Title: A Disturbance Rejection Based Control System Design for Z-Axis Vibratory Rate Gyroscopes

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A Disturbance Rejection Based Control System Design for Z-Axis Vibratory Rate Gyroscopes

Qing Zheng and Lili Dong

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I. INTRODUCTION

MEMS gyroscopes are inertial rate sensors. Most MEMS gyroscopes are vibratory. They use vibrating elements to sense rotation rates and can be easily batch fabricated on crystal silicon or polysilicon [1]. A Z-axis MEMS gyroscope is sensitive to the rotation rate about the axis normal to the plane of silicon chip. The operating principle of the MEMS gyroscope is based on the energy transfer from driving mode (or drive axis) to sensing mode (or sense axis) of the MEMS gyroscopes caused by Coriolis acceleration, which is generated by vibrating a proof mass in a rotating reference frame. With the advancement of micromachining technology, MEMS gyroscopes have been applied to automobiles, consumer electronics, GPS assisted inertial navigation, and so on [2]. However, fabrication imperfections and environmental variations produce undesirable coupling terms and frequency mismatch between two axes, input and measurement noises, and parameter variations, which degrade the performance of the gyroscopes. Consequently, a control system is essential to improve the performance and stability of MEMS gyroscopes.

Since the 1990’s, there has been substantial research on the control designs of MEMS gyroscopes. Both open-loop and feedback control methods are applied to the sense axis. Since feedback control greatly increases the bandwidth of the gyroscope and is able to compensate for fabrication imperfections, it has replaced open loop control in most MEMS gyroscopes [3]. However, almost all of the reported closed-loop control approaches [3-7] are based on precise...
mathematical models of the gyroscopes and assume constant rotation rates. In reality, however, the rotation rate is time varying. The adaptive controller reported in [8, 9] is designed to approximate a time-varying rotation rate, but it is also a model-based controller, which is sensitive to all fabrication imperfections and parameter variations. In addition, the multiple tuning parameters in [8, 9] make it difficult to implement in real world situation. Dealing with such time-varying uncertain dynamics of MEMS gyroscopes makes the control problem challenging and critically important. Since the system dynamics are only partially known, a solution that is insensitive to the uncertainties in system dynamics and is able to accurately determine the rotation rate is needed.

In this paper, a new control solution known as active disturbance rejection control (ADRC) is applied to the sense axis of MEMS gyroscopes [10, 11]. ADRC as a practical design method has been successfully applied to motion control [12, 13], aircraft flight control [14], voltage regulation in DC-DC power converter [15], and other mechanical systems. The basic idea of this control strategy is to use an extended state observer (ESO) to estimate the generalized disturbance, which is the combination of the internal dynamics and external disturbances, in real time and dynamically compensate for it. The use of ADRC is extended to MEMS. The force-to-rebalance control scheme on the sense axis is chosen because of the general success of the nulling-the-output approach in a broad range of precise sensing applications [3]. Under this control scheme, the output of the sense axis is driven to zero by ADRC. In addition, the ESO provides an estimate of the generalized disturbance which includes the unknown time varying rotation rate and unknown Quadrature error terms arising from mechanical imperfections. With the accurate estimation of ESO, the time-varying rotation rate is obtained through demodulation.

This paper is organized as follows. The dynamics of MEMS gyroscope is described in Section II. The ADRC approach is presented in Section III and the rotation rate estimation law is developed in Section IV. The simulation results are shown in Section V and the robustness performance is analyzed in Section VI. Finally, some concluding remarks are given in Section VII.

II. THE DYNAMICS OF MEMS GYROSCOPES

The mechanical structure of a Z-axis MEMS gyroscope is shown in Fig. 1. It can be understood as a proof mass attached to the rigid frame through dampers and springs. As the rigid frame is subject to a rotation rate (Ω) about the rotation axis (Z axis), a Coriolis acceleration is produced along the sense axis (Y axis) while the proof mass is driven to resonance along the drive axis (X axis), which is perpendicular to both the sense and rotation axes. The Coriolis acceleration is proportional to both the applied rotation rate and the amplitude of the velocity of the moving mass along the drive axis. Therefore the rotation rate can be determined through sensing the vibration of the sense axis.

