On Model Errors

- Plant models tend to be less reliable at high frequencies.

- In mechanical plants, higher resonant modes are both uncertain (difficult to measure due to small motion) and variable (susceptible to shifts due to temperature, mounting, etc).

- Even if a repeatable frequency response can be captured using a dynamic signal analyzer (HP 3265A, etc), high-order models are inconvenient.

- Truncation to low-order models is always necessary.

- Spillover effect: unstable response of true plant when a nominally stabilizing controller is deployed. Occurs due to model errors associated with high-frequency unmodeled dynamics.

- Need to have a practical way to check a designed controller against model errors.
Feedback Loop with Additive Model Error

\[ G_t = G + A \]

Feedback Loop with Multiplicative Model Error

\[ G_t = (1 + \Delta_1)G \] (relative to nominal model)

\[ G_t = (1 - \Delta_2)^{-1}G \] (relative to true system)
Equivalence Btw Model Errors

\[ A = \Delta_1 G \]
\[ A = (1 - \Delta_2)^{-1} \Delta_2 G \]
\[ \Delta_2 = 1 - (1 + \Delta_1)^{-1} \]

Note: The disadvantage of \( A \) is that it doesn’t measure the error in \( GK ((G+A)K \neq GK) \), while \((1+\Delta_1)G\) \( K = (1 + \Delta_1)GK \) and similarly with \( \Delta_2 \).

Robust Stability Condition for \( A \) error

The condition below has been specialized to the SISO case from the multi-input, multi-output case (transfer matrices), which is expressed in terms of singular values.

Assumptions:

1. The controller \( K \) stabilizes the nominal model \( G \).
2. The true plant and the model have the same number of unstable (r.h.p.) poles.

The closed-loop system is stable provided:

\[ \frac{1}{|A(s)|} > |K(s)(1 - G(s)K(s))^{-1}| \]

for all complex \( s \) in a Nyquist contour in the complex plane.

Simplification for stable systems: If neither \( G \) or \( G_t \) have unstable poles, then we just take \( s = jw \) and test for all \( w \geq 0 \).
Robust Stability Condition for $\Delta_1$ error

1. The controller $K$ stabilizes the nominal model $G$.
2. The true plant and the model have the same number of unstable (r.h.p.) poles.

The closed-loop system is stable provided:

$$\frac{1}{|\Delta_1(s)|} > |G(s)K(s)(1 - G(s)K(s))^{-1}|$$

for all complex $s$ in a Nyquist contour in the complex plane.

Simplification for stable systems: If neither $G$ or $G_t$ have unstable poles, then we just take $s = jw$ and test for all $w \geq 0$.

Robust Stability Condition for $\Delta_2$ error

1. The controller $K$ stabilizes the nominal model $G$.
2. The true plant and the model have the same number of unstable (r.h.p.) poles.

The closed-loop system is stable provided:

$$|\Delta_2(s)| < \min\{1, \frac{1}{|(1 - G(s)K(s))^{-1}|}\}$$

for all complex $s$ in a Nyquist contour in the complex plane.

Simplification for stable systems: If neither $G$ or $G_t$ have unstable poles, then we just take $s = jw$ and test for all $w \geq 0$. 
Interpretation in a Magnitude Plot

In the case of the $A$ and $\Delta_1$ errors, assuming we can evaluate their frequency magnitude, the magnitude of $1/A(j\omega)$ (respectively $1/\Delta_1(j\omega)$) must stay above the magnitude of $K(j\omega)/(1 - G(j\omega)K(j\omega))$ (respectively $G(j\omega)K(j\omega)/(1 - G(j\omega)K(j\omega))$).

Practical Evaluation Using Spectral Analyzer Data

1. Run a frequency sweep capturing all features in a sufficiently wide frequency range.
2. Fit and validate a nominal model $G$ using any method.
3. The spectral analyzer gives real/imaginary pairs corresponding to $G_t$.
4. Numerically compute the frequency response of $G$ as real/imaginary pairs using the same frequency range.
5. Compute the frequency data for the desired model error $(A(j\omega) = G_t(j\omega) - G(j\omega))$ and perform the calculations necessary to construct the plot.
We obtained an experimental frequency response in the 10-5000 Hz range using the HP 3265A.

The instructor fitted a 2nd-order model to match the magnitude and phase in a limited frequency range.

A proportional controller \((K = 0.0036)\) was selected to achieve nominal stability and performance. In class, we demonstrate robust stability.

Task1: Design a controller to reduce the settling time by a factor of 10, with an overshoot of less than 5%. Do not worry about control input levels for now.

Evaluate the robust stability of your nominal loop by using model errors \(A\) and \(\Delta_1\) and the data from the HP 3265A. If your controller fails, derate it so that the robust stability test is passed with a settling time as low as possible and less than 5% overshoot.