Types of Analysis:

1. One Way ANOVA
2. Randomized Blocked ANOVA
3. Two Factor ANOVA
4. Multifactor ANOVA
5. Repeated Measures ANOVA
   a. crossover factors
   b. nested factors

Approach to analysis:

I. Visualize the data if possible
   A. One Way ANOVA – Side by Side box plots
   B. Randomized Block ANOVA – Line Graph with blocking variable on X-axis. Look to see if there are gaps between the lines
   C. 2 Way ANOVA – Line Graph – look to see if gaps between lines, if lines are not flat, and if lines are not parallel. Sometimes helpful to plot with Factor 1 on X axis and then reverse so factor 2 on X-axis.
   D. Repeated Measures ANOVA – do a line graph line blocking if it is one factor crossover factor with subject on the X-axis.

II. Perform the Analysis
   A. Identify Fixed or Random Effects
   B. One Way ANOVA
      1. If two groups in treatment – equivalent to two independent sample ttest
      2. Fixed effects Model:  \( Y_{ij} = \mu + \alpha_i + E_{ij} \), \( E_{ij} \sim N(0, \sigma^2) \), \( i=1..k, j=1..n \)
         a. Estimate \( \hat{\alpha_i} = \overline{Y}_i - \overline{Y} \)
         b. \( H_0 : \mu_1 = \mu_2 = ... = \mu_k \)
      3. Random effects Model:
         \( Y_{ij} = \mu + A_j + E_{ij} \), \( A_j \sim N(0, \sigma^2_A) \), \( E_{ij} \sim N(0, \sigma^2) \), \( i=1..k, j=1..n \)
         a. \( H_0 : \sigma^2_A = 0 \)
         b. We did not go over estimation of \( \sigma^2_A \)
      4. Analysis the same for Fixed and Random Effects
      5. TotalSS=SST+SSE
      6. F=MST/MSE main test
      7. If Null is Rejected, We perform Multiple Comparison Procedure
         a. LSD→Duncan→SNK→Tukey→Bonferroni
      8. Kruskal Wallis is the nonparametric test
C. Randomized Block ANOVA

1. If two groups in treatment – equivalent to Paired t-test.
2. Fixed effects Model:
   \[ Y_{ij} = \mu + \alpha_i + \beta_j + E_{ij}, E_{ij} \sim N(0, \sigma^2), i = 1..k, j = 1..b \]
   a. Estimate \( \hat{\alpha_i} = \bar{Y}_{..i} - \bar{Y}_{..} \)
   b. Estimate \( \hat{\beta_j} = \bar{Y}_{..j} - \bar{Y}_{..} \)
   c. \( H_0: \mu_1 = \mu_2 = ... = \mu_k \)
3. Random effects Model:
   \[ Y_{ij} = \mu + A_i + B_j + E_{ij}, A_i \sim N(0, \sigma^2_A), B_j \sim N(0, \sigma^2_B), E_{ij} \sim N(0, \sigma^2), i = 1..k, j = 1..n \]
   a. \( H_0: \sigma^2_A = 0 \)
   b. \( H_0: \sigma^2_B = 0 \)
   c. We did not go over estimation of \( \sigma^2_A \), or \( \sigma^2_B \)
4. Mixed Models as well.
5. Analysis the same for Fixed and Random Effects
6. TotalSS=SST+SSB+SSE
7. F=MST/MSE is the main test statistic of interest
8. F=MSB/MSE is a posteriori check on the blocking – if not significant, keep in the model.
9. If Null is Rejected, We perform Multiple Comparison Procedure on the treatment variable
10. Friedman Rank test is the nonparametric test of interest.

D. 2 Way ANOVA

1. Fixed effects Model:
   \[ Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + E_{ijk}, E_{ijk} \sim N(0, \sigma^2), i = 1..r, j = 1..c, k = 1..n \]
   d. Estimate \( \hat{\alpha_i} = \bar{Y}_{..i} - \bar{Y}_{..} \)
   e. Estimate \( \hat{\beta_j} = \bar{Y}_{..j} - \bar{Y}_{..} \)
   f. Estimate \( \hat{\gamma_{ij}} = \bar{Y}_{ij} - \bar{Y}_{..i} - \bar{Y}_{..j} + \bar{Y}_{..} \)
   g. Treatment R means \( H_0: \mu_1 = \mu_2 = ... = \mu_r \)
   h. Treatment C means \( H_0: \mu_1 = \mu_2 = ... = \mu_c \)
   i. Interaction \( H_0 \): there is no interaction
2. Random effects Model:
   \[ Y_{ijk} = \mu + A_i + B_j + C_k + E_{ijk}, A_i \sim N(0, \sigma^2_A), B_j \sim N(0, \sigma^2_B), C_k \sim N(0, \sigma^2_C), E_{ijk} \sim N(0, \sigma^2), i = 1..r, j = 1..c, k = 1..n \]
   a. \( H_0: \sigma^2_A = 0 \)
   b. \( H_0: \sigma^2_C = 0 \)
c. \( H_0 : \sigma_{rc}^2 = 0 \)
d. We did not go over estimation of \( \sigma_{r}^2, \sigma_{c}^2, \sigma_{rc}^2 \)

