REINFORCED MASONRY BEAMS

The following figure depicts several examples where reinforced masonry components are subjected to flexure.

(a) Masonry Beams andLintels

(b) Brick Masonry Beams and Lintels

(c) Concrete Masonry Beams and Lintels
When masonry beams are loaded, several different response regimes can be identified:

- Elastic stresses, section uncracked
- Elastic stresses, section cracked
- Inelastic stresses, section cracked

We study the first two cases in this section since ACI 530 covers these two situations in the working stress design (WSD) method. The third case is dealt with in limit state analysis.

These three regimes present themselves in the following moment-curvature graph:

For elastic stresses in uncracked sections, the section properties are determined by transforming steel into concrete through the ratio of the elastic moduli:

\[ n = \frac{E_s}{E_m} \]
The geometry and stress distributions as well as strain distributions are depicted below.

\[ \frac{nA_s}{2} \]

Note that when

\[ \frac{nA_s}{2} \]

is added to the sides of the uncracked section that the real holes must be “filled.” Thus the total amount of transformed area added to either side is

\[ \frac{nA_s}{2} - \frac{A_s}{2} = \frac{(n-1)A_s}{2} \]

Thus the moment of inertia of the uncracked transformed section is

\[ I_{TR} = \left( \frac{1}{12} \right) b h^3 + \frac{(n-1)A_s y_b^2}{2} \]

Where

\[ h = \text{full depth of the cross section} \]

The masonry bending stress is

\[ f_m = \frac{M_y}{I_{TR}} \]

And the stress in the steel is

\[ f_s = \eta \left( \frac{M y_b}{I_{TR}} \right) \]
Typically one assumes that the section is cracked and stresses are computed based on a cracked section geometry, which is depicted in the next figure.

NOTE THAT THE GEOMETRIC PARAMETERS K AND J DEFINED IN THE FIGURE ABOVE ARE USED FREQUENTLY IN ANALYSES.

The compressive resultant force is

\[ C = \left( \frac{f_m}{2} \right) (k_0 d) b \quad f_m < f_m' \]

The tensile resultant force is

\[ T = A_s f_s \]

\[ = \rho b d f_s \]

WHERE

\[ \rho = \text{STEEL RATIO} \]

\[ = \frac{A_s}{b d} \]

Based on equilibrium

\[ C = T \]

\[ \left( \frac{f_m}{2} \right) (k_0 d) b = \rho b d f_s \]

WITH

\[ f_m = \frac{E_m E_m}{E_s} \]

\[ f_s = \frac{E_s E_s}{E_s} \]
THEN

\[
\left( \frac{E_m E_n}{2} \right) K = \rho \left( \frac{E_E}{E_m} \right)
\]

\[
K = \frac{Z \rho}{E_m} \left( \frac{E_E}{E_m} \right)
\]

\[
= \frac{Z \rho n}{E_m} \left( \frac{E_S}{E_m} \right)
\]

BY SIMILAR TRIANGLES IN THE PREVIOUS FIGURE

\[
\frac{E_S}{E_m} = \frac{d - kd}{kd} = \frac{1 - k}{k}
\]

THUS FOR A CRACKED SECTION

\[
K = \frac{Z \rho n}{E_m} \left( \frac{1 - k}{k} \right)
\]

\[
K^2 + 2\rho n K - 2\rho n = 0
\]

USING THE QUADRATIC FORMULA TO FIND ROOTS, FIRST

\[
(2\rho n)^2 + 8\rho n > 0
\]

ALWAYS. THUS

\[
K = \frac{-2\rho n \pm \left[ 4\rho^2 n^2 + 8\rho n \right]^{1/2}}{2}
\]

\[
= -\rho n \pm \left[ \rho^2 n^2 + 2\rho n \right]^{1/2}
\]

SINCE K MUST ALWAYS BE POSITIVE

\[
K = \left[ (\rho n)^2 + 2\rho n \right]^{1/2} - \rho n
\]

NEXT SUM MOMENTS ABOUT THE TENSILE RESULTANT FORCE IN THE FOLLOWING FREE BODY DIAGRAM

\[
M
\]

\[
\rightarrow \quad t
\]

\[
\uparrow
\]

\[
j t d
\]
\[ \Sigma M_t = 0 \]
\[ = -M + C_{j'd} \]
\[ M = C_{j'd} \]

**Substituting for \( C \)**

\[ M = \left( \frac{f_m}{2} \right)(1 - \alpha) j' \alpha \]
\[ = \frac{f_m j' k b d^2}{2} \]

**OR**

\[ f_m = \frac{2M}{j' k b d^2} \]

**In a similar fashion, summing moments about \( C \)**

\[ \Sigma M_c = 0 \]
\[ = -M + T j' \alpha \]

**Substituting for \( T \)**

\[ M = \left( \rho b d f_s \right) j' \alpha \]
\[ = \rho f_s j' b d^2 \]

**OR**

\[ f_s = \frac{M}{\rho j' b d^2} \]

**Note that from the geometry of the cross section**

\[ j' \alpha = \alpha - \frac{K \alpha}{3} \]
\[ j' = 1 - \frac{K}{3} \]
**BALANCED DESIGN**

In AS50 the balanced condition occurs where both the steel stress and the masonry stress reach the specified allowable concurrently, i.e.,

\[ f_s = F_b \text{ (allowable steel stress)} \]
\[ f_m = F_b \text{ (allowable bending stress)} \]

Based on similar stress triangles depicted in the following figure:

\[ \frac{k_b \cdot d}{a} = \frac{F_b}{F_b + (F_b \cdot n)} \]

\[ k_b = \frac{F_b}{F_b + (F_b \cdot n)} \]

Based on equilibrium:

\[ C_b = T_b \]
\[ \frac{F_b \cdot k_b \cdot b \cdot d}{2} = \rho_b \cdot b \cdot d \cdot F_s \]
\[ \frac{F_b \cdot k_b}{2} = \rho_b \cdot F_s \]
\[ \left( \frac{F_b}{2} \right) \left[ \frac{F_b}{F_b + (F_b \cdot n)} \right] = \rho_b \cdot F_s \]
\[ \frac{1}{2} \left( \frac{F_0}{F_s} \right) \left\{ \frac{F_0}{nF_0 + F_s} \right\} = \rho_b \]

\[ \rho_b = \left( \frac{1}{2} \right) \left( \frac{F_0}{F_s} \right) \left[ \frac{nF_0}{nF_0 + F_s} \right] \]

\[ \rho_b = \left( \frac{1}{2} \right) \left( \frac{F_0}{F_s} \right) \left\{ \frac{nF_0}{F_0 \left[ n + (F_s/F_0) \right]} \right\} \]

\[ \rho_b = \frac{nF_0}{2F_s \left[ n + (F_s/F_0) \right]} \]

This steel ratio is used to indicate whether tension or compression controls design. Thus if

\[ \rho < \rho_b \]

then

\[ f_b \rightarrow F_s \]

first. If

\[ \rho > \rho_b \]

then

\[ f_m \rightarrow F_0 \]

first.
Consider the following arbitrarily loaded beam.

If we isolate a differential element on the neutral axis of the beam, we would see the following state of stress:

\[ \tau = \frac{VQ}{It} = \tau_{\text{max}} \]

Mohr's circle is as follows:

If we consider an element on the tensile (bottom) face, we would see the following state of stress:

\[ \sigma = \frac{Mc}{I} = \sigma_{\text{max}} \]

Mohr's circle for this stress state is located at the origin as above.
Since masonry units are weak in tension, shear cracks appear on the tensile side and turn 45° by the time they reach the neutral axis. In essence, they are not shear cracks in the sense that the material is shearing in two directions, instead they are tension cracks that are caused by beam shear.

If we isolate a section of the beam above the neutral axis,

\[ F \cdot dx \]

Applying equilibrium in the x-direction yields

\[ \Sigma F_x = 0 \]

\[ = C - (C + dC) + f_v b \quad dx \]

\[ dC = f_v b \quad dx \]

Recall that

\[ j = j(k) = j(p, n) \]

\[ M = c \cdot j \cdot d \]

\[ = j(A, b, d, E_m, E_s) \]

Since for a given cross section \( j \) and \( d \) are fixed, i.e., constants, then taking the differential on both sides of this expression yields

\[ dM = (j \cdot d) \cdot dC \]
Thus

\[ dC = \frac{dM}{j'd} \]

and

\[ \frac{dM}{j'd} = \int_0^b f_v b \, dx \]

\[ f_v = \left( \frac{1}{j'bd} \right) \left( \frac{dM}{dx} \right) \]

\[ = \frac{V}{j'bd} \]

Section 2.3.5.2.1 of ACI 530-11 allows the following approximate equation for use in computing shear stress in a reinforced masonry beam

\[ f_v = \frac{V}{bd} \]

The approximation is a result in dropping \( j' \) from the previous expression. As the student gains experience with the calculation of \( j' \) and \( k' \), he/she will find for most practical combinations of \( f_m' \) and masonry element dimensions that

\[ k' \approx \frac{3}{B} \]

\[ j' \approx \frac{2}{B} \]

Since \( j \) is approximately equal to one, the justification for dropping this parameter from the expression above becomes transparent.
Shear Reinforcement

Shear reinforcements (stirrups) are used to control diagonal cracking when the applied shear stresses exceed the allowances stipulated in ACI 530. Consider the following beam reinforced with vertical stirrups.

\[ S = \frac{V}{F_y} \]

Summing forces in the y-direction yields

\[ \sum F_y = 0 \]

\[ = V - \left( \frac{d}{S} \right) T_s \]

\[ = V - \left( \frac{d}{S} \right) F_s A_v \]

\[ A_v = \frac{V S}{F_s d} \quad \text{(For standard block, } S = 8", 12", 24") \]

The maximum spacing is limited to

\[ \frac{d}{2} < 48" \quad \text{(Section 2.3.5.3.1, ACI 530)} \]

So that each diagonal crack is crossed by a stirrup before it can propagate to fail into the beam.

