Development of structural analysis program for non-linear elasticity by continuum damage mechanics

Yoshiyuki Kaji a,*, Wenwei Gu b, Masahiro Ishihara a, Taketoshi Arai a, Hitoshi Nakamura b

a Japan Atomic Energy Research Institute, Tokai-mura, Ibaraki, 319-1195, Japan
b CRC Research Institute, Inc., Koto-ku, Tokyo 136, Japan

Received 10 July 2000; accepted 2 January 2001

Abstract

The brittle damage constitutive equation developed by Chow and Yang is used to simulate the non-linear elastic deformation behavior of graphite using finite element method (FEM). This model is achieved by introducing a damage surface that is similar to the yield function in the conventional theory of plasticity. A special form of damage surfaces is constructed to illustrate the application of the model. For verifying the FEM program including the Chow and Yang model, the predicted deformations by this model are compared with both the experimental ones in the graphite structural model and the calculated ones without the continuum damage mechanics. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

It is well known that the deformation of most engineering materials is often accompanied by irreversible changes in their internal structures. The nucleation and the growth of distributed microscopic cavities and cracks will not only induce the occurrence of macro-cracks, but also lead to the deterioration of material properties due to internal microstructural changes. Therefore proper understanding and knowledge of the damaging process and its effects on the macroscopic behavior of materials are very important prerequisites for the analysis of structural integrity of practical problems. An approach to these problems can be provided by the recently developed theory of continuum damage mechanics (CDM), which involves irreversible microstructural changes implicitly.

In the Japan Atomic Energy Research Institute (JAERI), we are developing structural analytical methods considering the existence of mesoscopic cracks for brittle material’s structures. For one of them, we try to describe the non-linear elastic constitutive equation using CDM and evaluate the structural integrity of brittle material components by the damage parameter introduced in this type of theory. Several types of CDM models
have been proposed for brittle materials. For example, the Chow and Yang model (Chow and Yang, 1991) is based on the hypothesis of damage surface that is similar to the yield function in plasticity theory and uses one scalar valued internal variable to represent the damage state. The Chaboche model (Chaboche et al., 1995) assumes anisotropic damage basically and introduces tensorial damage variables. Therefore, it can be considered that there exists a relationship between small cracks (damage) and strain direction. However, a detailed method to apply this model to anisotropic damage in any direction is still under development. Moreover CDM theories, which incorporate vectors (Krajcinovic and Fonseka, 1981), scalars (Ladeveze and Lemaitre, 1984), second-order tensors (Cordebois and Sidoroff, 1979; Ramtani, 1990), and fourth-order tensors (Ju, 1989) have been proposed. But these theories show either a discontinuous stress–strain response when the unilateral condition takes place or an unacceptable non-symmetric elastic behavior for some loading conditions (Chaboche, 1992).

In this paper, the brittle damage constitutive equation developed by Chow and Yang is used to simulate the non-linear elastic deformation behavior of graphite, using finite element method (FEM). This model introduces a damage surface that is similar to the yield function in the conventional theory of plasticity. A special form of damage surfaces is constructed to illustrate the application of the model.

For verifying the FEM program including the Chow and Yang model, the deformations predicted by this model are compared with both the experimental ones in the graphite structural model and the calculated ones without using CDM.

2. Chow and Yang's constitutive equation (Chow and Yang, 1991)

Chow and Yang proposed a constitutive equation that can easily account for damage in brittle materials. This is achieved by introducing a damage surface that is similar to the yield function in the conventional theory of plasticity. In order to establish the model in a relatively general form, they employed the concept of internal state variables and a phenomenological method to describe the state of a material element. For simplicity, they only considered the rate-independent behavior of the element in an isothermal process. Consequently, time and temperature will not appear in the formulation.

The damage surface, which represents initial and subsequent damage surfaces in stress space, may be expressed analytically by

$$ F(\sigma_{ij}, D) = 0 $$

in which $D$ is a measure of damaged deformation and hence changes only when damage occurs.

The stress and strain relation, at any certain damage state, can be expressed as

$$ \varepsilon_{ij}^e = A_{ijkl} \sigma_{kl} $$

in which $A_{ijkl}$ ($A_{ijkl} = A_{ijlk} = A_{iklj} = A_{klij}$) is the elastic compliance at the current state, and $\varepsilon_{ij}^e$ are reversible strain components.

