Life Prediction of Structural Components

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**SCOPE**

- Fast Fracture (Time- Independent)
- Multi-axial Reliability Models
- Fracture Strength Data And Parameter Estimates
- Life Prediction (Time- Dependent)
- Time-dependent Strength Data & Parameter Estimates
- Life Prediction Examples
FAST FRACTURE (TIME- INDEPENDENT)

Most generic format for the probability of failure of a component can be stated as:

\[ P_f = 1 - \exp\left(-\int \psi \, dV\right) \]

A number of multi-axial models are available, including:

- Principle of Independent Action (PIA)
- Batdorf’s Model
- Lamon – Evans Model
- Muest Model

Differences between the models lie in the formulation of \( \psi \), the failure function per unit volume.
MULTIAXIAL RELIABILITY MODELS

Principal of Independent Action (PIA) Model

\[ \psi = \left( \frac{\sigma_1}{\sigma_0} \right)^m + \left( \frac{\sigma_2}{\sigma_0} \right)^m + \left( \frac{\sigma_3}{\sigma_0} \right)^m \]

Batdorf/Lamon-Evans Model

\[ \psi = m k_B \int_0^{(\sigma_e)_{\text{max}}} \frac{\Omega(\Sigma, \sigma_{cr})}{4\pi} \sigma_{cr}^{m-1} d\sigma_{cr} \]

Muest Model

\[ \psi = \left( \frac{\sigma_1}{\sigma_{0v}} \right)^{m_v} I_v \left( m_v, \frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{\sigma_1} \right) \]
In order to obtain point estimates of the two unknown Weibull distribution parameters, well-defined functions that incorporate the failure data and specimen geometry are utilized. These functions are referred to as estimators.

It is desirable that an estimator be consistent and efficient. In addition, the estimator should produce unique, unbiased estimates of the distribution parameters.

Different types of estimators exist, including moment estimators, least-squares estimators, and maximum likelihood estimators (MLE). Maximum likelihood estimators are preferred due to their efficiency and the ease of application when censored failure populations are encountered.

\[
\frac{\sum_{i=1}^{n} (\sigma_i)^\tilde{m} \ln(\sigma_i)}{\sum_{i=1}^{n} (\sigma_i)^\tilde{m}} - \frac{1}{n} \sum_{i=1}^{n} \ln(\sigma_i) - \frac{1}{\tilde{m}} = 0
\]

\[
\tilde{\sigma}_\theta = \left[ \frac{\sum_{i=1}^{n} (\sigma_i)^\tilde{m}}{n} \right]^\frac{1}{\tilde{m}}
\]
Principle Stress Contour Map
Effective Volume as a Function of Mesh Size for $m = 10$

![Effective Volume Graph](image)

Effective Area as a Function of Mesh Size for $m = 10$

![Effective Area Graph](image)
Maximum Circumferential Tensile Stress
as a Function of Specimen Geometry
Ceramics exhibit the phenomenon of delayed fracture or fatigue. With load histories at stress levels that do not induce fast fracture, there is a regime where subcritical crack growth (SCG) occurs. Subcritical crack growth involves a combination of simultaneous, deleterious and synergistic failure mechanisms. These can be grouped into two categories:

1. crack growth due to stress corrosion, and

2. crack growth due to mechanical effects arising from variable load histories

Stress corrosion reflects a stress-dependent chemical interaction between the material and its environment. Water, for example, has a pronounced deleterious effect on the strength of ceramics. Higher temperatures also tend to accelerate this process.
Models for SCG that have been developed tend to be semi-empirical and approximate the behavior of crack growth phenomenologically. Experimental data indicates that crack growth rate is a function of the applied stress intensity factor.

Experimental data show three distinct regimes or regions of growth when the data is graphically depicted as the logarithm of the rate of crack growth versus the logarithm of the mode I stress intensity factor.
The second region typically dominates the life of the material. For the stress corrosion failure mechanism, these curves are material and environment sensitive. Models cited in the literature are based on power law formulations for the evolutionary equations. This mechanistic power law formulation is expressed as

\[
\frac{da(x, y, z, t)}{dt} = A K_{\text{eq}}^N(x, y, z, t) = A \sigma_{\text{eq}}^N(x, y, z, t) Y^N a(x, y, z, t)
\]

Mode I stress \( \sigma_{\text{eq}}(x, y, z, t_f) \) at the time of failure \( (t = t_f) \) is transformed to its critical effective stress distribution at time \( t = 0 \). The transformation is obtained by solving the following classical fracture mechanics expression for the crack length \( a \)

\[
K_{\text{eq}}(x, y, z, t) = \sigma_{\text{eq}}(x, y, z, t) Y a(x, y, z, t)
\]
Differentiating the resulting expression for $a$ with respect to time yields the following expression

$$\sigma_{\text{eq,0}}(x, y, z, t_f) = \left[ \int_0^{t_f} \frac{\sigma_{\text{eq}}^N(x, y, z, t)}{B} + \sigma_{\text{eq}}^{N-2}(x, y, z, t_f) \right]^{1/(N-2)}$$

where

$$B = \frac{2}{A Y^2 K_{IC}^{N-2}(N-2)}$$

For a constant load applied over time (creep or “static fatigue”) the expression above yields

$$\sigma_{\text{eq,0}}(x, y, z, t_f) = \sigma_{\text{eq}}(x, y, z) \left[ t_f \frac{\sigma_{\text{eq}}^2(x, y, z)}{B} + 1 \right]^{1/(N-2)}$$