This is why stirrups are designed for a value of \( V \) that is all from the face of the support \( (ACI 530, \text{ Section 2.3.5.5}) \).

Stirrups should be provided a distance of beyond where they are needed.
DESIGN A MASOYURI CINTEC TO SPAN OVER A WALL OPENING FOR THE FOLLOWING CONDITIONS

UNIFORM DEAD AND LIVE LOAD = 650 LBS/FT
BEAM WEIGHT = 87 LBS/FT² (NORMAL WEIGHT, 8" BLOCK)
CLEAR SPAN = 15' 8"

\[ b = 7-5/8'' \]

\[ f_m' = 1500 \text{ PSI} \]

GRADE 60 STEEL

\[ d = 29-5/8'' \]

Beam Section

CALCULATIONS

\[ \text{BEAM WEIGHT} = 87 \left( \frac{31.625}{12} \right) = 229.3 \text{ LBS/FT} \]

TOTAL LOAD = 229.3 + 650 = 879.3 LBS/FT

ASSUME SPAN IS SIMPLY SUPPORTED. BY SPEC 2.3.3.3 AND SPEC 1.13.1.2

\[ L = 15' 8'' + 2(2'') = 16' \]

\[ M = \frac{wL^2}{8} = \left( \frac{879.3}{1000 \text{ LBS/FT}} \right) \left( 16 \right)^2 = 337.6 \text{ k·ft} \]

THUS

FOR A BALANCED BEAM

\[ f_5 = F_5 \]

USING THE MOMENT EXPRESSION FOR A CYLINDRICAL CROSS SECTION

\[ f_s = F_s = \frac{M}{\rho J b d^2} \]
Thus

\[ F_s = \frac{M}{\rho j b d^2} \]

\[ \rho = \frac{M}{F_s j b d^2} \]

\[ \frac{A_3}{b d} = \frac{M}{F_s j b d^2} \]

\[ A_3 = \frac{M}{F_s j d} \]

With

\[ F_s = 24,000 \text{ PSI} \quad \text{(Spec. 2,3,2.1)} \]

\[ F_b = \left(\frac{1}{3}\right) f' m = \left(\frac{1}{3}\right) 1500 = 500 \text{ PSI} \quad \text{(Spec. 2,3,2.2)} \]

\[ n = \frac{F_2}{E m} = \frac{29,000,000}{900 (f'm)} = \frac{29,000,000}{900 (1500)} = 21.5 \quad \text{Spec. 1,8,2,2.1} \]

\[ K_b = \frac{F_b}{F_b + (F_s/n)} \]

\[ = \frac{500}{500 + \left(\frac{24,000}{21.5}\right)} \]

\[ = 0.309 \]

Thus

\[ j' = 1 - \frac{K_b}{3} = 1 - \frac{0.309}{3} = 0.897 \]

And

\[ (A_3)_b = \frac{337.6 \times (1000)}{(24,000) (0.897) (29,625)} \]

\[ = 0.529 \text{ in}^2 \]
USE 2 45 bars

\[ A_s = 2 \times (0.31) = 0.62 \text{ in}^2 \]
\[ > 0.529 \text{ in}^2 \]

The steel stress will be smaller than the allowable now.

\[ \rho = \frac{A_s}{bd} = \frac{0.62}{(7.625)(29.625)} = 0.0027 \]
\[ np = 2.5(0.0027) = 0.058 \]

And

\[ \kappa = \left[ (np)^2 + 2\rho n \right]^{\frac{1}{2}} - np \]
\[ = \left[ (0.058)^2 + 2(0.058) \right]^{\frac{1}{2}} - 0.058 \]
\[ = 0.29 \]

\[ j^* = 1 - \frac{\kappa}{3} = 1 - \frac{0.29}{3} = 0.90 \]

Now check masonry stresses

\[ f_m = \frac{2M}{j^* K d^2 b} \]
\[ = \frac{2(3.776)(1000)}{(0.90)(0.29)(29.625)^2(7.625)} \]
\[ = 386.6 \text{ PSI} \]
\[ < 500 \text{ PSI} \quad \text{OK} \]

Now locate the cracked section neutral axis

\[ kd = 0.29 \times (29.625) = 8.59" \]

And compute the moment of inertia

\[ I_{EL} = \frac{b(kd)^3}{3} + nA_s (d-kd)^2 \]
\[ I_{xx} = \frac{7.625 (8.59)^3}{3} + 21.5 (0.62) (29.625 - 8.59)^2 \]
\[ = 7512 \text{ in}^4 \]

Assume allowable deflection (Spec 1.13.3.1)
\[ D_{\text{allow}} = \frac{6}{600} = \frac{16 (12)}{600} = 0.32 \text{ in} \]

Now compute beam deflection
\[ D = \frac{5WL^4}{384EI} \]
\[ = \frac{5 (879.3) (12)^4 (12)^3}{384 (900) (1500) (7512)} \]
\[ = 0.13 \text{ in} \]
\[ < 0.32 \text{ OK} \]

Check shear stresses
\[ V = \frac{WL}{2} = \frac{879.3 (12)}{2} = 7034.4 \text{ lbs} \]
\[ f_v = \frac{V}{bd} = \frac{7034.4}{(7.625)(29.625)} = 31.14 \text{ psi} \]

Can check shear at a distance 0.12 from support (Spec 2.3.5.9.1)
Not necessary here since by Spec 2.3.5.2.2 (9)
\[ f_v = \sqrt{f_{im}} = (1500)^{1/2} \]
\[ = 38.7 \text{ psi} \quad (< 50 \text{ psi}) \]
\[ > 31.14 \text{ psi} \quad \text{OK} \]

And Spec 2.3.5.2.2 assumes shear "reinforcement is not provided."
For the beam depicted below select flexural reinforcement and shear reinforcement.

\[ W = \frac{1210 \text{ lbs/ft}}{\text{BEAM NOT INCLUDED}} \]

\[ f' = 1500 \text{ PSI} \quad \text{GRADE WO STEEL} \]

15' - 8"  \[ \text{Lintel Block} L = 2" \]

\[ \text{FACE SHELL} L = 1\frac{1}{4}" \]

\[ W_{\text{BEAM}} = \frac{87 (31.625)}{12} = 229.3 \text{ lbs/ft} \]

For flexural steel try 2\# B A365, assume 3 stirrups.

\[ d = 31.625 - 2 - 0.5 - 0.375 - 0.5 \]

\[ \text{LINTEL} \quad \# 3 \quad \# 3 \quad \text{SPEC 1.15.3.5} \]

\[ = 28.25" \]

Estimated steel ratio:

\[ \rho = \frac{A_s}{b_0d} = \frac{2(0.78)}{(7.625)(28.25)} = 0.0072 \]

Calculate \( n \):

\[ n = \frac{E_s}{E_m} = \frac{29,000,000}{900(1500)} = 21.5 \]

Calculate \( k \) and \( j' \):

\[ k = \left[ \left( \frac{\eta \rho}{2} \right)^2 + 2 \rho n \right]^{1/2} - \eta \rho \]

\[ = \left[ \left( \frac{(21.5)(0.0072)}{2} \right)^2 + 2(0.0072)(21.5) \right]^{1/2} - (21.5)(0.0072) \]

\[ = 0.4227 \]

\[ j' = 1 - \frac{k}{3} = 1 - \frac{0.4227}{3} = 0.8591 \]
Compute flexural stresses in the masonry and check

\[ f_b = \frac{2M}{J \cdot K \cdot d^2} = \frac{2(1210 + 229.3)(14)^2(12)}{8(0.8591)(0.4127)(18.25)^2(9.625)} \]

= 500 PSI

\[ = \left( \frac{1}{3} \right) f_m' \quad \text{OK} \]

Check deflections - homework

Compute steel stresses and check

\[ f_s = \frac{M}{\rho \cdot J \cdot b \cdot d} = \frac{(1210 + 229.3)(14)^2(12)}{8(0.0072)(0.8591)(9.625)(18.25)^2} \]