It is assumed there are no damping couplings and allow for the frequency mismatch between both axes. The governing equations of the Z-Axis MEMS gyroscope [4] are
\[
\begin{aligned}
\ddot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x + \omega_{s} y - 2\Omega y &= \frac{K}{m} u_d(t) \\
\ddot{y} + 2\zeta_s \omega_s \dot{y} + \omega_s^2 y + \omega_{xy} x + 2\Omega \dot{x} &= \frac{K}{m} u_s(t)
\end{aligned}
\]  
(1)

where \(x(t)\) and \(y(t)\) are the drive axis and sense axis outputs respectively, \(\omega_n\) and \(\omega_s\) are natural frequencies of drive and sense axes, \(\zeta\) and \(\zeta_s\) are damping coefficients, \(u_d\) and \(u_s\) are control inputs for the drive and sense axes, \(m\) is the proof mass, \(2\Omega \ddot{x}\) and \(2\Omega \ddot{y}\) are Coriolis accelerations, \(\Omega\) is an unknown time-varying rotation rate, \(\omega_{xy} y\) and \(\omega_{sx} x\) are constant unknown Quadrature error terms caused by stiffness couplings between two axes, and \(K\) is a constant that accounts for sensor, actuator, and amplifier gains. There are also mechanical-thermal noises added to the input of the sense axis produced by the dampers. The power spectrum density (PSD) of the noise is

\[
S_n(f) = 4K_B T R N^2 \ \text{s}
\]  
(2)

where \(K_B\) is a Boltzman constant \((K_B = 1.38 \times 10^{-23} \text{JK}^{-1})\), \(T\) is the absolute temperature of the resistor, and \(R\) is a term associated with damping coefficient \((R = 2\zeta \omega_s m)\). Since the PSD of the noise is constant at all frequencies, the mechanical-thermal noise can be modeled as a zero-mean white noise \([3]\). Because the PSD \(S_n(f) = \delta^2 = 4K_B T R\) according to \([16]\), the mechanical thermal noise is represented by \(d \sim (0, \delta^2) = (0, S_n)\).

Rotation sensing is achieved by forcing the drive axis into a fixed amplitude vibration at the resonant frequency, and measuring the displacement \(y(t)\) of the sense axis. The operation of a conventional Z-axis MEMS gyroscope, where the movement along the drive axis is much larger than the one along sense axis, is based on the following equations:

\[
\begin{aligned}
x &= A \cos(\omega t) \\
y &= \frac{k}{m} u_s - \omega_{xy} x - 2\Omega \dot{x}
\end{aligned}
\]  
(3)

where \(A\) is the amplitude of X-axis vibration, and \(\omega\) is the resonant frequency of the X-axis. The basic idea of force-to-rebalance control strategy is that, if the output amplitude of the sense axis is driven to zero by feedback control, and since \(\dot{y} \approx \dot{y} \approx y \approx 0\) at steady state, then (3) becomes

\[
\frac{1}{m} u_s \approx \omega_{xy} x + 2\Omega \dot{x}
\]  
(4)

at steady state. This implies that the control signal is a part of the measurement of the rotation rate \(\Omega\). Based on the fact that Coriolis acceleration and Quadrature error are 90 degree out-of-phase with respect to each other, the useful Coriolis acceleration signal can be separated from the undesirable Quadrature error term and the rotation rate can be determined. Therefore our control tasks are to force the output of the sense axis to zero and to estimate the rotation rate in the presence of noises.

### III. ACTIVE DISTURBANCE REJECTION CONTROL

Most existing control approaches for MEMS gyroscope require the accurate model information of the plant. However, in practice, the factors such as the mechanical-thermal noise, the measurement noise, the unknown time varying rotation rate, and the unknown Quadrature error terms, introduce modeling errors and structural uncertainties in the MEMS gyroscope. The mechanical imperfection and environmental variations also introduce the parameter variations to the gyroscope model. ADRC is a natural fit for the MEMS gyroscope control due to its inherent disturbance rejection
characteristics. The idea of ADRC is briefly introduced as follows.

The sense axis of MEMS gyroscope can be taken as a slightly damped second-order system. The practical system of the sense axis can be rewritten as

\[ \ddot{y} = -\left(2\zeta_y \omega_y \dot{y} + \omega_y^2 y + \omega_y x + 2\Omega \dot{x}\right) + b_y u \]  

(5)

where \( b_y = \frac{K}{m} \). Define

\[ f_y = -\left(2\zeta_y \omega_y \dot{y} + \omega_y^2 y + \omega_y x + 2\Omega \dot{x}\right) \]

(6)

where \( f_y \) is referred to as the generalized disturbance. Then the system (5) becomes

\[ \ddot{y} = f_y + b_y u \]

(7)

The basic idea of the ADRC is to obtain the estimated \( f_y \), i.e., \( \hat{f}_y \), and to compensate for it in the control law in real time without an explicit mathematical expression of it. The ESO for the estimation purpose is presented as follows.