Total incremental strain at any stage of the deformation process may be linearly divided into elastic and damaged components, within the limitation of small deformation, as

$$ d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^D $$

where the superscripts $e$ and $D$ refer to the elastic and damage strain components respectively. The elastic strain increment may be related to the stress increment $d\sigma_{ij}$ through the generalized Hooke’s Law as

$$ d\varepsilon_{ij}^e = A_{ijkl} d\sigma_{kl} $$

The incremental damaged strains are evaluated based on the damage function $F$, using the following equations, similar to the associative flow rule in plasticity theory:

$$ d\varepsilon_{ij}^D = \begin{cases} 
\frac{d\lambda \partial F}{\partial \sigma_{ij}}, & \text{if } F > 0 \\
0, & \text{otherwise}
\end{cases} $$

where $\partial F/\partial \sigma_{ij}$ is a scalar of proportionality and can be determined from the consistency condition, $dF = 0$. 

For verifying the FEM program including the Chow and Yang model, the deformations predicted by this model are compared with both the experimental ones in the graphite structural model and the calculated ones without using CDM.
In many brittle materials, the stress–strain behavior for tensile loading shows a different one for compressive loading. Therefore the different threshold damage stress values for tensile and compressive deformation are introduced to simulate the asymmetric behavior between tensile and compression in this program.

3. Verification of functions of program

Three different types of non-linearity observed in graphite materials were formulated and examined with the experiments for simple loading and unloading processes. The finite element types of these analyses are axial symmetric, plane stress and plane strain elements. The boundary condition of the model is that one end is rigidly constrained and the constrained displacement is applied successively with three basic deformation patterns, which are described in the following at the other end. For all elements, the comparison between calculated and experimental results was carried out for simple loading and unloading processes.

3.1. Non-linear characterization

For example, Fig. 1 shows relationship between stress and strain for two-dimensional axial symmetric problem. Elastic, damage and total strain are plotted against stress as compared with the results of polynomial expression in this figure. It is confirmed that non-linear stress-strain relationship of calculated results is in good agreement with the experimental results (Arai et al., 1992) for all elements. Moreover, damage strain does not increase and this model reproduce the elastic unloading process. Therefore, it is obvious that the Chow and Yang’s model can predict fairly well the observed non-linear behavior in PGX graphite, which is a medium-grained near-anisotropic nuclear grade graphite.

3.2. Residual strain characterization

The “shift stress” can be given for input data in the case of initiating the residual strain for unloading process in this program. Fig. 2 shows calculated results for an axial symmetric problem with experimental results of Gilscocarbon graphites (Everett and Ridealgh, 1972). The analytical results after using the shift stress are coincident with the experimental results for Gilscocarbon graphites and the residual strain is predicted successfully. Therefore, it is found that the residual strain in unloading process can be modeled by introducing the “shift stress” parameter.

3.3. Asymmetric deformation in tension and compression

Fig. 3 shows the calculated asymmetric behav-
ior between tensile and compression for axial symmetric problem. In this case, the threshold damage stress values for tensile and compression are 0 and $-10 \text{ MPa}$, respectively. It is confirmed that asymmetric behavior can be successfully reproduced by establishing different threshold stress values for tensile and compression sides.

4. Application to structural integrity tests

4.1. Structural integrity tests (Ishihara et al., 1995)

The core support graphite component in the high temperature engineering test reactor (HTTR), which supports the core graphite components, are mainly made of PGX graphite, medium-to-fine grained molded graphite, as shown in Fig. 4. The core support graphite components are the hot plenum blocks, permanent reflector blocks and so forth, which are connected with each other by key–keyway elements. These connecting elements are designed so as to transfer the shear force between connected blocks during an earthquake. In order to estimate the structural integrity for the connecting elements, the component test was carried out with full-size key–keyway elements.

For the key–keyway test two types of test specimens (the full-size hot plenum and permanent reflector blocks in the HTTR) were used. A test specimen made of PGX graphite was set with two key elements as shown in Fig. 5. Loads were applied to the specimen up to 13 kN for the hot plenum block and 6.5 kN for the permanent reflector block. Strain was measured by a triaxial strain gauge, whose gauge length was 1 mm, near the bottom corner on the keyway. In these tests, the applied loads were critical loads and cracking occurred in these loading conditions. We investigate the stress states of two keyway elements under the critical load conditions in the next section.

4.2. Damage mechanics analysis

4.2.1. Material constants

The material properties of PGX graphite are Young’s modulus $E = 8.3 \text{ GPa}$ and Poisson’s ra-
The stress–strain curves for PGX graphite are non-linear relations as shown in Fig. 6. The relationship between strain ($\varepsilon$) and stress ($\sigma$: MPa) is presented in Fig. 6, which can be expressed as (Arai et al., 1992)

$$
\varepsilon = 1.263 \times 10^{-4} \sigma + 1.761 \times 10^{-6} \sigma^2 + 7.791 \times 10^{-9} \sigma^3
$$

where $\varepsilon$: strain, $\sigma$: stress (MPa).