= 14,683 PSI

\[ < 29,000 \text{ PSI} \quad \text{OK} \quad (\text{SPEL 2.3.2.1}) \]

Check if stirrups are necessary

\[ f_u = \text{allowable shear stress} \]

\[ = (f_m')^{1/2} \]

\[ = (1500)^{1/2} \]

\[ = 38.7 \text{ PSI} \quad < 50 \text{ PSI} \quad (\text{SPEL 2.3.5.2.2}) \]

Compute applied shear stress

\[ f_u = \frac{V}{bd} = \frac{(1210 + 229.3)(14)}{2(7.625)(18.25)} \]

= 53.4 PSI

\[ > 38.7 \text{ PSI} \quad \text{NO GOOD} \]

Need stirrups. Try #3 and check beam width (SPEL 1.15.3.1)

Minimum beam width = 2(1.25) + 2(0.975) + 2(1.0) + 2(0.5) + 1.0

\[ \text{Face shell} \quad 3 \quad \text{SPEC} \quad \text{SPEC} \quad \text{SPEC} \quad 1,15.3,5 \quad 1,15.3,7 \]

= 7.25"  

\[ < 7.625" \quad \text{OK} \]
CALCULATE SHEAR AT

\[ \frac{d}{2} = \frac{28.25}{2} = 14.125'' \]

FROM Fact

\[ V = \frac{WL}{2} - \frac{wod}{2} = (1210 + 229.3) \left[ \frac{16}{2} - \frac{14.125}{2(12)} \right] \]

\[ = 10,667 \text{ LBS} \]

WHERE DOES THE SHEAR REINFORCEMENT STOP

\[ V^* = F_v b d \]

\[ = (1500)^{\frac{1}{2}} (7.625) (28.25) \]

\[ = 8343 \text{ LBS} \]

\[ d^* = \left( \frac{10,667 - 8343}{1210 + 229.3} \right) (12) \]

\[ = 19.379'' \]

CHECK BOL SPACING. LOOK AT INCREMENTS OF 8'', BUT CAN "START" AT UP TO 12'' BECAUSE

\[ 12'' < \frac{d}{2} = \frac{28.25}{2} = 14.125'' \]

FACE OF OPENING

CHECK IF #3 BARS ARE APPROPRIATE FOR STIRRUPS IF 12'' SPACING IS USED.

\[ A_v = \frac{V^*}{F_y d} = \frac{(10,667)(12)}{24 (1000)(28.25)} \]

\[ = 0.18 \text{ in}^2 \]

\[ < 2(0.11) = 0.22 \text{ in}^2 \]

#3 STIRRUPS OK

MUST USE 8'' STIRRUP SPACING AFTER THE INITIAL 12'' IF 16'' LONG BLOCK IS USED ABOVE THE BOND BEAM COURSE. #3 BARS ARE STILL APPROPRIATE FOR AN 8'' SPACING.
Finally check SPEC 2.3.5.3.2

$$A_b > \left( \frac{1}{3} \right) A_V$$

2 - #8 Bars > \left( \frac{1}{3} \right) 2 - #3 Stirrups

Although not specifically stated in the code or commentary, flexure steel can be used to satisfy this specification.
DEVELOPMENT LENGTH

Once again consider the following differential section through an arbitrarily loaded beam.

\[ M \rightarrow V \rightarrow M + dM \]
\[ T \rightarrow T + dT \]
\[ V + dV \]

Recall that
\[ M = T \frac{dV}{dL} \]

Taking the differential of both sides yields
\[ dM = (\frac{d}{dx}) \frac{dT}{dT} \]

Now consider a free body diagram of the representative element.

\[ T \rightarrow T + dT \]
\[ \frac{dT}{dx} \]

Define
\[ U = \text{Bond force per unit length} \]

From equilibrium
\[ \sum F_x = 0 \]
\[ = -T - Ud\xi + (T + dT) \]
\[ U \frac{d\xi}{dx} = dT \]
\[ = \frac{dM}{j \cdot d\xi} \]
Thus

\[ U = \left( \frac{1}{f_d} \right) \left( \frac{\partial M}{\partial x} \right) \]

\[ = \frac{V}{f_d} \]

Now define

\[ U = \text{FLEXURAL BOND STRESS} \]

\[ = \frac{U}{E_b} \]

where

\[ E_b = \text{PARAMETER OF REBAR} \]

Then

\[ U = \frac{V}{E_b f_d} \]

ACI 530 permits an allowable bond stress of 160 psi (Code Commentary, Section 2.1.9.2).

Now consider a section of rebar a distance \( l_d \) long, where one end of the rebar is stress free.

\[ \Sigma F_x = 0 \]

\[ = f_s A_s - U \pi d_b l_d \]

If we allow the steel to reach its yield value, and limit the bond stress to 160 ksi, then

\[ F_s A_s = (160) \pi d_b l_d \]
SOLVING FOR THE DEVELOPMENT LENGTH UNDER THESE CONDITIONS YIELD

\[ \lambda_d = \frac{F_3 A_5}{(140) \pi d_b} \]

\[ = \frac{F_3 \pi d_b^2}{4(140) \pi d_b} \]

\[ = (0.0015) d_b F_3 \]

THIRD IS EQUATION 2.11 IN SECTION 2.1.9.2 OF ALL 530, BUT IT IS ONLY GOOD FOR WIRE FABRIC. FOR DEVELOPMENT LENGTHS OF BARS, SEE SECTION 2.1.9.3

\[ \lambda_d = \frac{(0.13) d_b^2 f_y}{K (f_m')^{3/2}} \]

WHERE

- \( K \leq \) MASONRY COLUMNS
- \( \leq \) CORR. SPACING BETWEEN ADJACENT BARS
- \( \leq 5 d_b \)

- \( \gamma = 1.2 \quad ^{#3 \text{ THROUGH } #5} \)
- \( = 1.3 \quad ^{#6 \text{ THROUGH } #11} \)
- \( = 1.5 \quad ^{#12} \)

WHEN EPOXY COATED BARS ARE USED, THIS LENGTH IS INCREASED BY 50%.
DETERMINE THE EMERGMENT LENGTH FOR THE REBAR SHOWN IN THE FIGURE BELOW. THE REBAR IS A FOUNDATION DOWEL EMBEDDED VERTICALLY INTO THE FOUNDATION AND ONE OF ITS WALLS.

#4 REBAR
GRADE 60 STEEL
f'_m = 1500 PSI

\[ L_d = \frac{(0.13) d_b^2 f'_m}{K (f'_m)'^2} = \frac{(0.13)(0.5)^2(1500)}{5(0.5)(1500)^{0.5}} \]

\[ = 8.06" \text{ USE} \]

\[ > 12d_b = 12(0.5) = 6" \text{ SPEC 2.1.9.4.1.3} \]

THE EMERGMENT LENGTH IS SHOWN INTO THE FOUNDATION. A SIMILAR EMERGMENT LENGTH WOULD BE REQUIRED INTO THE WALL.
VARIOUS REQUIREMENTS FOR DEVELOPMENT LENGTH STIPULATED IN SPECIFICATION 2.1.9 ARE SUMMARIZED IN THE FOLLOWING FIGURE.

THIS FIGURE 2-1-8 IN THE CODE COMMENTARY.
The 8.06 inch embedment depth in the previous example is too large for the foundation pad. Use a standard hook and redesign the embedment depth.

\[ f' = 1500 \text{ psi} \]

#4 REBAR

GRADE 60 STEEL

8.06" SPEC 1.18.5 dictates what constitutes hook and by this SPEC

\[ L' = 12 \, d_b \]
\[ = 12 \times (0.5) \]
\[ = 6" \]

So \( L' \) is the actual physical dimension of the bent bar for it to be considered a hook.

SPEC 2.1.9.5.1 dictates what is obtained in the way of "embedment length" from the hook

\[ L_e = \text{equivalent embedment length of a hook} \]
\[ = (11.25) \, d_b \]
\[ = (11.25)(0.5) \]
\[ = 5.625 \text{ in} \]

By SPEC 2.1.9.3

\[ L_d = \frac{10.13 \, d_b^2 \, f_{y} \, \delta}{k \, V'_{mi}} \]

WITH a #4 BAR

\( \delta = 1 \)

AND

\[ k = 50d_b \]
\[ = 5(0.5) = 2.5" \]
\[ l_d = \frac{(0.13)(0.5)^2(24,000)(1)}{(2.5)(1500)^{1/2}} \]

\[ = 8.06\text{"} \quad \text{(AS BEFORE)} \quad \text{USE} \]

\[ > 12 \cdot \frac{1}{12} = 12(0.5) = 6\text{"} \quad \text{OK (SPEC 2.1.9.4.1.3)} \]

The embedment dimension from the wall/sub interface to the beginning of the hook is

\[ l_d' = l_d - l_e \]

\[ = 8.06 - 5.625 \]

\[ = 2.43\text{"} \]

Say 2\text{1/2}"
LOAD DISTRIBUTION ON LINTELS

Lintel beams (steel or masonry) are used to span wall openings in structures. These beams are subjected to two types of loads:

1. Distributed loads

2. Concentrated loads

Because of arching action, lintels may not carry the full load above the lintel. If arching action occurs, then only a triangular portion of the wall above the lintel would collapse if the lintel fails. In essence, the masonry forms an arch over the opening.