Let \( \xi_y = y, \xi_{y1} = y, \xi_{y2} = f_y, \) and \( \xi_{y3} = [\xi_{y1}, \xi_{y2}, \xi_{y3}]^T \).

Assuming \( f_y \) is differentiable and the derivative of \( f_y \) \( (h_y = \dot{f}_y) \) is bounded, the state space form of (5) is

\[
\begin{align*}
\dot{\xi}_y &= A \xi_y + Bu_y + Eh_y \\
y &= C \xi_y
\end{align*}
\]

(8)

where

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b_y \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\]

An ESO for (8) is designed as

\[
\begin{align*}
\dot{\xi}_y &= A \xi_y + Bu_y + L_y \left( \xi_{y1} - \hat{\xi}_{y1} \right) \\
\hat{y} &= C \hat{\xi}_y
\end{align*}
\]

(9)

where \( L_y = \begin{bmatrix} l_{y1}, l_{y2}, l_{y3} \end{bmatrix}^T \) is the observer gain. The observer gains are chosen such that the characteristic polynomial \( s^3 + l_{y1}s^2 + l_{y2}s + l_{y3} \) is Hurwitz. For tuning simplicity, all the observer poles are placed at \(-\omega_{oy} \), which results in the characteristic polynomial of (9) to be

\[ \lambda_{oy}(s) = s^3 + l_{y1}s^2 + l_{y2}s + l_{y3} = (s + \omega_{oy})^3 \]

(10)

where \( \omega_{oy} \) is the observer bandwidth of the sense axis and \( l_{y1} = 3\omega_{oy}, l_{y2} = 3\omega_{oy}^2, l_{y3} = \omega_{oy}^3 \). This makes \( \omega_{oy} \) the only tuning parameter for the observer. Hence the implementation process of the observer is much simplified.

Once the observer is designed and well tuned, its outputs will track \( y, \dot{y}, f_y \) respectively. By canceling the effect of \( f_y \) using \( \hat{\xi}_{y3} \), ADRC actively compensates for \( f_y \) in real time.

The control law is designed as follows. First, the control law

\[ u_y = \frac{u_0 - \hat{\xi}_{y3}}{b_y} \]

(11)

approximately reduces the original plant (5) to

\[ \ddot{y} \approx u_y \]

(12)

which is a simple control problem to deal with. A simple PD controller

\[ u_0 = k_{y1} (r_y - \hat{\xi}_{y1}) - k_{y2} \hat{\xi}_{y2} \]

(13)

is usually sufficient, where \( r_y \) is the desired trajectory of the sense axis. The controller gains are selected so that the closed-
loop characteristic polynomial $s^2 + k_2 s + k_1$ is Hurwitz. For tuning simplicity, all the controller poles are placed at $-\omega_y$.

Then the approximate closed-loop characteristic polynomial is

$$\lambda_y(s) = s^2 + k_2 s + k_1 = \left(s + \omega_y\right)^2$$  \hspace{1cm} (14)

where $k_1 = \omega_y^2$, $k_2 = 2\omega_y$. This makes $\omega_y$, the controller bandwidth, the only tuning parameter for the controller of the sense axis. This process is called bandwidth parameterization [14], which greatly simplifies the ADRC tuning. The ESO (9) and the control law (11) and (13) constitute the force-to-rebalance control of the sense axis of the MEMS gyroscope.

IV. ROTATION RATE ESTIMATION

On the sense axis of the MEMS gyroscope, both Coriolis acceleration and Quadrature error terms are amplitude modulated signals centered at the resonant frequency of the drive axis. The only distinguishing characteristic between the two signals is that they have a relative phase shift of $90^\circ$. Therefore the advantage of this characteristic can be taken to separate the undesired Quadrature errors from the useful Coriolis acceleration through the demodulation technique.

The block diagram of ADRC for the sense axis control and rate estimation is shown in Fig. 2, where a demodulation block is used for estimating the rotation rate. In Fig. 2, $N_y$ represents the mechanical-thermal noise input to the sense axis and $N_m$ represents the measurement noise (position noise) at the output of the sense axis [7].

