Since both \((AB, AB)\) and \((A, B)\) are subgame perfect, the government’s selection of \((A, B)\) is effective in that the chosen configuration is fully sustainable and self-enforcing. However, if \((AB, AB)\) is the only equilibrium in this zone \((\Delta_F(a, c) < \Delta \leq \min\{\Delta_R(a, c), \Delta_L(a, c)\})\), then the government’s role is much more active. Since the mutually beneficial \((A, B)\) is not subgame-perfect, it is not sustainable without the continued government coercion.

5. Generalization to 2N-Oligopoly

In this section, I generalize the duopoly model formulated in previous sections to the case of oligopoly with more than two firms. My aim is not to characterize all subgame perfect equilibria as I had done in the case of duopoly, but only to show that the zone of excess diversification dilemma exists and is non-empty for all numbers of firms.

Suppose there exists a total of 2N firms with two potential markets, A and B, where N is a positive integer. Let \(m\) be the profile of market-selection strategies such that \(m \equiv (m_1, m_2, \cdots, m_i, \cdots, m_{2N})\). Also define \(m_{-i} \equiv m - m_i\), where \(i = 1, \cdots, 2N\). The total stage-1 profit for firm \(i\) given the profile of \(m\) is \(\Pi_i(m_i, m_{-i})\), which is the sum of profits from both markets, A and B. A strategy profile \(m^*\) is then a subgame perfect equilibrium if \(\Pi_i(m_i^*, m_{-i}^*) \geq \Pi_i(m_i, m_{-i}^*)\) for all \(m_i \in \{A, B, AB\}\) for all \(i\). Instead of writing \(\Pi_i(m_i, m_{-i})\), I will use for convenience an equivalent
notation $\widehat{\Pi}_i(m_i; x, y, z)$ as firm $i$'s stage-1 total profit from choosing $m_i \in \{A, B, AB\}$ given the rational anticipation of Cournot-Nash equilibrium in both markets in stage 2, when $x$ number of rivals focus on $A$ only, $y$ number of rivals focus on $B$ only, and $z$ number of rivals choose to enter both $A$ and $B$ simultaneously. Note that $x + y + z = 2N - 1$.

The next theorem identifies the condition under which all firms choose broad strategy in equilibrium.

**Theorem 5.1.** There exists $\Delta_\phi(N) > 0$ such that $m_i^* = AB$ for all $\Delta \in \left[-(a - c), \Delta_\phi(N)\right]$, $i = 1, 2, \cdots, 2N$.

**Proof.** To prove that $m_i^* = AB$ for all $i$, one needs to show that $\widehat{\Pi}_i(AB; 0, 0, 2N - 1) \geq \widehat{\Pi}_i(A; 0, 0, 2N - 1)$. First consider $\widehat{\Pi}_i(AB; 0, 0, 2N - 1)$. Given that all $2N$ firms produce in both markets at the marginal cost of $c$, it is straightforward to show that $\widehat{\Pi}_i(AB; 0, 0, 2N - 1) = \frac{2(a - 6c)^2}{b(2N + 1)^2}$. Likewise, for $\widehat{\Pi}_i(A; 0, 0, 2N - 1)$ note that in market $A$ firm $i$ incurs a marginal cost of $c$ and competes against $(2N - 1)$ rivals with the marginal cost of $\widehat{c}$. The equilibrium total profit for firm $i$ is then $\widehat{\Pi}_i(A; 0, 0, 2N - 1) = \frac{(a - 2Nc + (2N - 1)c)^2}{b(2N + 1)^2}$. It follows that $\widehat{\Pi}_i(AB; 0, 0, 2N - 1) \geq \widehat{\Pi}_i(A; 0, 0, 2N - 1)$, if and only if

$$\Phi(\Delta) \equiv (a - c)^2 - 4\Delta(a - c) - 2(2N - 1)\Delta(a - c) + 2\Delta^2 - (2N - 1)^2\Delta^2 \geq 0. \quad (5.1)$$