However, for arching action to take place, there must be sufficient enough masonry on either side of the arch to resist lateral thrusts. Alternatively, tension ties across the bottom of the opening will resist horizontal thrust provided that the ties extend beyond the edge of the wall opening and is sufficiently anchored in the wall.

Assuming arching action takes place, the lintel beam will see the following loads:

As long as the apex of the triangular area above the tie does not intersect an applied distributed load to the wall (e.g., floor load or a roof load from concrete planks), then the designer can use the triangular loading.
If the apex of the triangle lies above an applied distributed load,

\[ w = \text{wall load} \]

\[ \text{Roof or floor load, } P \]

\[ \text{Load from wall above} = wh \]

\[ \text{Load from self-weight, } w_a \]

Then the full distributed load is used to design the beam.

For concentrated loads the triangular loading distribution is slightly different.

\[ w_p = \frac{P}{a} \]

\[ a = 1.15h \]

Load from wall above

Load, \( w_p \), from concentrated load

Load from self-weight

There the distributed load from \( P \) is included if the two triangles defined in the figure above intersect
The Halfen concealed lintel system is designed to provide hidden supports for brickwork above openings, allowing the brickwork to remain the architectural focal point of the opening.

The concealed lintel system has three basic elements. (1) Spine is the load-bearing portion spanning between support piers. (2) Horseshoe plates straddle the spine at every third mortar joint. (3) Stitching rods pass through the slots in the horseshoe plates and through the mortar-filled cored holes in the bricks.

The simple system illustrated above is suitable for supporting almost any flat arch or jack arch. Curved or specially shaped spines can also be manufactured to support any of the other shapes of arches illustrated on this page, or to suit your specific design. Further details of the system are shown on the following pages.
Example

Design a single wythe reinforced clay brick lintel for the service entrance depicted below.

Due to the short distance between the service entrance opening and the first floor load line, everything above the opening will be made into a lintel beam.

Note that

\[ L = 6' - 8" = 72 + 8 = 80" \]

If we use 8" (wide) x 12" (long) solid shell hollow bricks (Jumbo)

Bricks spanning opening = \( \frac{80}{12} \rightarrow 7 \) blocks

\( L \) = 6'-8" + 16"

= 8.00'

To account for bearing

\( L \) = 6'-8" + 16"
## Hollow Brick Units

### a) Solid Shell Hollow Brick Units

### b) Double Shell Hollow Brick Units

### c) Cored Shell Hollow Brick Units

<table>
<thead>
<tr>
<th>Nominal width of units, in. (mm)</th>
<th>Minimum solid face shell thickness, in. (mm)</th>
<th>Minimum cored or double face shell thickness¹, in. (mm)</th>
<th>Minimum end shell or end web thickness², in. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 &amp; 4 (75 &amp; 100)</td>
<td>3/4 (19)</td>
<td>-</td>
<td>3/4 (19)</td>
</tr>
<tr>
<td>6 (150)</td>
<td>1 (25)</td>
<td>1 1/2 (38)</td>
<td>1 (25)</td>
</tr>
<tr>
<td>8 (200)</td>
<td>1 1/4 (32)</td>
<td>1 1/2 (38)</td>
<td>1 (25)</td>
</tr>
<tr>
<td>10 (250)</td>
<td>1 3/8 (35)</td>
<td>1 5/8 (41)</td>
<td>1 1/8 (30)</td>
</tr>
<tr>
<td>12 (300)</td>
<td>1 1/2 (38)</td>
<td>2 (50)</td>
<td>1 1/8 (30)</td>
</tr>
</tbody>
</table>
Compute Uniform Load Assuming a Fully Grouted Brick Bond Beam

\[ W = 2250 + 2.0(79) = 2408 \text{ kN} \]

Compute Moment from Uniform Load and the Triangular Loading from Masonry Arching

\[ \frac{W}{2} + \frac{79L^3}{24} \]

Triangular Load is Masonry Wall Load Above the First Floor

\[ 7 = \text{Wall Height} = 79 \text{ psf} \]

\[ \begin{align*}
M_{\text{max}} &= \frac{W(L)}{2} + \frac{79L^3}{24} \\
&= \frac{(2408)(8)^2(12)}{8(1000)} + \frac{(79)(8)^3(12)}{24(1000)} \\
&= 2311 + 202.2 \\
&= 2513 \text{ k-N m}
\end{align*} \]

Check for Stirrups

\[ V = \frac{W}{2} + \frac{79L^2}{B} = \frac{2408(8)}{2} + \frac{79(8)^2}{8} \]

\[ = 9632 + 632 = 10,264 \text{ kN} \]

Compute Cover Assuming No Stirrups. Assume 4" Bars for Flexure, Assume a 2" Thick Face Shell for Clay Bond Beam

\[ d = 24 - 2 - 0.5 - 0.75 = 21.125" \]

Compute Applied Shear Stress

\[ f_v = \frac{V}{bd} = \frac{10,264}{7.425(21.125)} = 63.72 \text{ psi} \]

\[ > 50 \text{ psi NO STIRRUPS} \]
RECOMPUTE COVER WITH 

\[ d = 24 - 2 - 0.5 - 0.375 - \frac{0.75}{2} = 20.75 \] " 

ESTIMATE STEEL RATIO 

\[ \rho = \frac{A_s}{b d} = \frac{2(0.44)}{(7.625)(20.75)} = 0.0056 \]

CALCULATE \( n \) 

\[ n = \frac{E_s}{E_m} = \frac{21,000,000}{700(2500)} = 16.57 \]

CALCULATE \( k \) AND \( j^' \)

\[ k = \left[ (n \rho)^2 + 2 \rho \right]^{1/2} - n \rho \]

\[ = \left[ (16.57)(0.0056)^2 + 2(0.0056)(16.57) \right]^{1/2} - 16.57(0.0056) \]

\[ = (0.0086 + 0.1856)^{1/2} - 0.0928 \]

\[ = 0.3479 \]

\[ j^' = 1 - \frac{k}{3} = 1 - \frac{0.3479}{3} = 0.884 \]

COMPUTE FLEXURAL STRESS IN THE MASONRY AND CHECK 

\[ \sigma_b = \frac{2M}{j^' K d^2} = \frac{2(251.3)(1000)}{(0.884)(0.3479)(20.75)^2} \]

\[ = 497.8 \text{ PSI} \]

\[ < (1/3) f_m = (1/3)(2500) = 833 \text{ PSI} \quad \text{OK} \]

COMPUTE STEEL STRESS AND CHECK 

\[ f_s = \frac{M}{\rho j^' d^2} = \frac{(251.3)(1000)}{(0.0056)(0.884)(7.625)(20.75)^2} \]

\[ = 15,462 \text{ PSI} \]

\[ < 24,000 \text{ PSI} \quad \text{OK} \]
CHECK BEAM WIDTH (SPEC 1.15.3) ASSUME COURSE GROUT

Minimum Width = 2(1.25) + 2(0.375) + 2(0.75) + 2(0.5) + 1.0
= 6.75"
< 7.625" OK

DETERMINE WIRE STIRRUPS END

\[ V^* = F \cdot b \cdot d \]
= (2500) \( \frac{3}{4} \) (7.625) (20.75)
= 7911 LBS

NEGLECT TRIANGULAR LOADING (CONSERVATIVE)

\[ d^* = \left( \frac{10,264 - 7911}{2408} \right) 12 = 11.72" \]

REINFORCE FIRST TWO CORES WITH #3 STIRRUPS, COMPUTE EMBEYMENT LENGTH FOR FLEXURE 12A5, BY SPEC 2.1.9.4.1.3

\[ d_A = d = 20.75" \]

\[ d_A = 12 (0.75) = 12 (0.75) = 9.0" \]
< 20.75" — USE

CHECK DEFLECTIONS, FOR TRIANGULAR LOADING, USE BEAM TABLES FROM STEEL CODE, WHERE

\[ \frac{W}{L} = \frac{9L^3}{24} \]
\[ W = \frac{3L^2}{4} \]

NEXT, COMPUTE CRACKED SECTION NEUTRAL AXIS

\[ k_d = (0.2479)(20.75) = 7.2189" \]
\[ \frac{160 \text{ psi}}{0.116 \text{ in}} \leq 625 \text{ psi} = 160 \text{ psi} \]

**Endurance Stress**: 
\[ f = 0.25 (2500) = 0.25 \times 0.25 \text{ in} \]

**Result**: 
\[ f = 1.25 \text{ in} \]

**Calculation**: 
\[ \frac{10 \text{ psi}}{0.25 \text{ in}} = 40 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.02 \text{ in}} = 100 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.06 \text{ in}} = 33 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.04 \text{ in}} = 50 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.03 \text{ in}} = 67 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.07 \text{ in}} = 29 \text{ psi} \]

**Calculation**: 
\[ \frac{2 \text{ psi}}{0.08 \text{ in}} = 25 \text{ psi} \]

**Conclusion**:

**Compute Actual Beam Stress**

**Conclusion**:

**Compute the Critical Cross Section**

**Conclusion**:

**Conclusion**:

**Conclusion**:

**Conclusion**:

**Conclusion**:

**Conclusion**:
COMPRESSION STEEL

Compression steel is used to increase the compressive resistance of the beam, reduce deflections, and minimize creep.