In (5), one gets $x = A\cos(\omega t)$ and $\dot{x} = -A\omega\sin(\omega t)$. From (6), it follows that

$$\omega_y x + 2\Omega \dot{x} = -\left(f_y + 2\zeta_y \omega_y \dot{y} + \omega_y^2 y\right).$$  \hspace{1cm} (15)

Let $q = \omega_y x + 2\Omega \dot{x}$. It is assumed that the rotation rate is a constant or low frequency signal. Then one has

$$q \cdot \sin(\omega t) = \left(\omega_y x + 2\Omega \dot{x}\right) \cdot \sin(\omega t)$$

$$= \frac{1}{2} \omega_y A \sin(2\omega t) - 2\Omega A\omega \sin^2(\omega t)$$

$$= \frac{1}{2} \omega_y A \sin(2\omega t) - \Omega A\omega \cos(2\omega t) - \Omega A\omega$$

where $\omega$ is much larger than the frequency of the rotation rate. In (16), the high frequency signals $\frac{1}{2} \omega_y A \sin(2\omega t)$ and $\Omega A\omega \cos(2\omega t)$ will be filtered out through a low pass filter (LPF). Therefore the rotation rate $\Omega$ can be demodulated from the signal $q$ by multiplying $\sin(\omega t)$, dividing by a gain introduced from modulation/demodulation, and filtering the resultant signal with a LPF, that is

$$\Omega = F_{LPF} \left(-\frac{q \cdot \sin(\omega t)}{A\omega}\right)$$  \hspace{1cm} (17)

where $F_{LPF} (\cdot)$ represents the function of the LPF. With the information of the ESO, according to (15), the signal $q$ can be estimated by

$$\dot{q} = -\left(f_y + 2\zeta_y \omega_y \dot{y} + \omega_y^2 y\right).$$  \hspace{1cm} (18)

Therefore the rotation rate can be determined by

$$\hat{\Omega} = F_{LPF} \left(-\frac{\dot{q} \cdot \sin(\omega t)}{A\omega}\right).$$  \hspace{1cm} (19)

The transfer function of the low-pass filter is chosen as

$$G_{LPF}(s) = \frac{1}{(\tau s + 1)^2}$$  \hspace{1cm} (20)
where $\tau$ is the time constant of the filter.

V. SIMULATION RESULTS

A control system based on ADRC is designed and simulated on a model of the Berkeley Z-axis vibratory gyroscope [17]. The key parameters are $K=0.8338$, $\omega_y=80864.6$ rad/sec, $\zeta_y=3.125\times10^{-4}$, $\omega_{xy}=6000$ rad/sec$^2$, $m=2\times10^{-9}$ kg and $x=A\cos(\omega t)$ where $\omega=84194.7$ rad/sec. Typically $A=10^{-6}$ m. $A=50$ in “simulation units” is used to represent this [3]. The design parameter $b_y=\frac{K}{m}=4.169\times10^8$. The reference signal of the sense axis is $r_y=0$. In the simulation, the mechanical-thermal noise and the measurement noise are applied to the sense axis. The PSD of mechanical-thermal noise for the sense axis is $1.63\times10^{-27} N^2$ sec. The PSD of measurement noise for the sense axis is $1.49\times10^{-27} N^2$ sec as reported in [7]. The controller and observer parameters for the sense axis are: $\omega_{sy}=5\times10^5$ rad/sec, $\omega_{sy}=2\times10^7$ rad/sec. The time constant of LPF is $\tau=6.7\times10^{-3}$ sec.

The output of the sense axis under the control of the ADRC is shown in Fig. 3. The stabilized output is around 0.01% of the uncontrolled amplitude of $y$, which shows that the sense axis is driven to almost zero. For rotation rate estimation, the following cases are considered: 1) $\Omega=0.1$; 2) $\Omega=0.1\sin(2\pi f_{rate}t)$, and $f_{rate}=50$Hz; 3) $\Omega=0.1\sin(2\pi f_{rate}t)$, and $f_{rate}=200$Hz. Without changing any tuning parameters, the rotation rate estimations for different cases are shown in Fig. 4 – Fig. 6, which show the fast and accurate rotation rate estimation.

To demonstrate the robustness of ADRC against parameter variations, the system parameters are changed as follows: the natural frequency of the sense axis $\omega_y$ is increased by 20% and the magnitude of the Quadrature error term is increased by 20%. With the changed plant parameters, the output of the sense axis and the rotation rate estimations for different cases are shown in Fig. 7 - Fig. 9 respectively. It can be observed that the sense axis output and rate estimations are almost the same as those without parameter variations.