The following properties are to be noted:

\begin{align*}
\Phi(0) &= (a - c)^2 > 0 \\
\Phi(-(a - c)) &= (2N + 6)(a - c)^2 > 0 \\
\Phi(a - c) &= (2 - 6N)(a - c)^2 < 0 \\
\Phi'(\Delta) &= -4(a - c) - 2(2N - 1)(a - c) + 4\Delta - 2(2N - 1)^2\Delta < 0 \quad \text{for all } \Delta.
\end{align*} \quad (5.2)

These properties together imply that there exists $\Delta_\phi(N) > 0$ such that $\Phi(\Delta_\phi(N)) = 0$ and $\Phi(\Delta) \geq 0$ for all $\Delta \in \left[-(a - c), \Delta_\phi(N)\right]$.

It is straightforward, though tedious, to derive $\Delta_\phi(N)$:

$$\Delta_\phi(N) = \frac{2 \left(1 - \sqrt[4]{2}\right)N + 1}{1 + 4N - 4N^2}(a - c) \quad \text{for all } N \geq 1. \quad (5.3)$$
Hence, for \(-(a - c) \leq \Delta \leq \Delta_\Phi(N)\), there exists a symmetric, though not necessarily unique, subgame perfect equilibrium, in which all 2N firms pursue the broad strategy and engage in both markets. Note that \(\Delta_\Phi(1) = (3 - 2\sqrt{2})(a - c)\), which is identical to \(\Delta_R(a, c)\) derived previously for the case of duopoly.

**Corollary 5.2.** \(\Delta_\Phi(N) > \Delta_\Phi(N + 1)\) for all \(N \geq 1\).

**Proof.** For \(N = 1\), it is straightforward to show that \(\Delta_\Phi(1) \equiv (3 - 2\sqrt{2})(a - c) > \frac{5 - 4\sqrt{2}}{1 + 4(N + 1)^2} \equiv \Delta_\Phi(2)\). For \(N > 1\), \(\Delta_\Phi(N) > \Delta_\Phi(N + 1)\) if and only if

\[
\frac{2(1 - \sqrt{2})N + 1}{1 + 4N - 4N^2} > \frac{2(1 - \sqrt{2})(N + 1) + 1}{1 + 4(N + 1) - 4(N + 1)^2}.
\] (5.4)

Note that for \(N > 1\) the denominators on both sides of the inequality are strictly less than zero; \(1 + 4N - 4N^2 < 0\) and \(1 + 4(N + 1) - 4(N + 1)^2 < 0\). Hence, (5.4) can be rewritten as

\[
\left[2(1 - \sqrt{2})N + 1\right] \left[1 + 4(N + 1) - 4(N + 1)^2\right] > \left[2(1 - \sqrt{2})(N + 1) + 1\right] \left[1 + 4N - 4N^2\right],
\] (5.5)

which simplifies to

\[
4N(N + 1) + 1 > \left(\frac{4}{\sqrt{2} - 1}\right) N.
\] (5.6)

Since \(4N(N + 1) > \left(\frac{4}{\sqrt{2} - 1}\right) N\) for all \(N > \sqrt{2}\), this is sufficient to prove that (5.6) holds and, hence, \(\Delta_\Phi(N) > \Delta_\Phi(N + 1)\) for all \(N > 1\). ■

**Corollary 5.3.** \(\lim_{N \to \infty} \Delta_\Phi(N) = 0\).

The above corollaries show that \(\Delta_\Phi(N)\) is strictly positive for all finite \(N\), is monotonically declining in \(N\), and approaches zero from above in the limit as \(N\) grows. For a very large \(N\) so that the competition is very severe, diseconomies of scope (i.e., \(\Delta > 0\)) is incompatible with the uniform adoptions of broad strategy as the subgame perfect equilibrium.