The total moment capacity for a doubly reinforced beam can be expressed as

\[ M = M_0 + M_2 \]

Where

\[ M_0 = \text{moment capacity for the beam under balanced conditions with no compression steel} \]

\[ M_2 = \text{additional moment capacity developed by } A_{s2} \text{ and } A_{s2} \]

Note that

\[ A_{sb} = \text{tensile reinforcement needed to develop } M_b \]

\[ A_s = \text{total tensile steel} \]

\[ A_{s'} = \text{total compression steel} \]

\[ A_{s2} = A_s - A_{sb} \]

Consider the following doubly reinforced beam section.

![Beam Cross-Section](image1)

![Stress Diagram](image2)

![Transformed Section](image3)
Based on similar triangles from the previous figure

\[ \frac{f_s'}{f_s} = \frac{k d - d'}{d - k d} \]

Solving this expression for \( f_s' \) yields

\[ f_s' = f_s \frac{d' \left( k - (d'/d) \right)}{d \left( 1 - k \right)} \]

\[ = f_s \left[ \frac{k - (d'/d)}{1 - k} \right] \]

Once again, based on similar triangles from the previous figure

\[ \frac{f_h}{f_s} = \frac{k d}{d - k d} \]

\[ = \frac{k}{1 - k} \]

Solving this expression for \( f_s \) yields

\[ f_s = f_h n \left( \frac{1 - k}{1 - k' \frac{d'}{d}} \right) \]

Substitution in the expression above for \( f_s' \) yields

\[ f_s' = n f_h \left( \frac{1 - k}{1 - k} \right) \left[ \frac{k - (d'/d)}{1 - k} \right] \]

\[ = n f_h \left[ \frac{k - (d'/d)}{k} \right] \]

Recalculating the free body diagram of the cross section

\[ \text{F.B.D.} \]

\[ \text{F.B.D.} \]

\[ \text{F.B.D.} \]
\[ \sum M_{rb} = 0 \]
\[ = C_m j_b d - M_b \]
\[ M_b = \left( \frac{1}{4B} \right) f_{rb} (r_b d) j_b d \]

Since this free body diagram represents balanced conditions,

\[ f_m = F_b \]

And

\[ M_b = \frac{F_b j_b l_b b d^2}{2} \]

\[ \ldots \ldots \ldots \ldots (3) \]

Also from F.B.D. #2

\[ \sum M_{cm} = 0 \]
\[ = T_{sb} j_b d - M_b \]
\[ M_b = (A_{sb} f_{sb}) j_b d \]

Again, for balanced conditions

\[ f_{sb} = F_s \]

And

\[ A_{sb} = \frac{M_b}{F_s j_b d} \]

\[ \ldots \ldots \ldots (4) \]

From F.B.D. #3

\[ \sum M_{c_b} = 0 \]
\[ = T_{sb} (d - d') - M_b \]
\[ M_b = (f_{sb} A_{sb}) (d - d') \]
\[ A_{sb} = \frac{M_b}{f_{sb} (d - d')} \]

\[ \ldots \ldots \ldots (5) \]
Also from F.B.D. #3

\[ \sum M_{T_{52}} = 0 \]
\[ = C_5 (d - d') - M_2 \]
\[ M_2 = \left[ \left( \frac{f_a'}{n} \right) (n - 1) A_{52}' \right] (d - d') \]

Solving for \( A_{52}' \) yields

\[ A_{52}' = \frac{M_2}{\left( \frac{f_a'}{n} \right) (n - 1)} \] \hspace{1cm} (6)

Given a applied moment (M) for a design problem, the procedure for determining \( A_3 \) and \( A_{52}' \) is as follows:

1. Assume a reasonable value for \( d' \)
2. Compute \( k_0 \) and \( J_0 \) (Knowing \( f_0 \) & \( F_0 \))
3. Compute \( M_3 \) from equation (3)
4. Compute \( A_{52}' \) from equation (4)
5. Compute \( M_2 \)

\[ M_2 = M - M_3 \]
6. Compute \( A_{52} \) from equation (5), assume

\[ f_{52} = F_0 \]

In making this calculation, this limits the steel stress associated with \( A_{52} \). Must also assume a reasonable value for \( d' \) in order to make this calculation.

7. Compute \( f_a' \) from equation (1) where steel stresses control and equation (2) where masonry stresses control. Adopt the smaller value.
8. Compute \( A_s \) from Equation (6)

9. Revise \( c \) and \( c' \) after selecting appropriate amounts of steel

10. Compute \( k \) and \( j \) based on selected amounts of steel and appropriate values of \( c \) and \( c' \)

11. Check revised values of \( f_b \), \( f_s \) and \( f_s' \) against allowable.

In order to execute Step 10, a "\( k \)" expression must be developed that includes both tension and compression steel, from force equilibrium:

\[ C_s + C_m = T_s \]

Define:

\[ \rho' = \frac{A_s'}{b'd} \]

Then, from the first figure:

\[ \left( \frac{f_s'}{n} \right) (n-1) A_s' + \left( \frac{1}{2} \right) b (kd) f_b = A_s f_s \]

\[ \left( \frac{f_s'}{n} \right) (n-1) \rho' b'd + \left( \frac{1}{2} \right) b (kd) f_b = \rho b'd f_s \]

\[ \left( \frac{f_s'}{n} \right) (n-1) \rho' + \left( \frac{1}{2} \right) k f_b = \rho f_s \]

\[ n f_b \left[ \frac{k - (d'd'd)}{k} \right] \left( \frac{n-1}{n} \right) \rho' + \left( \frac{1}{2} \right) k f_b = \rho f_b \eta \left( \frac{1-k}{k} \right) \]

\[ \left[ \frac{k - (d'd'd)}{k} \right] (n-1) \rho' + \left( \frac{1}{2} \right) k = \rho \eta \left( \frac{1-k}{k} \right) \]

\[ 2 \left[ k - (d'd'd) \right] (n-1) \rho' + \eta^2 = 2 \rho \eta \left( 1-k \right) \]

\[ \frac{k^2}{2} + k \left[ (n-1) \rho' + \rho \eta \right] - \left[ \left( \frac{f_s'}{n} \right) (n-1) \rho' + \rho \eta \right] = 0 \]
This expression is quadratic in \( \kappa \), with

\[ a = 1/2 \]

\[ b = n \rho + (n-1) \rho' \]

\[ c = - \left[ \left( \frac{d''}{d} \right)(n-1) \rho' + \rho n \right] \]

Then

\[ \kappa = \left\{ \left[ n \rho + (n-1) \rho' \right]^2 + 2 \left[ \left( \frac{d''}{d} \right)(n-1) \rho' + \rho n \right] \right\}^{1/2} - \left[ n \rho + (n-1) \rho' \right] \]
This page depicts the plan and the following page depicts all four elevations for a single story 10,000 ft² shopping center.
Consider the concrete masonry unit at a width of 10 x 10 opening in the wall depicted below. Design the unit with compression steel.

\[ F' = 1500 \text{ PSI} \sqrt{w} \]

grade 60 rebar

8" CAN BLOCK - NORMAL WEIGHT

Load from \( W = 16 \times 31 = 15.72 \text{ klf} \) (centered on wall opening A)

Check weight of the triangular loading at apex

\[ h_A = 14' 7\frac{3}{8}'' - 10'' = 4' 7\frac{3}{8}'' \]

\[ < \frac{(12)(10)}{2} = 5' \]

Cannot use triangular loading. Compute uniform line load from the width 31.

\[ w_p = \frac{P}{Q} = \frac{P}{(1.15)h_A} = \frac{15.720}{(1.15)(4.615)} = 2972 \text{ pcf} \]

Compute uniform wall load assuming wall is fully grouted

\[ w_w = (87 \text{ pcf})(19 - 10) = 783 \text{ pcf} \]

Note that this is a conservative estimate. Can be refined.
AT THIS POINT WE WILL ASSUME THAT THE LINTEL IS CONSTRUCTED FROM FOUR COURSES OF BLOCK

\[ h_2 = 4(8) = 32'' \]

THIS WILL BE COMMENTED ON LATER IN THE PROBLEM. COMPUTE COUL BENDING STRESS ASSUMING 6 BARS FOR FLEXURE. ASSUME BOTTOM FACE SHELL OF LINTEL BLOCK IS 2'' THICK.

\[ d_1 = 32 - 2 - 0.5 - \frac{3}{8} - 0.75/2 \]
\[ = 28.75'' \]

