Continuous beams considered here are prismatic, rigidly connected to each beam segment and supported at various points along the beam. Joints are selected at points of support, at any free end, and changes in cross section occur at support points (i.e., the beam is prismatic).

A continuous beam having m members and m+1 joints is depicted in figure (a) to the left.

Support restraints of two types may exist at any joint in a continuous beam. These are restraints against rotation and/or restraints against translations.

We will only consider flexural deformations. Torsion and axial displacements are not considered. Thus only two displacements can occur at each joint.
Given the numbering system in figure (b) the translation at a particular joint is numbered prior to a rotation and it follows that the number of translations is equal to the number of joints minus one, while the rotation is twice the joint number. Thus at joint \( j \) the translations and rotation are number \( 2j-1 \) and \( 2j \) respectively.

It is evident that the total number of possible joint displacements is twice the joints (or \( 2n_j \)). If the total number of support restraints against translation and rotations is denoted \( n_r \), then the actual degrees of freedom are

\[
\begin{align*}
\text{n} &= 2n_j - n_r \\
&= 2m + 2 - n_r
\end{align*}
\]

Here \( n \) is the number of degrees of freedom.
To relate the end displacements of a particular member to the displacements of a joint, consider a typical member in figure (c) below. The member end displacements are numbered \( j_1, j_2, k_1 \) and \( k_3 \) and correspond to end displacements 1, 2, 3 and 4 in figure (b).

The new notation helps facilitate computer programming. The four end displacements correspond to the four joint displacements as follows:

\[
\begin{align*}
  j_1 &= 2j - 1 \\
  k_1 &= 2k - 1 \\
  j_2 &= 2j \\
  k_2 &= 2k
\end{align*}
\]

Since \( j \) and \( k \) are equal numerically to \( i \) and \((i+1)\), then:

\[
\begin{align*}
  j_1 &= 2i - 1 \\
  k_1 &= 2i + 1 \\
  j_2 &= 2i \\
  k_2 &= 2i + 2
\end{align*}
\]

This indexing system is necessary to construct the joint stiffness matrix \([S_j]\)
The analysis of continuous beams consists of establishing the stiffness matrix and the load matrix. The most important matrix generated is the overall joint stiffness matrix \([S_j]\). The joint stiffness matrix consists of contributions from the beam stiffness matrix \([S_m]\).

It is convenient to assess the contributions for one typical member \(i\) and repeat the process for members 1 through \(m\).

So the next step involves expressing the stiffness coefficients shown in the figure to the left in terms of the various member stiffnesses the contribute to the joint stiffnesses.
This next step requires that the member stiffnesses be obtained from the matrix below:

\[
S_M = \begin{bmatrix}
\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} & -\frac{12EI_z}{L^3} & \frac{6EI_z}{L^2} \\
\frac{6EI_z}{L^2} & \frac{4EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{2EI_z}{L} \\
-\frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} & \frac{12EI_z}{L^3} & -\frac{6EI_z}{L^2} \\
\frac{6EI_z}{L^2} & \frac{2EI_z}{L} & -\frac{6EI_z}{L^2} & \frac{4EI_z}{L}
\end{bmatrix}
\]

For example the contribution to the joint stiffness \((S_{j})_{j1,j1}\) from member \(i-1\) is the stiffness \(S_{m33}\) for that member. Similarly, the contribution to \((S_{j})_{j1,j1}\) from member \(i\) is the stiffness \(S_{m11}\) from member \(i\).
In general the contribution of one member to a particular joint stiffness will be denoted by appending the member subscript to the member stiffness itself. From this discussion one can see that the joint stiffness matrix coefficients are generated by the following expressions:

\[
(S_J)_{j1,j1} = (S_{M33})_{i-1} + (S_{M11})_i \\
(S_J)_{j2,j1} = (S_{M43})_{i-1} + (S_{M21})_i \\
(S_J)_{k1,j1} = (S_{M31})_i \\
(S_J)_{k2,j1} = (S_{M41})_i 
\]

which represent the transfer of elements of the first column of the member stiffness matrix \([S_m]\) to the appropriate location in the joint stiffness matrix \([S_j]\)
Expressions analogous to the previous expressions are easily obtained for a unit rotation about the $z$ axis at joint $j$:

\[
(S_J)_{j1,j2} = (S_{M34})_{i-1} + (S_{M12})_i \\
(S_J)_{j2,j2} = (S_{M44})_{i-1} + (S_{M22})_i \\
(S_J)_{k1,j2} = (S_{M32})_i \\
(S_J)_{k2,j2} = (S_{M42})_i
\]

Expressions analogous to a unit $y$ displacement at joint $k$ are:

\[
(S_J)_{j1,k2} = (S_{M13})_i \\
(S_J)_{j2,k1} = (S_{M23})_i \\
(S_J)_{k1,k1} = (S_{M33})_i + (S_{M11})_{i+1} \\
(S_J)_{k2,k1} = (S_{M43})_i + (S_{M21})_{i+1}
\]
Finally the expressions for a unit $z$ rotation at joint $k$ are:

\[
\begin{align*}
(S_J)_{j_1,k_2} &= (S_{M14})_i \\
(S_J)_{j_2,k_2} &= (S_{M24})_i \\
(S_J)_{k_1,k_1} &= (S_{M34})_i + (S_{M12})_{i+1} \\
(S_J)_{k_2,k_2} &= (S_{M44})_i + (S_{M22})_{i+1}
\end{align*}
\]

The last 4 sets of equations show that the sixteen elements of the 4x4 member stiffness matrix $[S_M]_i$ for member I contribute to the sixteen of the stiffness matrix $[S_J]$ coefficients in a very regular pattern. This pattern can be observed in the figure on the next overhead.
For this structure the number of joints is seven, the number of possible joint displacements is fourteen, and the joint stiffness matrix \([S_J]\) is dimensionally \(14 \times 14\).

The indexing scheme is shown down the left hand edge and across the top. The contributions of individual members are indicated in the hatched block., each of which is dimensionally \(4 \times 4\).

The blocks are numbered in the upper right corner to indicated the member associated with the block.

The overlapping blocks are dimensionally \(2 \times 2\) and denote elements that receive contributions from adjacent members.
Suppose that the actual beam has simple supports at all the joints as indicated in the figure below. The rearranged and partitioned joint stiffness matrix is shown at the lower right.

To obtain this rearranged matrix, rows and columns of the original matrix have been switched in proper sequence in order to place the stiffnesses pertaining to the actual degrees of freedom in the first seven rows and columns. As an aid in the rearranging process, the new row and column designations are listed in the previous figure for the matrix along the right hand side and across the bottom. The rearranging process is consistent with the numbering system in the figure above.
In summary, the procedure followed in generating the joint stiffness matrix \([S_J]\) consists of taking the members in sequence and evaluating their contributions one at a time. Then the stiffness matrix \([S_M]\)_i is generated, and the elements of this matrix are transferred to the \([S_J]\) as indicated in the previous overheads. After all members have been processed in this manner, the \([S_J]\) matrix is complete. This matrix can be rearranged and partitioned in order to isolate the \([S]\) matrix. The inverse of this matrix is then determined and the unknown displacements are computed.