In order to demonstrate that there exists a non-empty zone of excess diversification dilemma, we now show that the above subgame perfect equilibrium in which all firms choose broad strategy is payoff-dominated by an alternative configuration in which the first \(N\) firms focus only on market \(A\) and the remaining \(N\) firms focus only on market \(B\).
Theorem 5.4. There exists $\Delta_\Omega(N) < 0$ such that $\hat{\Pi}_i(AB; 0, 0, 2N - 1) \leq \hat{\Pi}_i(A; N - 1, N, 0) = \hat{\Pi}_i(B; N, N - 1, 0)$ for all $\Delta \in [\Delta_\Omega(N), a - c]$, $i = 1, 2, \ldots, 2N$.

Proof. That $\hat{\Pi}_i(AB; 0, 0, 2N - 1) = \frac{2(a-c)^2}{(2N+1)^2}$ is already known. For the alternative configuration that involves two groups of firms with (non-overlapping) focused strategies, one only needs to solve for the Cournot equilibrium in two $N$-firm oligopolies with $c$ as the common marginal costs. It is straightforward to derive $\hat{\Pi}_i(A; N-1, N, 0) = \hat{\Pi}_i(B; N, N-1, 0) = \frac{(a-c)^2}{(N+1)^2}$. Given these equilibrium profits, it is easy to show that $\hat{\Pi}_i(AB; 0, 0, 2N - 1) \leq \hat{\Pi}_i(A; N - 1, N, 0) = \hat{\Pi}_i(B; N, N - 1, 0)$ if and only if $\Omega(\Delta) = 2(N+1)^2 \Delta^2 - 4(N+1)^2(a-c)\Delta + (-2N^2+1)(a-c)^2 \leq 0$. The following properties can be shown about $\Omega(\Delta)$:

$$\begin{align*}
\Omega(-a-c) &= (4N^2 + 12N + 6)(a-c)^2 > 0, \\
\Omega(0) &= (1 - 2N^2)(a-c)^2 < 0, \\
\Omega(a-c) &= (-4N^2 - 4N - 1)(a-c)^2 < 0, \\
\Omega'(\Delta) &= 4(N+1)^2 \Delta - 4(N+1)^2(a-c) < 0.
\end{align*}
$$

(5.7)

These properties together prove that there exists $\Delta_\Omega(N) < 0$ such that $\Omega(\Delta) \leq 0$ for all $\Delta \in [\Delta_\Omega(N), a - c]$. ■

For $\Delta_\Omega(N) \leq \Delta \leq a - c$, the payoff to a firm in a configuration entailing symmetric pursuits of focused strategies - $N$ firms specializing in market $A$ and $N$ firms specializing in market $B$ - dominates the payoff to a firm in a configuration involving uniform pursuits of the broad strategy.

Again, it is possible to derive a closed-form expression for $\Delta_\Omega(N)$:

$$\Delta_\Omega(N) = \left[\frac{2(1-\sqrt{2})N + 2 - \sqrt{2}}{2(N+1)}\right](a-c) \quad \text{for all } N \geq 1. \quad (5.8)$$

Note that $\Delta_\Omega(1) = \left(1 - \frac{3\sqrt{2}}{4}\right)(a-c)$, which is identical to $\Delta_\Gamma(a, c)$ in the case of duopoly.

Corollary 5.5. $\Delta_\Omega(N) > \Delta_\Omega(N + 1)$ for all $N \geq 1$.

Proof. $\Delta_\Omega(N) > \Delta_\Omega(N + 1)$ if and only if:

$$\frac{2(1-\sqrt{2})N + 2 - \sqrt{2}}{2(N+1)}(a-c) > \frac{2(1-\sqrt{2})(N + 1) + 2 - \sqrt{2}}{2(N+2)}(a-c), \quad (5.9)$$

which is easily shown to be true. ■
Corollary 5.6. \( \lim_{N \to \infty} \Delta_{\Omega}(N) = (1 - \sqrt{2}) (a - c) < 0. \)

Hence, \( \Delta_{\Omega}(N) \) is strictly negative for all \( N \) and monotonically declines in \( N \). Since \( \Delta_{\Omega}(N) < 0 \) and \( \Delta_{\Phi}(N) \geq 0 \) for all \( N \), it follows that for any \( a - c > 0 \) there exists a non-empty set of \( \Delta \)s, \([\Delta_{\Omega}(N), \Delta_{\Phi}(N)]\), such that the broad strategy is chosen by all firms in equilibrium, even though an alternative configuration involving specialized focused strategy generates higher profits for all firms. The zone of excess diversification dilemma exists for all \( N \geq 1 \).

