On Optimization of Sensor Selection for Aircraft Gas Turbine Engines

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Overview of presentation

• Background
• Sensor selection optimization
  • Probabilistic approach
  • Genetic algorithm approach
• Results
• Conclusions and future work
Background
Background

• Reliable health monitoring of an engine
  • Maintenance scheduling
  • Real time control
• Health parameter estimation goals
  • Accuracy
  • Low cost
Background

• MAPSS (Modular Aerospace Propulsion System Simulation)
  • Aircraft turbofan engine simulation
  • Developed using Matlab at NASA GRC
  • 3 States
  • 8 Health Parameters
  • 11 Sensors
Background – Engine sensors

1. Core rotor speed
2. Percent low pressure spool rotor speed
3. Fan exit pressure
4. Booster inlet pressure
5. HPC (high pressure compressor) inlet temperature
6. HPC exit temperature
7. Bypass duct pressure
8. HPC exit pressure
9. LPT (low pressure turbine) blade temperature
10. LPT exit temperature
11. LPT exit pressure
Background – Health parameters

1. Fan airflow capacity
2. Fan efficiency
3. Booster hub airflow capacity
4. Booster hub efficiency
5. High pressure turbine airflow capacity
6. High pressure turbine efficiency
7. Low pressure turbine airflow capacity
8. Low pressure turbine efficiency
Background

- Linearized system model

\[
\tilde{x}_{k+1} = A \tilde{x}_k + B \tilde{u}_k + v_k \quad E[v_k v_k^T] = Q
\]
\[
\tilde{y}_k = C \tilde{x}_k + D \tilde{u}_k + e_k \quad E[e_k e_k^T] = R
\]

\(\tilde{x} = \text{state}, \quad \tilde{y} = \text{measurement}, \quad \tilde{u} = \text{input}\)
\(v = \text{process noise}, \quad e = \text{measurement noise}\)
Background - Riccati Equation

• Kalman filter estimation
• $K$ is the Kalman gain for the given sensor set
• Steady state error covariance $P$ is the solution of the discrete time algebraic Riccati equation

\[
P = (A - AKC)P(A - AKC)^T + BQB^T + (AK)R(AK)^T
\]
Background – Cost function

\[ J = \sum_{i=1}^{n} w_i \sqrt{\frac{P_{ii}}{P_{ii}^{\text{ref}}}} + \frac{\text{financial cost}}{\text{ref financial cost}} \]

The cost function balances estimation accuracy with financial cost. Up to 2 sensors of each type allowed. 11 sensors total will be used. Reference set is \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}
Background – Brute force search

- One cost function evaluation requires 0.2 seconds of CPU time
- The number of sensor sets containing 11 sensors (with up to 2 sensors of each type) is equal to 25,653
  - $25,653 \times 0.2 \text{ s} = 86 \text{ minutes of CPU time}$
Sensor Selection Optimization

- Probabilistic Approach
- Genetic Algorithm Approach
Probabilistic Approach

• A random search estimates the probability of each individual sensor being in the best $x\%$ of all sensor sets
• We then use the probabilities to generate new sensor sets via a directed random search
• Use the best resulting sensor set as the approximately optimal solution
Probabilistic Approach

• Example
  • Generate 1000 random sensor sets
  • Observe that sensor \( #i \) has a \( p_i \) chance of being in the top 10% of the 1000 sensor sets
  • Generate 1000 more sensor sets, where sensor \( #i \) has a \( p_i \) chance of being selected
  • Use the best result from this process as an approximately optimal sensor set
Probabilistic Approach

- This is a simple, intelligent, directed approach to sensor set selection
- How many sensor sets should be randomly generated to obtain a certain confidence that the best resulting set is within some percentage of optimal?
Genetic Algorithms

- Developed by John Holland in the 1960s
- Search algorithms based on the mechanics of natural selection
- A highly simplified computational model of biological evolution
Genetic Algorithms

- A sensor set is a chromosome with 11 genes
- Each gene is a sensor number between 1 and 11

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
<th>S11</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>9</td>
<td>8</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>3</td>
<td>11</td>
<td>4</td>
</tr>
</tbody>
</table>

Example:
Genetic Algorithms

- Initialization: A population of distinct sensor sets is randomly generated
- Fitness is computed for each individual
- Elitism: Parents are randomly selected from the top 50% of the population to create a new population using crossover
- Mutation: Random changes are inserted into the population to preserve diversity
Genetic Algorithms – Crossover

- Two random crossover points are generated

**Before crossover:**

Parent 1: \[ S1 \ S2 \ S3 \ S4 \ S5 \ S6 \ S7 \ S8 \ S9 \ S10 \ S11 \]

Parent 2: \[ S'1 \ S'2 \ S'3 \ S'4 \ S'5 \ S'6 \ S'7 \ S'8 \ S'9 \ S'10 \ S'11 \]

**After crossover:**

Child 1: \[ S1 \ S2 \ S3 \ S4' \ S5' \ S6' \ S7' \ S8 \ S9 \ S10 \ S11 \]

Child 2: \[ S1' \ S2' \ S3' \ S4 \ S5 \ S6 \ S7 \ S8' \ S9' \ S10' \ S11' \]
## Results – Problem setup