Here the quantity $\sigma_{\text{eq,0}}(x, y, z, t_f)$ is used to compute component probability of failure using the fast fracture techniques presented earlier.
Expressions are derived for $\sigma_{\text{eq,0}}(x,y,z,t_f)$ for various load histories. These expressions could be utilized in the same fashion indicated in the previous overhead. However, a more compact computational approach can be found in the use of “g-factors.” For constant load

$$t_f = \frac{2}{(2-N)AY^N\sigma^N} \left[ a_f^{(2-N)/2} + a_i^{(2-N)/2} \right]$$

For other load histories

$$\left(t_f\right)^* = \left\{ \frac{2}{(2-N)AY^N\sigma^N} \left[ a_f^{(2-N)/2} + a_i^{(2-N)/2} \right] \right\} \frac{1}{g\text{-factor}}$$

$$= t_f / g\text{-factor}$$

Again this is a computational convenience. For example, the g-factor for monotonically increasing loads (dynamic fatigue) is:

$$g\text{-factor} = \frac{1}{N+1}$$
TIME-DEPENDENT STRENGTH DATA & PARAMETER ESTIMATES

The approach for computing the time dependent strength parameters $B$ and $N$ from failure data must be defined. The expressions below represent a uniaxial test specimen in order to simplify the development. Hence the stress state is uniform throughout the specimen, and is not spatially dependent.

Consider a test specimen subjected to constant load with respect to time. Under these conditions one can show

$$t_f = \frac{B (\sigma_{ov})^{N-2} (\sigma_{eq})^{-N}}{V^* \ln \left( \frac{1}{1 - P_f} \right)}$$

where

$$V^* = \left( \frac{\sigma_{eq}}{\sigma_f} \right)^{N m_y} \int dV$$
Taking

\[
D = \frac{B (\sigma_{ov})^{N-2}}{\left[ V^* \ln \left( \frac{1}{1-P_f} \right) \right]^{m_v}}^{N-2}
\]

then

\[
t_f = D (\sigma_{eq})^N
\]

Taking the logarithm of both sides yields

\[
\ln(t_f) = \ln(D) - N \ln(\sigma_{eq})
\]

Plotting the log of time to failure against the log of the applied creep failure stress should yield a straight line with a slope of \(N\). Typically linear regression techniques are used to determine the parameters \(N\) and \(D\). The parameter \(D\) can be evaluated from the asymptotic nature of the creep data. Once these parameters are determined from the time dependent failure data this information would be combined with the Weibull distribution parameter estimates and the parameter \(B\) would be computed from the expression above.
LIFE PREDICTION EXAMPLES

- Aeroshell for NASA Mars Lander
- Ingersoll-Rand Small Gas Turbine
- SAT HARM Missile
- ARL Ceramic Gun Barrel
- LLNL NIKE Codes
Mars Polar Lander Spacecraft with Microprobes

Microprobes free-fall through the Martian atmosphere.

Mission failed, but not due to aeroshell design.
Two microprobes were part of the recent Deep Space 2 Martian exploration experiment.

Each probe was housed in a ceramic Aeroshell which provided protection for the equipment during atmosphere entry.

- Ceramic Aeroshell was analyzed to determine Reliability for mission.
- Launch Loads were considered for worst case conditions - Equivalent g-load.
- Reliability under these loads was established at 5 nines (0.99999).
Finite Element Model of Aeroshell

Stress plot from launch induced load

Risk of rupture of attachment hole

60g acceleration load (into page)
CMT Rotor Steady State Temperatures

- TIT = 1000 °C
- Maximum adiabatic Wall Temperature = 905 °C
- Maximum rotor temperature difference = 60 °C (excluding cooled attachment)*

* low rotor temp gradients characteristic of low expansion ratio and high thermal conductivity - both serving lower thermal stress
CMT Rotor Worst Case Stresses

- **Worst Cases:**
  - **Vane**
    - 234 MPa @ 38 sec. into Cold Start
  - **Bore**
    - 282 MPa @ 54 sec. into Cold Start
  - **Back Wall**
    - 278 MPa @ Steady State

Max Vane Stress = 34.2 ksi
Max Bore Stress = 41.0 ksi
Max Back WallFillet Stress = 40.3 ksi
Radome - Flight Mission Loads
Maximum Principle Stresses

Max Principal Stress, Max–Max Combined Loads
Radome Load History for
Time Dependent Reliability Analysis

Max Principle Stress

1 year
Storage Load

24 hrs
Mission Load

Time
Ceramic Gun Barrel Program
Objective of the contract was to integrate the CARES design methods into the LLNL codes, Nike2d and Nike3d.

**Nike2d** - Successful integration for 2d elements including axisymmetry. Volume analysis only. PIA, Batdorf, and Weibull’s NSA criterion.

**Nike3d** - Integration successfully completed. Includes volume analysis capability for hexahedron elements. PIA, Batdorf, and Weibull’s NSA criterion.

Unique features include material model and Reliability evaluation as an option at FE run time. This is different and more efficient than the CARES approach which requires a separate execution outside the FE code.