6. Chaebols and Big Deals Revisited

I opened this paper with a brief look at the South Korean chaebols and the government’s big deal program. Three questions motivated my investigation. First, why did these chaebols drive themselves into the state of excessive diversification? My model shows that this outcome may obtain as a subgame perfect equilibrium when there exists cost-interdependency of certain degree between the markets. Second, why are the chaebols opposed to the big deal program which would allow them to specialize in niche markets - presumably a more profitable opportunity for them? Given full diversification as the subgame perfect equilibrium, a unilateral specialization would not be an optimal move from an individual firm’s perspective. Furthermore, even if mutual specialization could somehow be orchestrated, the resulting outcome would be far from stable unless they are in the range of parameter values that supports multiple equilibria, of which mutual pursuit of focused strategy is one. This observation leads to the third question - will the big deal reform be effective in the long run? The results obtained in this paper suggests that the answer to that question depends again on the direction and the extent of cross-unit cost-interdependency. If the diversified firms are in the region of multiple equilibria which supports both mutual diversification as well as mutual specialization, the role of the government is simply to facilitate a coordinated move from one equilibrium to another. However, if the firms are in the region in which full diversification is the unique equilibrium, an industrial policy such as big deal cannot be effective in the long run and, hence, lacks credibility. The chaebols’ uniform unwillingness to cooperate with the program so far
strongly suggests that they may be in this last region of parameter values. Unless the direction and the extent of cost-interdependency themselves are somehow controlled, it is unlikely that the big deal program will bear much fruit. One possibility is that the elimination of any existing political advantages coupled with the design of more objective and capable financial institutions will raise $\Delta$ for the chaebols, thereby pushing them into the region of multiple equilibria in which the government’s coordination attempt through ‘big deal’ can be made sustainable in the long run.

7. Conclusion

Even when a multi-business conglomerate is engaged in businesses that are entirely unrelated on the demand-side, it is often the case that their operations are interconnected in some subtle ways that the engagement in one business tends to influence the operational efficiency in another business. While this connectedness always introduces complexity considerations into the corporate headquarters’ coordination capacity, it also brings about many benefits that may be realized through complementarities arising from the existence of shared inputs and activities as well as mutual learning among various business units. My purpose in this paper was to take the existence of such cross-market (or cross-unit) linkages as given and examine how the aggregate effect of such linkages influences the firms’ decisions to diversify and enter into multiple businesses.

In the case of a duopoly, it was found that cross-market cost-complementarities do result in mutual pursuit of the broad strategy in equilibrium. However, if the cost advantage from multi-market operation is not sufficiently great, though positive, this equilibrium turns out to be less profitable than an alternative configuration in which each firm specializes in a niche market through the adoption of a focused strategy. In that the firms fail to support this mutually beneficial configuration as an equilibrium, they are viewed as being confronted with the excess diversification dilemma. It was further found that the pursuit of excess diversification can exist even when diversification generates competitive disadvantages. The zone of excess diversification dilemma was identified and fully characterized for a duopoly. This zone was shown to exist in a more general oligopoly setting.
Appendix

Proof of Proposition 3.1. We will consider the four cases separately.

Case 1: \((m_1, m_2) = (A, A)\) In this case, both firm 1 and firm 2 compete in market \(A\) only.