4.2.2. Calculated model

The two-dimensional models of hot plenum and permanent reflector blocks for damage mechanics analysis are shown in Fig. 7. The total number of nodes and elements of both models are 435 and 392 for hot plenum block and 754 and 700 for permanent reflector block, respectively. The element used here is 4-node plane strain element. Elastic analyses with and without taking account of damage mechanics were performed using this developed program.
4.3. Comparison between experiment and analysis

Figs. 8 and 9 show typical stress distributions around the keyway, as obtained by the elastic analyses with and without considering damage mechanics for hot plenum and permanent reflector blocks, respectively. The maximum value of maximum principal stress arises at around 40–70° from the bottom edge of the keyway. It is found that the state of the distributions of the maximum principal stress with and without considering damage mechanics is mostly the same for hot plenum and permanent reflector blocks. However, the maximum value of the maximum principal stress in the case of damage mechanics analysis is smaller than that in the case of elastic analysis; this result is reasonable since the nonlinearity in stress–strain relationship is considered in the damage mechanics analysis.

The maximum principal stresses $S_{\text{max}}$ measured using the triaxial strain gauges in the structural integrity tests were compared with the analytical results for hot plenum and permanent reflector blocks, as shown in Fig. 10. Assuming elastic deformation, the experimental value is obtained by the following equation:

$$
S_{\text{max}} = \frac{E}{2(1-\nu^2)}[(1+\nu)(\varepsilon_1 + \varepsilon_3) + (1-\nu)[2(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}] \\
$$  \hspace{1cm} (7)

where $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$ are the measured strains, $\nu$ is Poisson’s ratio and $E$ is Young’s modulus. Though the absolute analytical values are a little smaller than the experimental ones, these analytical results agree closely with the experimental ones. Moreover, the main crack was observed at the calculated maximum stress concentration area for hot plenum and permanent reflector blocks. The maximum principal stress distributions for hot plenum and permanent reflector blocks are shown in Fig. 11. In this figure, a, b and c show the line number in Fig. 10. Along these lines, the maximum principal stress distribution for the damage analysis is in good agreement with the structural integrity test results for both hot plenum and permanent reflector blocks, and the maximum stress value for permanent reflector blocks is higher than that of the hot plenum.
Fig. 8. Calculated maximum principal stress distribution in hot plenum block for elastic and damage analysis: (a) elastic analysis, (b) damage analysis.
Fig. 9. Calculated maximum principal stress distribution in permanent reflectors for elastic and damage analysis: (a) elastic analysis, (b) damage analysis.
Fig. 10. Comparison between damage analytical and experimental results for hot plenum and permanent reflector blocks: (a) hot plenum block, (b) permanent reflector block.

- Estimated principal stress by measured triaxial strain gages

(a) Hot plenum block

(b) Permanent reflector block
blocks. The difference of maximum principal stress is understandable from a viewpoint of so-called “stress distribution effect on strength”, specimens with higher stress distribution and/or stress gradient have higher strength because the stress gradient for the permanent reflector blocks is more than five times as large as that for the hot plenum blocks.

Fig. 12 shows the distribution of the calculated damage factor $D$ for hot plenum and permanent reflector blocks. Though the absolute value of damage valuable $D$ is small, the maximum point of $D$ is coincident with the crack initiation and the calculated stress concentration points. Furthermore assuming that the fracture stress of PGX graphite is about 35 MPa from Fig. 6, the calculated limiting damage factor $D$ is about $1.5507 \times 10^{-2}$ at most. Therefore, it is found that the irreversible real damage states evaluation is possible by using the damage value $D$ for the structural integrity evaluation of the brittle structures. In these example problems, the difference between elastic and damage analysis results is very small, because the fracture stress is small for the structural integrity tests and stress region is almost in linear portion in stress–strain curves as shown in Fig. 6.

Further work will be made to investigate the detailed verification of this CDM model under several conditions, for example, cyclic loading condition, because the calculated maximum $D$ values at fracture are different with both component model tests for hot plenum and permanent reflector blocks only under compression loading condition. Therefore, it is necessary to examine the cause of the difference of critical fracture condition, which is, for example, the problem of this CDM model itself or the two-dimensional analysis, or is the material’s specific problem.

5. Conclusion

The brittle damage constitutive equation developed by Chow and Yang is discussed and coupled to finite element method (FEM) calculations for non-linear elastic deformation behavior of graphite. The conclusions obtained are summarized as follows.

The finite element method program with continuum damage mechanics using Chow and Yang elastic damage constitutive equation was developed. In this program, additional functions for the residual strain and tensile/compression asymmetrical behavior were added to consider the various types of nonlinear stress–strain behaviors in graphite materials.

To validate the program, comparisons between the calculated and the structural integrity test results of the graphite component models were carried out. It was found that the irreversible real damage states evaluation was possible by using the damage valuable $D$ for the structural integrity evaluation of the brittle structures.
Fig. 12. Calculated results of damage valuable $D$ for hot plenum and permanent reflector blocks.
References