3. Mixed Models as well.

4. TotalSS=SSR+SSC+SSRC+SSE

5. Row  Fixed: \( F = \frac{MSR}{MSE} \) Mixed or Random: \( F = \frac{MSR}{MSRC} \)

6. Column Fixed: \( F = \frac{MSC}{MSE} \) Mixed or Random: \( F = \frac{MSC}{MSRC} \)

7. Interaction Fixed: \( F = \frac{MSRC}{MSE} \) Mixed or Random: \( F = \frac{MSRC}{MSE} \)

8. If interactions are not significant, we drop them from the model and refit.

9. If one of the factors is not significant after dropping interaction, we drop that factor and do One Way ANOVA

10. If interaction is significant and factors are not, we leave them in and this is called hierarchical modeling.

11. If Null is Rejected, We perform Multiple Comparison Procedure on Factor R or Factor C, depending on interest.

12. We did not go over a nonparametric test for this type of analysis.

E. High Order ANOVAS

1. The Analysis is very similar to 2 Way ANOVA

2. Gets very complicated very fast

3. High order interactions are often dropped.

F. Repeated Measures ANOVA

1. One Crossover Factor
   a. Similar to Blocked ANOVA analysis
   b. Model (Fixed Treatment):
      \[
      Y_{ij} = \mu + S_i + \tau_j + E_{ij}, E_{ij} \sim N(0, \sigma^2) \]
      \( i = 1..s, j = 1..t, \)
      s=# of subjects, t=# of treatments
      i. Estimate \( \hat{\tau}_i = \bar{Y}_{i*} - \bar{Y} \)
      ii. \( H_0 : \mu_1 = \mu_2 = \ldots = \mu_i \)
   c. Model (Random Treatment):
      \[
      Y_{ij} = \mu + S_i + T_j + E_{ij}, T_j \sim N(0, \sigma_T^2), E_{ij} \sim N(0, \sigma^2) \]
      \( i = 1..s, j = 1..t, \)
      i. \( H_0 : \sigma_T^2 = 0 \)
   d. Main Test Statistics for Fixed or Random Treatment Factors is
      \[
      F = \frac{MST}{MSTS} = \frac{MS_T}{MS_{TYS}}
      \]

2. Two Crossover Factors
   a. Model (Fixed Treatment):
\[ Y_{ijk} = \mu + S_i + \alpha_j + \beta_k + \delta_{jk} + S_{ij} + S_{ik} + E_{ijk}, E_{ijk} \sim N(0, \sigma^2) \]

\( i = 1..s, j = 1..a, k = 1..b \)

s=# of subjects, a=# of treatments for Factor a, b=# of treatments factor B

\[ H_0 : \text{Treatment A means are equal} \]

\[ H_0 : \text{Treatment B means are equal} \]

b. Main Test Statistics for Fixed Factors is

\[ F_A = \frac{MS_A}{MSSA}, F_B = \frac{MSB}{MSSB}, F_{AB} = \frac{MSAB}{MSE}, F_{MS} = \frac{MS_{Error}}{MS_{Error}} \]

3. One Nested Factor

a. Model (Fixed Treatment):

\[ Y_{ijk} = \mu + S_{i(j)} + \tau_j + E_{i(j)} \]

\( i = 1..s, j = 1..t, k = 1..r, s = \# \text{given each treatment} \)

\( t = \# \text{of treatments}, r = \# \text{of replicates measured on ith subject for a given treatment} \)

\( \tau_j = \text{fixed effect of treatment } j \)

\( S_{i(j)} = \text{Random effect of subject } i \text{ within treatment } j \)

\( E_{i(j)} = \text{Random error for repeat } k \text{ on subject } i \text{ at treatment } j \)

i. \( H_0 : \text{t treatment means are equal} \)

b. Main Test Statistics for Fixed Treatment Factor is

\[ F_A = \frac{MST}{MS_T} = \frac{MS_T}{MS_{S(i)}} \]

4. One Crossover and One Nested Factor

a. Model (Fixed Treatments):

\[ Y_{ijk} = \mu + S_{i(j)} + \alpha_j + \beta_k + \delta_{jk} + E_{k(i)} + E_{i(j)}, E_{k(i)}, E_{i(j)} \sim N(0, \sigma^2) \]

\( i = 1..s, j = 1..a, k = 1..b, s = \# \text{subjects observed at each level of Factor A} \)

a=# of treatments for Factor A (s subjects per level-Nested), b=# of treatments factor B –Crossover

i. \( H_0 : \text{Treatment A means are equal} \)

ii. \( H_0 : \text{Treatment B means are equal} \)

b. Main Test Statistics for Fixed Factors is

\[ F_A = \frac{MSA}{MSSA}, F_B = \frac{MSB}{MSE}, F_{AB} = \frac{MSAB}{MSE}, F_{MS} = \frac{MS_{Error}}{MS_{Error}} \]

4
III. Check the Normality Assumption
   A. Assume fixed effects
   B. Examine the residuals – the difference between the observed $Y$ and the predicted $\hat{Y}$ from the model $e = Y - \hat{Y}$
   C. Use SAS to generate the predicted and residual values
   D. Two issues
      1. Homogeneity of Variance – Look at a plot of residual vs. predicted values and see if it is a random scatter. If there is a trumpet effect, that is not good. We are violating the assumption of homogeneity.
      2. Normality of Residuals – Perform a Normality analysis on the residuals. Examine QQ plot, histogram, and the formal tests of Normality of the data.
   E. When assumptions are violated trying applying a transformation to the data –
      1. ln, 1/x, sqrt, or arsin or ln(x+1), or 1/(x+1) or sqrt(x+???) or arsin (1/x)
      2. perform analysis on transformed data and then see if residuals behave better.