The total profit of firm \(i\) is then simply \(\hat{\Pi}_i((m_1, q_1), (m_2, q_2)) = \pi_i^A((m_1, q_1^A), (m_2, q_2^A))\), where

\[
\begin{align*}
\pi_1^A((A, q_1^A), (A, q_2^A)) &= [a - b(q_1^A + q_2^A) - c]q_1^A, \\
\pi_2^A((A, q_1^A), (A, q_2^A)) &= [a - b(q_1^A + q_2^A) - c]q_2^A.
\end{align*}
\]

The corresponding first-order conditions are:

\[
\begin{align*}
\frac{\partial \pi_1^A}{\partial q_1^A} &= a - 2bq_1^A - bq_2^A - c = 0, \\
\frac{\partial \pi_2^A}{\partial q_2^A} &= a - bq_1^A - 2bq_2^A - c = 0.
\end{align*}
\]

Solving for \(q_1^A\) and \(q_2^A\), respectively, one derives the best response functions:

\[q_i^A = \frac{a - c}{2b} - \frac{1}{2}q_j^A, \quad i, j = 1, 2, \text{ and } i \neq j.\]

The Cournot-Nash equilibrium in quantities are derived by solving the best response functions simultaneously:

\[q_i^A = \frac{a - c}{3b} = q_2^A.\]

The equilibrium profits are:

\[\hat{\Pi}_1(A, A) = \frac{(a - c)^2}{9b} = \hat{\Pi}_2(A, A).\]

Case 2: \((m_1, m_2) = (A, B)\) Since each firm acts as a monopolist in its chosen market, firm \(i\)’s profit is \(\hat{\Pi}_i((m_1, q_1), (m_2, q_2)) = \pi_i^I(I, q_i^I)\), where

\[\pi_i^I(I, q_i^I) = (a - bq_i^I - c)q_i^I, \quad i = 1, 2 \text{ and } I = A, B.\]

The corresponding monopoly quantities are:

\[q_i^I = \frac{a - c}{2b}, \quad i = 1, 2 \text{ and } I = A, B.\]

The equilibrium profits are:

\[\hat{\Pi}_1(A, B) = \frac{(a - c)^2}{4b} = \hat{\Pi}_2(A, B).\]
Case 3: \((m_1, m_2) = (A, AB)\)  
Since firm 1 enters market A only and firm 2 enters both markets, there is a duopoly competition in market A and a monopoly for firm 2 in market B. The marginal costs are asymmetric between the firms in that firm 1 faces a marginal cost of \(c\), while firm 2 faces \(\hat{c}\). Let us consider the two markets separately. In market A, the duopoly profit functions are:

\[
\pi_1^A((A, q_1^A), (A, q_2^A)) = [a - b(q_1^A + q_2^A) - c]q_1^A,
\]
\[
\pi_2^A((A, q_1^A), (A, q_2^A)) = [a - b(q_1^A + q_2^A) - \hat{c}]q_2^A.
\]

The best response functions from the corresponding first-order conditions are:

\[
q_1^A = \frac{a - c - \hat{c}}{2b} - \frac{1}{2}q_2^A,
\]
\[
q_2^A = \frac{a - c - \hat{c}}{2b} - \frac{1}{2}q_1^A.
\]

Solving these best response functions simultaneously, we obtain the following asymmetric Cournot-Nash equilibrium:

\[
q_1^A = \frac{a - 2c + \hat{c}}{3b},
\]
\[
q_2^A = \frac{a + c - 2\hat{c}}{3b}.
\]

In market B, firm 2 has a sole monopoly control and, hence, its monopoly quantity given the marginal cost of \(\hat{c}\) is \(q_2^B = \frac{a - \hat{c}}{2b}\), while \(q_1^B = 0\). Given the equilibrium in both markets, the total profits for the firms are:

\[
\hat{\Pi}_1(A, AB) = \frac{(a - 2c + \hat{c})^2}{9b},
\]
\[
\hat{\Pi}_2(A, AB) = \frac{(a + c - 2\hat{c})^2}{9b} + \frac{(a - \hat{c})^2}{4b}.
\]

Case 4: \((m_1, m_2) = (AB, AB)\)  
Both firms engage in both markets. Consequently, the resulting competition entail two markets, each with duopoly competition among firms 1 and 2 with the common marginal cost of \(\hat{c}\). It is easy to see that the market A duopoly in Case 1 will be extended to market B as well with the marginal cost of \(c\) being replaced with \(\hat{c}\) in both markets:

\[
q_1^A = \frac{a - \hat{c}}{3b} = q_2^A.
\]