VI. PERFORMANCE ANALYSIS

In this section, the stability robustness and the consistent input disturbance rejection of the ADRC against the system parameter uncertainties are analyzed through frequency response. Taking $-\left(\omega_{xy}x+2\Omega \dot{x}\right)$ as a part of the input disturbance $D_i(s)$, the block diagram of the sense axis control system can be derived in the form of a two-degree-of freedom (2DOF) closed-loop system [18], as shown in Fig. 10, where $G_p(s)$ represents the transfer function of the plant; $R(s)$, $U_i(s)$, and $Y(s)$ represent the reference signal, control signal, and output respectively.

The transfer function from the control input $u_s$ to the output $y$ is given by

$$G_p(s)=\frac{Y(s)}{U_i(s)}=\frac{K}{ms^2+2\zeta_y\omega_y s+\omega_y^2}.$$ (21)
Converting the ADRC equations into the transfer function form, one obtains
\[
G_c(s) = \frac{1}{b_y} \left( \frac{(k_p l_{y1} + k_d l_{y2} + l_{y3}) s^2 + (k_p l_{y2} + k_d l_{y3}) s + k_p l_{y3}}{s^3 + (l_{y1} + k_d) s^2 + (l_{y2} + k_p + k_d l_{y1}) s}ight) \tag{22}
\]
and
\[
G_f(s) = \frac{k_p (s^3 + l_{y1} s^2 + l_{y2} s + l_{y3})}{(k_p l_{y1} + k_d l_{y2} + l_{y3}) s^2 + (k_p l_{y2} + k_d l_{y3}) s + k_p l_{y3}} \tag{23}
\]
The loop gain transfer function \(G_p(s)\) and the closed-loop system transfer function \(G_{c}\) are
\[
G_{c}(s) = G_c(s)G_p(s) \tag{24}
\]
\[
G_{c}(s) = \frac{G_f(s)G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \tag{25}
\]
Furthermore, the transfer function from the input disturbance to the output is given by
\[
G_{ID}(s) = \frac{G_p(s)}{1 + G_c(s)G_p(s)} \tag{26}
\]
The loop gain Bode plots at different \(\omega_y\) are shown in Fig. 11. Fig. 11 demonstrates that the gain margin, phase margin, and crossover frequency are almost immune to changes in \(\omega_y\). Similar insensitivity to changes in \(\phi_y\) is performed. The plots are not shown here due to the space limitation. These show the strong robustness of ADRC against the variations of system parameters.

Now the input disturbance rejection performance is analyzed as follows. The transfer function from the input disturbance to the output is
\[
G_{ID}(s) = \frac{b_y s^3 + (l_{y1} + k_d) s + (l_{y2} + k_p + k_d l_{y1}) s}{s^3 + D_{d0} s^4 + D_{d3} s^3 + D_{d2} s^2 + D_{d1} s + D_{d0}} \tag{27}
\]
where
\[
D_{d0} = k_p l_{y3}, D_{d1} = \omega_y^2 (l_{y2} + k_p + k_d l_{y1}) + k_p l_{y2} + k_d l_{y3}, \]
\[
D_{d2} = \omega_y^2 (l_{y1} + k_d) + 2 \zeta_y \omega_y (l_{y2} + k_p + k_d l_{y1}) + k_p l_{y1} + k_d l_{y2} + l_{y3}, \]
\[
D_{d3} = \omega_y^2 + 2 \zeta_y \omega_y (l_{y1} + k_d) + l_{y2} + k_p + k_d l_{y1}, D_{d4} = 2 \zeta_y \omega_y + l_{y1} + k_d. \]
Equation (27) shows that \(G_{ID}(s)\) tends to zero when \(s\) approaches zero and infinity. To find out how it behaves in the middle range frequencies and how it is affected by parametric uncertainties, the Bode plots of (27) with different \(\omega_y\) values are shown in Fig. 12, demonstrating excellent disturbance rejection properties that are unaffected by the plant parametric uncertainties. Similar insensitivity to changes in \(\phi_y\) is performed.

VII. CONCLUDING REMARKS

A novel control approach of active disturbance rejection is used to control the sense axes of a Z-axis MEMS gyroscope. Based on the accurate estimation of the generalized disturbance by ESO, a demodulation technique is used to estimate the rotation rate. Since ADRC does not require an accurate mathematical model of the system, it is a good fit for the control and rate estimation of the MEMS gyroscope. The simulation results demonstrate the high tracking performance of the sense axis as well as the fast and accurate estimation of the rotation rates in the presence of noises and parameter variations. The performance analysis shows a remarkable level of consistency in stability margins against significant plant parameter variations. Similar consistency is also found in
disturbance rejection performance. Since most MEMS sensors have similar control problems to the MEMS gyroscope, i.e., amplitude regulation, disturbance rejection, and minimizing the effects of fabrication imperfections, ADRC provides a new effective solution to the problems.