COMPUTE

\[ \eta = \frac{E_s}{E_m} = \frac{29,000,000}{900(1500)} = 21.48 \]

DETERMINE ALLOWABLE MASONRY BENDING STRESS

\[ F_b = \left( \frac{1}{3} \right) f_m' = \left( \frac{1}{3} \right)(1500) = 500 \text{ PSI} \]

DETERMINE ALLOWABLE TENSILE STEEL STRESS

\[ F_s = 24,000 \text{ PSI (Grade 60)} \]
Determine lintel span length

\[ L = 10' + 2(2")/12 \]
\[ = 10.33' \]

Compute max moment

\[ \begin{align*}
5.17 - 2.655 & = 2.51' = 30.12'' \\
\frac{5.17'}{2} & = 10.33''/2 \\
R_L & = 783(5.17) + 2.655(2972) \\
& = 4048 + 7891 = 11,939 \text{ lbs} \\
M & = \frac{-783(5.17)^2}{2} - \frac{2972(2.655)^2}{2} + (11,939)(5.17) \\
& = -10,464 - 10,475 + 61,725 \\
& = 40,786 \text{ lbs-ft} \\
& = 489,432 \text{ lbs-in} \\
& = 489 k-in
\]
Determine Moment Capacity of Beam Based on Balanced Conditions

\[
K_b = \frac{M}{n + \frac{F_3}{F_b}} = \frac{21.48}{21.48 + \frac{24}{0.5}} = 0.309
\]

\[
J_b = 1 - \frac{K_b}{3} = 1 - \frac{0.309}{3} = 0.897
\]

\[
M_b = \frac{F_3 K_b J_b}{2} d'^2 = \frac{500 (0.309) (0.897) (7.62)}{1000 (2)}
\]

\[= 436.2 \text{ k-in} \]

\[= 36.39 \text{ k-ft} \]

Compute Additional Moment Capacity

\[
M_2 = M - M_b
\]

\[= 40.79 - 36.39
\]

\[= 4.4 \text{ k-ft} \]

Design beam with compression steel and assume

\[d'^2 = 2'' \]

Compute Balanced steel Area

\[
A_{sb} = \frac{M_b}{F_3 J_b d} = \frac{12(36.39)}{24 (0.897) (28.75)} = 0.71 \text{ in}^2
\]

Compute required Additional Tensile steel

\[
A_{s2} = \frac{M_2}{F_3 (d - d')} = \frac{12(4.4)}{24 (28.75 - 2.0)} = 0.08 \text{ in}^2
\]

Thus total tensile steel area required is

\[
A_s = A_{sb} + A_{s2} = 0.71 + 0.08 = 0.79 \text{ in}^2
\]
USE TWO #6 BARS

\[ 2 \times 0.44 = 0.88 \text{ in}^2 \]
\[ > 0.79 \text{ in}^2 \ \ \ \text{OK} \]

NEXT, DETERMINE COMPRESSION STEEL AREA, INITIALLY TALCE

\[ K = K_0 = 0.309 \]

COMPUTE THE STEEL STRESS FROM

\[ f_s' = n \cdot f_0 \left( \frac{Kd - d'}{Kd} \right) \]
\[ = 21.48 \text{ (ksi)} \left[ \frac{0.309 (28.75) - 2}{(0.309) (28.75)} \right] \]
\[ = 8322 \text{ psi} \]
\[ < f_s = 24,000 \text{ psi} \]

OR USE

\[ f_s' = f_s \left[ \frac{K - d'd}{1 - \kappa} \right] \]
\[ = 24,000 \left[ \frac{0.309 - (2/28.75)}{1 - 0.309} \right] \]
\[ = 8316 \text{ psi} \]
\[ < f_s = 24,000 \text{ psi} \]

USE SMALLER \( f_s' \) TO COMPUTE \( A_s' \)

\[ A_s' = \frac{M_{pl}}{f_s' (d - d') (n - 1)} = \frac{4.4 (12) (1000)}{8316 (28.75 - 2) (21.48 - 1)} \]
\[ = 0.25 \text{ in}^2 \]
USE 2 4 X 8 BARS

\[ 2(0.40) = 0.8 \text{ in}^2 \]
\[ > 0.25 \text{ in}^2 \text{ OK} \]

REVIEW KING AND CHECK THE CAPACITY OF THE SECTION

\[ p = \frac{A}{bd} = \frac{0.80}{7.625(28.75)} = 0.004 \]
\[ p' = \frac{A_s'}{bd} = \frac{0.40}{7.625(28.75)} = 0.0018 \]

COMPUTE \( K \) FROM

\[ K = \left\{ \left[ n p + (n-1) p' \right]^2 + 2 \left[ \frac{A'}{d} (n-1) p' + pn \right] \right\}^{\frac{1}{2}} - \left[ n p + (n-1) p' \right] \]

FROM SPREAD SHEET

\[ K = 0.315 \]

COMPUTE THE STEEL STRESS ASSUMING MASONRY STRESSES CONTROL

\[ f_s' = n F_0 \left[ \frac{kd - d'}{kd} \right] \]
\[ = 21.48 (500) \left[ \frac{0.315(28.75) - 2}{0.315(28.75)} \right] \]
\[ = 8,360 \text{ psi} \text{ USE} \]
\[ < F_s = 24,000 \text{ psi} \text{ OK} \]

ALSO CHECK ASSUMING STEEL STRESSES CONTROL

\[ f_s' = F_s \left[ \frac{K - (d'/d)}{1-K} \right] \]
\[ = 24,000 \left[ \frac{0.315 - (2/28.75)}{1 - 0.315} \right] \]
\[ = 8,599 \text{ psi} \]
\[ < F_s = 24,000 \text{ psi} \text{ STILL OK} \]
Check Tensile Steel Stress. From Equation #1

\[ f_s = \frac{(1-k)f_s'}{k - \frac{a'}{a}} \]

\[ = \frac{(1 - 0.315)(8368)}{0.315 - \frac{2}{28.75}} \]

\[ = 23,354 \text{ psi} \]

\[ < 24,000 \text{ psi} \ \text{OK} \]

Check Masonry Stress. From Equation #2

\[ f_b = \frac{k f_s'}{\eta \left[k - \frac{a'}{a}\right]} \]

\[ = \frac{(0.315)(8368)}{21.48 \left[0.315 - \frac{2}{28.75}\right]} \]

\[ = 500 \text{ psi} \]

\[ \leq f_{\text{m}} = 500 \text{ psi} \ \text{OK} \]

With

\[ M_2 = \left[\frac{f_s'}{n}\right](n-1)A_s'(d-a') \]

\[ = \left(\frac{8368}{21.48}\right)(21.48 - 1)(0.40)(28.75 - 2) \]

\[ = 85,369.2 \text{ lbs.-in} \]

\[ = 85.4 \text{ k-in} \]
\[ M = M_2 + M_0 = 85.4 + 436.7 = 522 \text{ k-in} \]
\[ = 43.5 \text{ k-ft} \]
\[ > 40.8 \text{ k-ft} = M_{\text{app}} \quad \text{OK} \]
For the last example problem, check deflections

\[ \kappa = 0.315 \]

\[ f_m' = 1500 \text{ PSI} \]

\[ \eta = 21.48 \]

Compute the cracked moment of inertia

\[ I_{cr} = \frac{6 (kd)^3}{3} + n A_3 (d - kd) + (n-1) A'_3 (kd - d')^2 \]

\[ = \frac{7.625 (0.315)(28.75)^3}{3} + 21.48 (2)(0.44)(28.75 - 0.315(28.75))^2 \]

\[ + (21.48-1)(2)(0.20)(0.315)(28.75) - 2 \]

\[ = 247.58 + 7,331.18 + 209.43 \]

\[ = 7,888.19 \text{ in}^4 \]

Estimate deflection as follows

\[ \Delta = \frac{5nL^4}{384EI} + \frac{P L^2}{48EI} \]

\[ = \frac{5 (788)(10.32)(12)^4}{384 \times 1000 \times 1500 \times 788.19} + \frac{15,720 [(10.32)(12)]^2}{48 \times 1000 \times 1500 \times 788.19} \]

\[ = 0.229 + 0.06 \]

\[ = 0.289 \text{ in} \]
By SPEC 1.10

\[ \Delta_{\text{allow}} = \frac{L}{600} \]

\[ = \frac{10.33(12)}{600} \]

\[ = 0.207 \text{ in} \]

\[ < 0.289 \text{ in} \quad \text{No Good} \]

Recompute the uniform load from the weight of the lintel and check deflections. If this does not work, increase the cracked moment of inertia by increasing the grout by one more course, alternatively increase steel.
Toward a Unified Strength Design of Reinforced Masonry

A proposal for simplifying and unifying the current strength design provisions for reinforced masonry given in the 2000 International Building Code

BY JAVEED A. MUNSHI

With recent advances in state-of-the-art masonry research and practice, the design of reinforced masonry seems to be following in the footsteps of design developments in reinforced and prestressed concrete as given in ACI 318. This trend is clear in the new strength design provisions of the International Building Code (IBC), which were essentially based on the strength provisions introduced in the Uniform Building Code (UBC), and also in the efforts of the Masonry Standards Joint Committee (MSJC) to add strength design provisions to the MSJC masonry code. With reinforced and prestressed concrete moving toward a unified design approach, it seems logical for the design of masonry to follow a similar path.