The corresponding equilibrium total profits are:

\[
\hat{\Pi}_1(AB, AB) = \frac{2(a - \hat{c})^2}{9b} = \hat{\Pi}_2(AB, AB).
\]
References


### Table 1: Nash Equilibrium in Stage 2

<table>
<thead>
<tr>
<th>$(m_1, m_2) = (A, A)$</th>
<th>$(m_1, m_2) = (A, B)$</th>
<th>$(m_1, m_2) = (A, AB)$</th>
<th>$(m_1, m_2) = (AB, AB)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1^A = \frac{a-c}{3b}$; \hspace{1em} $q_2^A = \frac{a-c}{3b}$; \hspace{1em} $q_1^B = 0$; \hspace{1em} $q_2^B = 0$;</td>
<td>$q_1^A = \frac{a-c}{2b}$; \hspace{1em} $q_2^A = 0$; \hspace{1em} $q_1^B = 0$; \hspace{1em} $q_2^B = \frac{a-c}{2b}$;</td>
<td>$q_1^A = \frac{a+2c+c}{3b}$; \hspace{1em} $q_2^A = \frac{a+c-2c}{3b}$; \hspace{1em} $q_1^B = 0$; \hspace{1em} $q_2^B = \frac{a-c}{2b}$;</td>
<td>$q_1^A = \frac{a-c}{3b}$; \hspace{1em} $q_2^A = \frac{a-c}{3b}$; \hspace{1em} $q_1^B = \frac{a-c}{3b}$; \hspace{1em} $q_2^B = \frac{a-c}{3b}$;</td>
</tr>
<tr>
<td>$P^A = \left(\frac{1}{3}\right) a + \left(\frac{2}{3}\right) c$; \hspace{1em} $P^B = NA$;</td>
<td>$P^A = \left(\frac{1}{2}\right) (a + c)$; \hspace{1em} $P^B = \left(\frac{1}{2}\right) (a + c)$;</td>
<td>$P^A = \frac{(a-2c+c)2}{9b}$; \hspace{1em} $P^B = \frac{(a+c-2c)2}{9b}$; \hspace{1em} $P^B = \frac{(a-c)2}{4b}$;</td>
<td>$P^A = \left(\frac{1}{3}\right) a + \left(\frac{2}{3}\right) c$; \hspace{1em} $P^B = \left(\frac{1}{3}\right) a + \left(\frac{2}{3}\right) c$;</td>
</tr>
<tr>
<td>$\pi_1^A = \frac{(a-c)2}{9b}$; \hspace{1em} $\pi_2^A = \frac{(a-c)2}{9b}$; \hspace{1em} $\pi_1^B = 0$; \hspace{1em} $\pi_2^B = 0$;</td>
<td>$\pi_1^A = \frac{(a-c)2}{4b}$; \hspace{1em} $\pi_2^A = 0$; \hspace{1em} $\pi_1^B = 0$; \hspace{1em} $\pi_2^B = \frac{(a-c)2}{4b}$;</td>
<td>$\pi_1^A = \frac{(a-2c+c)2}{9b}$; \hspace{1em} $\pi_2^A = \frac{(a+c-2c)2}{9b}$; \hspace{1em} $\pi_2^B = \frac{(a-c)2}{4b}$;</td>
<td>$\pi_1^A = \frac{(a-c)2}{9b}$; \hspace{1em} $\pi_2^A = \frac{(a-c)2}{9b}$; \hspace{1em} $\pi_2^B = \frac{(a-c)2}{9b}$;</td>
</tr>
<tr>
<td>$\tilde{\Pi}_1 = \frac{(a-c)2}{9b}$; \hspace{1em} $\tilde{\Pi}_2 = \frac{(a-c)2}{9b}$;</td>
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<td>$\tilde{\Pi}_1 = \frac{2(a-c)2}{9b}$; \hspace{1em} $\tilde{\Pi}_2 = \frac{2(a-c)2}{9b}$;</td>
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</table>
Figure 1: Subgame-Perfect Equilibrium in Market Selection Strategy
Figure 2: Excess Diversification Dilemma