REFERENCES


Dear Editor and Reviewers of the paper JSCE428:

With many thanks for your time and consideration, I would like to address the corrections into the previous paper submission. In the new version of the paper, we have considered many of the comments as following:

Response to Comments of Reviewer 1:
Regarding the reviewer’s comments on \(b_y\), it is assumed in the paper that \(b_y\), which is the ratio between the spring constant \(k\) and the mass \(m\) of the MEMS gyroscope, is a fixed known constant. Therefore, the robustness test on \(b_y\) is not included in the revised version.

Responses to Comments of Reviewer 2:
1. Regarding the reviewer’s comments on the mathematical representation of mechanical thermal noise, the terms used in the representation are explained at the end of the second paragraph of section II. Another reference paper is also added to the paper in this paragraph.
2. Regarding the reviewer’s comments on the parameter \(k\) of equation (3), the value of \(k\) is not one. There is a typo in the equation (3). The coefficient of \(u_s\) has been changed from \(\frac{1}{m}\) to \(\frac{k}{m}\).

In addition to the corrections above, we also made the following changes.
1. Rewrite the paper with the style in the third person;
2. Figures are separated from the main body of the text. All the figures are included into the file entitled “figures”;
3. A list of figure captions is provided and included into a separate file entitled “List of figure captions”;
4. A list of notation is provided and included into a separate file entitle “List of notation”;
5. The spacing of manuscript (including references) is changed to double line spacing across the page.
6. The issues of Editorial Checklist are addressed.

Regards,

Lili Dong
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Cleveland State University
Figures:

Fig. 1 The mechanical structure of a Z-axis MEMS gyroscope.

Fig. 2 Block diagram of the ADRC and rate estimation.

Fig. 3 The output of the sense axis.
The estimated $\Omega$

Fig. 4 The rotation rate estimation when $\Omega=0.1$.

The stabilized estimated $\Omega$

Fig. 5 The rotation rate estimation as $f_{\text{rate}}=50\text{Hz}$.

The estimated $\Omega$

Fig. 6 The rotation rate estimation as $f_{\text{rate}}=200\text{Hz}$.
Fig. 7 The output of the sense axis with parameter variations.

Fig. 8 The estimation of $\Omega = 0.1$ with parameter variations.

Fig. 9 The rotation rate estimation as $f_{rate} = 200$Hz with parameter variations.
Fig. 10 Block diagram of the sense axis control system.

Fig. 11 The loop gain Bode plots with different \( \omega_y \) values.

Fig. 12 The Bode plots of \( G_{ID}(s) \) with different \( \omega_y \) values.
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Fig. 8 The estimation of $\Omega (=0.1)$ with parameter variations.

Fig. 9 The rotation rate estimation as $f_{rate}=200\text{Hz}$ with parameter variations.

Fig. 10 Block diagram of the sense axis control system.

Fig. 11 The loop gain Bode plots with different $\omega_y$ values.

Fig. 12 The Bode plots of $G_{iDG}(s)$ with different $\omega_y$ values.

List of notation:

$b_y$: coefficient of the control law;
$f_y$: generalized disturbance;
$h_y$: derivative of generalized disturbance;
m: proof mass;
r_y: desired trajectory of the sense axis;
u_d: control input for the drive axis;
u_s: control input for the drive axis;
x(t): drive axis output;
y(t): sense axis output;
$K_B$: Boltzman constant;
$L_y=\begin{bmatrix}l_{y1}, l_{y2}, l_{y3}\end{bmatrix}^T$: observer gain;
$N_y$: the mechanical-thermal noise input to the sense axis;
$N_m$: the measurement noise at the output of the sense axis;
$R$: a term associated with damping coefficient;
$S_n$: power spectrum density (PSD) of the noise;
$T$: absolute temperature of the resistor;
$\omega_y$: controller bandwidth;
$\omega_d$: natural frequency of the drive axis;
$\omega_{oy}$: observer bandwidth of the sense axis;
$\omega_y$: natural frequency of the sense axis;
$\varsigma$: damping coefficient of the drive axis;
$\varsigma_y$: damping coefficient of the sense axis;
$\Omega$: rotation rate.