<table>
<thead>
<tr>
<th>SENSOR</th>
<th>RELATIVE COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core rotor speed</td>
<td>1.0</td>
</tr>
<tr>
<td>Pct low pressure spool rotor speed</td>
<td>1.0</td>
</tr>
<tr>
<td>Fan exit pressure</td>
<td>2.0</td>
</tr>
<tr>
<td>Booster inlet pressure</td>
<td>2.0</td>
</tr>
<tr>
<td>HPC inlet temperature</td>
<td>1.5</td>
</tr>
<tr>
<td>HPC exit temperature</td>
<td>1.5</td>
</tr>
<tr>
<td>Bypass duct pressure</td>
<td>2.0</td>
</tr>
<tr>
<td>HPC exit pressure</td>
<td>2.5</td>
</tr>
<tr>
<td>LPT blade temperature</td>
<td>2.5</td>
</tr>
<tr>
<td>LPT exit temperature</td>
<td>2.0</td>
</tr>
<tr>
<td>LPT exit pressure</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Results – Exhaustive search

Nominal set (no duplicates)
## Results – Exhaustive search

<table>
<thead>
<tr>
<th>SENSOR SET</th>
<th>COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1, 2, 2, 4, 5, 5, 6, 7, 8, 10</td>
<td>2.0957</td>
</tr>
<tr>
<td>1, 1, 2, 2, 3, 5, 5, 6, 7, 8, 10</td>
<td>2.1040</td>
</tr>
</tbody>
</table>
## Results – Probabilistic Approach

<table>
<thead>
<tr>
<th>SENSOR</th>
<th>PROBABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.118</td>
</tr>
<tr>
<td>2</td>
<td>0.095</td>
</tr>
<tr>
<td>3</td>
<td>0.082</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
</tr>
<tr>
<td>5</td>
<td>0.091</td>
</tr>
<tr>
<td>6</td>
<td>0.083</td>
</tr>
<tr>
<td>7</td>
<td>0.087</td>
</tr>
<tr>
<td>8</td>
<td>0.118</td>
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<tr>
<td>9</td>
<td>0.076</td>
</tr>
<tr>
<td>10</td>
<td>0.072</td>
</tr>
<tr>
<td>11</td>
<td>0.085</td>
</tr>
</tbody>
</table>
Results – Probabilistic Approach
### Results – Probabilistic Approach

<table>
<thead>
<tr>
<th>Sensor Set</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Search Sensor Set</td>
<td></td>
</tr>
<tr>
<td>1, 1, 2, 4, 5, 7, 8, 9, 10, 11, 11</td>
<td>2.306</td>
</tr>
<tr>
<td>1, 1, 3, 3, 4, 4, 5, 6, 8, 9, 11</td>
<td>2.310</td>
</tr>
<tr>
<td>Probabilistic Search Sensor Set</td>
<td></td>
</tr>
<tr>
<td>1, 2, 4, 5, 6, 7, 8, 8, 9, 10, 11</td>
<td>2.203</td>
</tr>
<tr>
<td>1, 1, 2, 3, 4, 5, 5, 6, 7, 8, 11</td>
<td>2.206</td>
</tr>
</tbody>
</table>

(Recall the best sensor set cost = 2.0957)
Results – GA Parameters

• GA parameters determined by manual tuning
  Initial population size = 100
  Population size = 50
  Crossover Probability = 0.9
  Mutation Probability = 0.003 per sensor
  Maximum Generations = 15
Results – GA Approach
Results – GA Approach

<table>
<thead>
<tr>
<th>GA Sensor Set</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1, 2, 2, 4, 5, 5, 6, 7, 8, 10</td>
<td>2.0957 *</td>
</tr>
<tr>
<td>1, 1, 2, 2, 3, 5, 5, 6, 7, 8, 10</td>
<td>2.1040 **</td>
</tr>
</tbody>
</table>

* Best sensor set
** Second best sensor set
Conclusions

- Exhaustive search
  - 25,653 distinct sets, 86 minutes of CPU time
  - Best sensor set cost = 2.0957

- Probabilistic approach
  - 10,000 sensor sets, 34 minutes of CPU time
  - Best sensor set cost = 2.203

- GA approach
  - 850 sensor sets, 3 minutes of CPU time
  - Best set cost = 2.0957 (relative cost)
Future Work

• The confidence in the quality of the probabilistically obtained sensor set can be quantified
• Joint probabilities can be obtained and used in the probabilistic search
• A variable chromosome length can be used in the GA to search for the optimal sensor set with a variable number of sensors
• Other evolutionary algorithms (e.g., particle swarm optimization and ant colony optimization) can be applied for sensor selection
• Eigenvector approach results need to be obtained
Thank you

• Any questions?
Background – Brute force search

- The number of distinct sets with \( p \) elements and no more than \( r \) repetitions of each element can be found from \((1 + x + x^2 + \cdots + x^r)^p\)

- The distinct number of sets with \( m \) elements in each set is the coefficient of \( x^m \) in the above expression after expansion

Example: given the set = \{1,1,2,2,3,3\}

\# of sets with 3 elements = coefficient of \( x^3 \) in the equation

\[
(1 + x + x^2)^3 = 1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6
\]

\( Sets = \{(1, 2, 3), (1,1, 2), (1,1, 3), (2, 2, 3), (2, 2,1), (3, 3,1), (3, 3, 2)\}\)
Background – Brute force search

- One cost function evaluation requires 0.2 seconds of CPU time
- The number of sensor sets containing 11 sensors (with up to 2 sensors of each type) is the coefficient of $x^{11}$ in $(1+x+x^2)^{11}$, which is equal to 25,653
  - $25,653 \times 0.2 \text{ s} = 86 \text{ minutes of CPU time}$
Genetic Algorithms – Selection

Roulette Wheel

Pointer