Background information

The design of reinforced and prestressed concrete per ACI 318, having evolved through many changes, has reached a point where it is increasingly believed that the procedures could be simplified and unified. The design methods for beams, columns, and other sections of reinforced concrete and prestressed concrete were developed and modified independently at different times through the history of the code development. This has created a lack of uniformity in the current concrete code provisions. In order to simplify and unify concrete design provisions, the 1995 edition of ACI 318 included new design provisions in Appendix B of the Code, which have become known as the Unified Design Provisions. These provisions are being moved into the main body of ACI 318 in the 2002 code cycle. The masonry industry should benefit from the developments that are taking place in reinforced concrete design and move toward a similar unified design approach. The need for unification became even more important with the inclusion of prestressed masonry in the 1999 edition of the MSJC code. Separate provisions for nonprestressed and prestressed sections create a situation of nonuniformity similar to the one that exists in ACI 318-99. A unified approach can handle all these design situations and remove many of the inconsistencies that currently exist in masonry code provisions.
The current situation

The strength design provisions of the IBC presumably present the state of the art in reinforced-masonry design. In this Code, separate design provisions are given for beams, columns, and piers; out-of-plane walls; in-plane walls; and elements of wall-frames. I will highlight some of the discrepancies and nonuniformities in these provisions.

The design assumptions for the strength provisions of masonry in the IBC, as stated in section 2108.9.1, are similar to those for reinforced concrete (RC). The masonry material is assumed to be homogeneous, and the principles of equilibrium and compatibility are deemed applicable. As in RC design, the strain in the steel is assumed to be directly proportional to the distance from the neutral axis. Two different maximum usable strain limits of 0.0025 and 0.0035, however, are specified for concrete and clay masonry, respectively. This differs from the single value of 0.003 used for reinforced concrete.

As in reinforced concrete, the masonry compression stress of $0.85f_m'$ is assumed to be uniformly distributed over an equivalent compression zone of depth $c = \beta_f c = 0.85c$. You will note that a constant value of $\beta_f = 0.85$ is used in masonry. In RC design, $\beta_f$ is a function of concrete strength $f'_c$, and varies from 0.85 to 0.60 for $f'_c$ between 4 ksi (27.6 MPa) and 8 ksi (55.2 MPa). Figure 1 shows the stress block and strain idealization of a typical rectangular masonry section.

Section 2108.9.2.13 of the 2000 IBC, which states maximum reinforcement limits, places a very stringent limit on the reinforcement ratio for masonry elements based on a critical strain condition. The critical strain condition corresponds to a specific magnitude of strain in the tension steel when the masonry reaches its usable compressive strain limit. For beams, columns, and walls subjected to in-plane forces, the maximum reinforcement ratio corresponds to a strain in the extreme tension reinforcement equal to 5 times the yield strain of steel when the masonry reaches its usable compressive strain. Because concrete and clay masonry have different usable strain limits in compression, this results in two different reinforcement ratios. The strain diagrams corresponding to these maximum reinforcement limits for concrete and clay masonry are shown in Fig. 2a. For walls subjected to out-of-plane forces, the maximum reinforcement limit corresponds to a strain of 1.3 times the yield strain in the tension reinforcement. Figure 2b shows the strain diagram corresponding to the maximum reinforcement ratio for concrete and clay masonry for out-of-plane walls.

To further complicate and limit the maximum reinforcement ratio, the stress in the tension reinforcement is assumed to be 1.25 times the yield stress of the steel reinforcing, $f_y$, and the strength of compression zone is taken as 80% of $f'_m$ times 80% of the area of compression zone, as shown in Fig. 2. This requirement is inconsistent with the design assumptions stated in Section 2108.9.1 of the IBC and can be a potential source of confusion. Figure 3 shows the reinforcement limits based on the above criteria for flexural members. The restrictions are likely to result in even more stringent maximum reinforcement ratios for sections in compression.

Design of beams, columns, and piers

Table 1 gives a summary comparison of various requirements for beams, columns, and piers, as stated in section 2108.9.3 of the 2000 IBC. There are different requirements for dimensions, axial load, longitudinal reinforcement, and transverse reinforcement for these types of members. Beams have to be a minimum of 6 x 8 in. (152 x 203 mm), columns 12 x 12 in. (305 x 305 mm), and piers 6 in. (152 mm). The maximum axial load on beams is limited to $0.05A_f f_m$. 

Fig. 1: Stress block idealization and usable strain limits for concrete and clay masonry

Fig. 2a: Strain conditions for reinforcement limits for beams, columns, and in-plane walls
while for piers it is limited to $0.3A_{f_{m}}$. There is no axial load limit on columns. For beams, the minimum longitudinal reinforcement is limited such that $M_{r}$ of the beam is at least $1.3M_{c}$. There is no such requirement for columns or piers. Columns, however, must have a minimum area of steel of $0.005A_{s}$. For columns, there is an additional requirement that the maximum steel area cannot exceed $0.03A_{s}$. This requirement is in addition to the maximum reinforcement limit specified in Section 2108.9.2.13 of the IBC. For piers, the minimum reinforcement ratio is neither in terms of $M_{r}$ nor $A_{s}$ but directly required as 0.0007. These variations in dimensions, axial load, and longitudinal reinforcement complicate the design process. The only thing uniform in the design of beams, columns, and piers is the strength reduction factor $\phi$ (see Table 1).

**Design of walls**

Table 2 summarizes the requirements for in-plane and out-of-plane walls, as stated in sections 2108.9.4-5 of the 2000 IBC. By comparing the requirements for walls, as shown in Table 2, with those for beams, columns, and piers, as shown in Table 1, you can see the nonuniformity of the provisions in dimension requirements, axial load, $\phi$-factor, and minimum and maximum longitudinal reinforcement. There are no thickness requirements for walls, while beams, columns, and piers do have dimensional requirements. The axial load limit is not given in terms of $A_{s}$, as it was with beams and piers. Also, the lower limit of the strength reduction factor for walls is 0.65, as opposed to 0.60 for beams, piers, and columns, as shown in Table 1. The longitudinal reinforcement limits for in-plane walls are also different from those listed for beams, columns, and piers in Table 1.

**Design of wall-frames**

Wall-frames require special reinforcement detailing, both in the members and connections, to provide resistance to both lateral and gravity load, much like a moment-resisting RC frame. Section 2108.9.6 of the 2000 IBC has separate provisions for beams and columns of

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**TABLE 1: REQUIREMENTS FOR BEAMS, COLUMNS, AND PIERS (2108.9.3, IBC)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Beams</th>
<th>Columns</th>
<th>Piers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension requirement</td>
<td>B ≥ 6 in. (152 mm) D ≥ 8 in. (203 mm) L ≥ 0.32B</td>
<td>B ≥ 12 in. (305 mm) D ≥ 12 in. (305 mm) L ≥ 0.30B</td>
<td>6 in. (152 mm) B ≥ 16 in. (406 mm) 3B ≤ D ≤ 6B L ≥ 30B L ≥ 5D</td>
</tr>
<tr>
<td>Axial load limit</td>
<td>$P_{u} = 0.05A_{f_{m}}$</td>
<td>None</td>
<td>$P_{u} = 0.3A_{f_{m}}$</td>
</tr>
<tr>
<td>$\phi$-factor</td>
<td>$\phi = 0.8 - P/A_{f_{m}}$</td>
<td>$\phi = 0.8 - P/A_{f_{m}}$</td>
<td>$\phi = 0.8 - P/A_{f_{m}}$</td>
</tr>
<tr>
<td>Longitudinal reinforcement</td>
<td>Max. (2108.9.2.13) Min. $M_{r} / 1.3M_{u}$</td>
<td>Max. (2108.9.2.13) Max. $A_{s} / 0.03A_{s}$ Min. $A_{s} / 0.005A_{s}$</td>
<td>Max. (2108.9.2.13) Min. ($\phi = 0.0007$)</td>
</tr>
<tr>
<td>Other</td>
<td>Minimum shear reinforcement ratio = 0.0007</td>
<td>Minimum transverse $A_{t} = 0.0018A_{s}$</td>
<td>None</td>
</tr>
</tbody>
</table>

![Fig. 2b: Strain conditions for reinforcement limits for out-of-plane walls](image)

**Fig. 2b: Strain conditions for reinforcement limits for out-of-plane walls**

![Fig. 3: Maximum reinforcement limits for flexural sections](image)

**Fig. 3: Maximum reinforcement limits for flexural sections**

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### TABLE 2:
**Requirements for Walls (2108.9.4-5, IBC)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Out-of-plane</th>
<th>In-plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension requirement</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Axial load limit</td>
<td>Factored axial load stress $\leq 0.2 f_{m}$</td>
<td>$P_a \leq 0.80 P_e$ for axial load only</td>
</tr>
<tr>
<td>$\phi$-factor</td>
<td>$\phi = 0.8$ for flexure</td>
<td>$0.65 \leq \phi \leq 0.8$ as $P_a$ decreases from $0.10 f_{m} A_e$ or $0.25 P_e$ to zero</td>
</tr>
<tr>
<td>Longitudinal reinforcement limits</td>
<td>Max. (2108.9.2.13)</td>
<td>Minimum horizontal (2106.4.2.6) $M_n \geq 1.8 M_{n,0}$ (fully grouted) $M_n \geq 3.0 M_{n,0}$ (partially grouted) Vertical reinforcement $\geq 1/2$ horizontal reinforcement</td>
</tr>
<tr>
<td>Other</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Columns
- The minimum column width in a wall-frame system is 8 in. (203 mm) or 1/14 of the clear length (see Table 3), as opposed to 12 in. (305 mm) required in Section 2108.9.3 (see Table 1).
- The minimum depth has to be the greater of 2 masonry-units or 32 in. (813 mm), as shown in Table 3, as opposed to the minimum of 12 in. (305 mm) required in Section 2108.9.3.
- There is no limit on the unbraced length of the column, but column length must be limited to 5 times its depth. Section 2108.9.3 (see Table 1) requires that the unbraced length of a column be less than or equal to 30 times the column width.
- The axial load on a column of a wall-frame system is limited to $0.15 A_n f_{m}^*$ (see Table 3), while there is no such limit for a column designed per Section 2108.9.3.
- The equations for the strength reduction factor $\phi$ are also different (compare Table 1 and 3).
- The maximum reinforcement ratio for columns of wall-frames is $0.15 f_{m}^*/f_0$ and that given by Section 11.6.2.2 (see Table 3). For columns designed under Section 2108.9.3, this limit is given by Section 2108.9.2.13 (see Table 1).
- The minimum transverse reinforcement ratio is given as 0.0015, as shown in Table 3, as opposed to 0.0018 $A_n^*$ for columns designed per Section 2108.9.3 (see Table 1).

### Strength reduction factors

Tables 1, 2, and 3 show the different strength reduction factors for beams, columns, piers, walls, and beams and columns of wall-frames. In the absence of scientific rationale or comprehensive test data, it doesn’t make sense to have so many different values of $\phi$ factors for these elements. I believe the factors can be unified and simplified for ease of design without jeopardizing safety.

### Proposed unified approach

The above discussion on the state of current design provisions underscores the need for simplification and unification. The rationale for a unified approach is based on the fact that whatever the size, shape, and reinforcement type in a section, its ultimate behavior is likely to be controlled either by significant yielding of steel in tension, referred to as tension-controlled; by compression failure of masonry, referred to as compression-controlled; or by a combined tension yielding and compression failure of masonry that may not be distinctly clear as either tension-controlled or compression-controlled, referred to as a transition region. If you examine the three possible limit states, it becomes possible to unify the design provisions based on the
strain condition associated with these three behavior limit states.

I would also point out that the general assumptions for design of reinforced masonry (Section 2108.9.1) are similar to those of reinforced concrete and, thus, become a basis for the proposed unified approach.

The proposed unified approach is similar to the one used for reinforced concrete design (see Appendix B of ACI 318). All design requirements for dimensions, axial load levels, reinforcement-ratio limits, and the strength reduction factors (φ) would be unified and equally applicable to any size and shape of element, whether a beam, column, wall, or beam-column. Special consideration will, however, be required for design of lateral force-resisting elements in seismic regions. The design provisions would also have to allow for limitations on reinforcement quantity and placement due to masonry cell dimensions and cover requirements.

**The new method**

The Unified Design Approach uses the unifying concept of cross-sectional behavior such as tension-controlled and compression-controlled for design of both nonprestressed and prestressed members. The behavior limits are defined based on the magnitude of strain in the extreme tension steel of the section at nominal strength, ε_s, as shown in Fig. 4. The strain limits, as shown in Fig. 4, are defined for the extreme tension steel depth d_t and not at the center of gravity of the tension steel d, as in conventional design. This is because the strain in the extreme tension steel is considered to be a better measure of the ductility and the ultimate behavior of the section.

As with the conventional strength design method, the design strength of the section is determined by multiplying the nominal

<table>
<thead>
<tr>
<th>TABLE 3: REQUIREMENTS FOR WALL-FRAMES (2108.9.6, IBC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-------------------------------------</td>
</tr>
<tr>
<td>Dimension requirement</td>
</tr>
<tr>
<td>B ≥ 8 in. (203 mm)</td>
</tr>
<tr>
<td>½ width by Section 8.8.1</td>
</tr>
<tr>
<td>≥ l_s/26</td>
</tr>
<tr>
<td>D ≥ 2D</td>
</tr>
<tr>
<td>D ≥ 2 masonry units</td>
</tr>
<tr>
<td>≥ 16 in. (406 mm)</td>
</tr>
<tr>
<td>Axial load limit</td>
</tr>
<tr>
<td>P ≤ 0.10A_s f_m</td>
</tr>
<tr>
<td>ϕ-factor</td>
</tr>
<tr>
<td>ϕ = 0.85 - 2(P_u/A_s f_m)</td>
</tr>
<tr>
<td>Longitudinal reinforcement limits</td>
</tr>
<tr>
<td>Max. (ρ ≤ 0.15 f_m/f_y or</td>
</tr>
<tr>
<td>2108.9.2.13)</td>
</tr>
<tr>
<td>Min. (ρ=130/f_y)</td>
</tr>
<tr>
<td>Other</td>
</tr>
<tr>
<td>reinforcement ratio = 0.0015</td>
</tr>
</tbody>
</table>

**Fig. 4a:** Strain limits and behavior regions for concrete masonry flexural section with ρ = 0.5 ρ_s

**Fig. 4b:** Strain limits and behavior regions for clay masonry flexural section with ρ = 0.5 ρ_s

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strength by the strength reduction factor (ϕ). But the value of ϕ, in the new approach, depends upon the magnitude of strain in the extreme tension steel, ε_t, or c/d, (see Fig. 5).

**Tension-controlled design**

I propose that sections that reach a net tensile strain ε_t of 0.0075 or more in the extreme tension steel at nominal strength (c/d ≤ 0.28, see Fig. 4) be designated as tension-controlled sections for both concrete and clay masonry. This strain limit is based on the reinforcement ratio limitation of 0.5% given in the 1997 UBC code. A strength reduction factor of 0.8 may be used in this case to determine the design strength $\phi M_u$. This strain condition will result in a maximum reinforcement ratio of 1.35% for tension-controlled design of a rectangular section with $f'_m = 4000$ psi (27.6 MPa) and a single layer of steel. Sections predominately under flexure or tension with negligible axial compressive loads should be designed for tension-controlled behavior.

**Compression-controlled design**

Compression-controlled behavior would be defined for a net tensile strain ε_t of 0.002 or less in the extreme tension Grade 60 steel. This results in c/d ≥ 0.56 for concrete masonry, and c/d ≥ 0.64 for clay masonry (see Fig. 4). This strain condition will result in reinforcement ratios of 2.7 and 3%, respectively, for concrete masonry and clay masonry flexural sections having $f'_m = 4000$ psi (27.6 MPa) and a single layer of steel. A strength reduction factor of 0.6 can be used in this case. Typically, compression members having negligible or zero moment, where small or no tensile strains are likely to develop, should be designed for compression-controlled behavior.

**Design in transition region**

For values of net tensile strain in the extreme tension steel ε_t between 0.002 (c/d = 0.56 and 0.64) and 0.0075 (c/d = 0.28), the value of ϕ varies linearly from 0.6 to 0.8 for determining the design strength of the section (see Fig. 5). Sections under combined axial load and flexure may typically fall into this behavior range.

Using Fig. 4 and 5, the c/d value can be expressed in terms of ultimate strain of masonry at nominal strength and strain in extreme tension steel ε_t, as follows:

$$\frac{c}{d} = \frac{0.0025}{0.0025 + \varepsilon_t} \text{ for concrete masonry}$$  \hspace{1cm} (Eq. 1)

$$\frac{c}{d} = \frac{0.0035}{0.0035 + \varepsilon_t} \text{ for clay masonry}$$  \hspace{1cm} (Eq. 2)

Design in the transition region will involve an iterative procedure. This is because ϕ varies linearly with ε_t, which is not known in the beginning of the design process for sections likely to fall in this behavior category. Design aids similar to those for reinforced concrete can facilitate design of sections in the transition region.

**Uniformity is the goal**

The current strength design provisions for reinforced masonry given in the 2000 IBC are complicated and non-uniform. There is a need to simplify and unify the provisions. The Unified Design Approach proposed here will, in general, simplify and unify the design provisions. However, some issues specific to masonry design would have to be addressed to make such a method widely applicable. I hope this article will stimulate discussion, which will hopefully lead to simpler and more rational design provisions for reinforced masonry.

**References**


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