ENVIRONMENTAL PHYSICS COMPUTER LAB. #1: TUTORIAL AND EXPONENTIAL CONSUMPTION MODEL

We learn first how to do simple computations with MathCad.

(1) Type:3*4= The answer is: 12. $3 \cdot 4 = 12$

(2) Type 24-35= The answer is -11 $24 - 35 = -11$

(3) Type: \3= (this is square root of 3). The answer is 1.73 $\sqrt{3} = 1.73205$

(4) Type 2.3^4.5= (this is 2.3 to the power 4.5). The answer is 42.44

$$2.3^{4.5} = 42.43999$$

(5) Type: (3-4)*4-6= The answer is -10 $(3-4) \cdot 4 - 6 = -10$

(6) Type: 7/(2.3-8.6)space bar+3=. The answer is:1.889

$$\frac{7}{2.3 - 8.6} + 3 = 1.88889$$

(7) Type: ln(e)= The answer is 1 $\ln(e) = 1$

(8) Type: log(10^3.9)= The answer is 3.9 since log is logarithm in basis 10.

$$\log\left(10^{3.9}\right) = 3.9$$

(9) Type: ln(10)= The answer is 2.30 $\ln(10) = 2.30259$

(10) Type: log(e)= The answer is 0.43 $\log(e) = 0.43429$

Next we learn how to define a function. Type x:1;50. It means: assign to x values from 1 to 50. : colon is assign ; semicolon is until

$$x := 1 .. 50$$

We next define the function f(x) = (1+1/x)^x

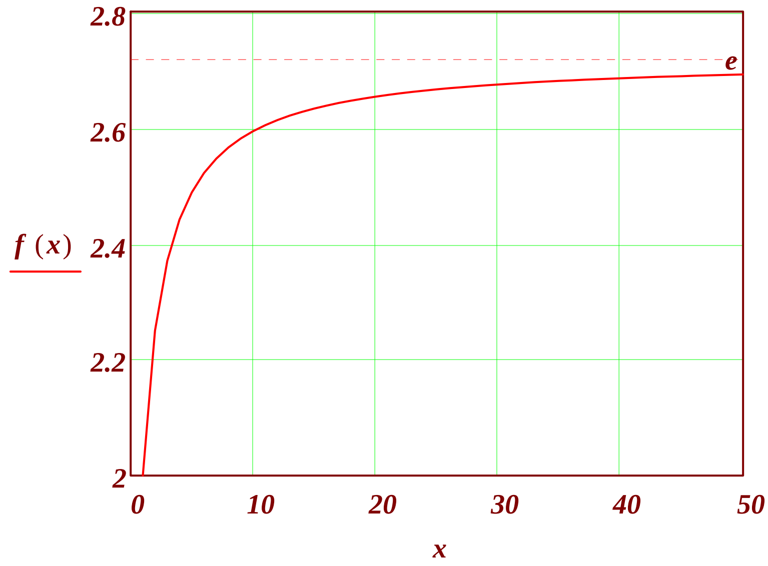
$$f(x) := \left(1 + \frac{1}{x}\right)^x$$

To get the function values just type f(x)=

$f(x) =$

2
2.25
2.37037
2.44141
2.48832
2.52163
2.5465
2.56578
2.58117
2.59374
2.6042
2.61304
2.6206
2.62715
2.63288
...

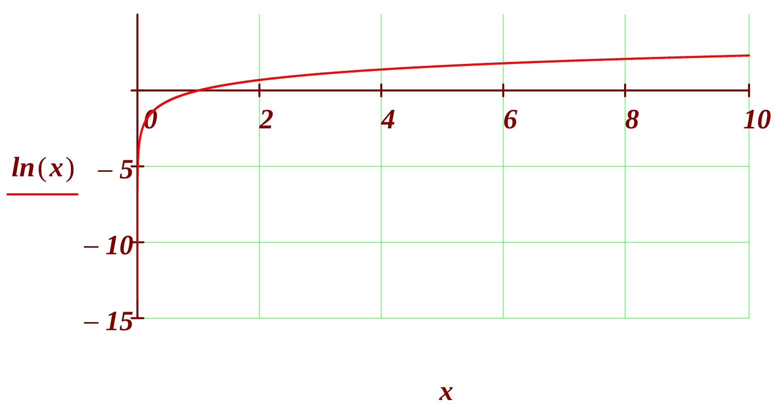
We graph the function f(x). Note that as x gets bigger and bigger f(x) approaches the natural number e = 2.718.



We can graph also by typing the function in the y placeholder as you can see below for ln(x) versus x.

$x := 10^{-6}, 0.001001 .. 10$

This means the first value of x is 0.000001, the second is 0.001001 and the last is 10.

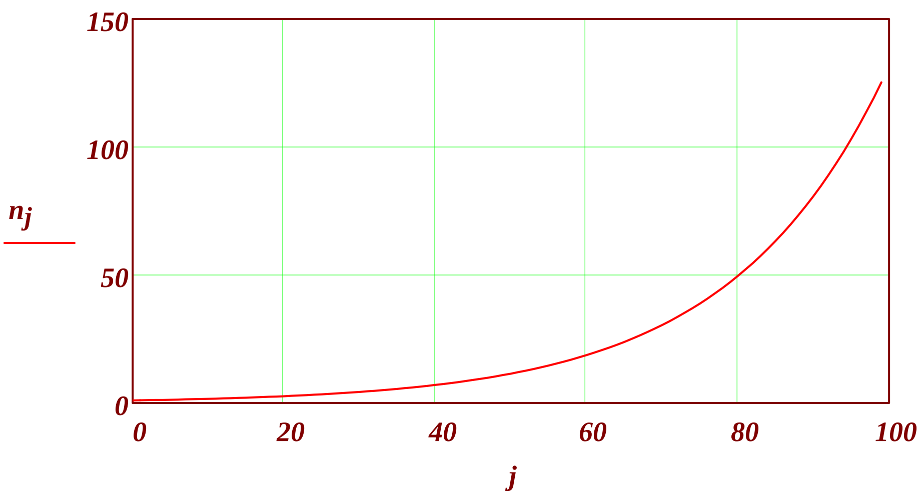


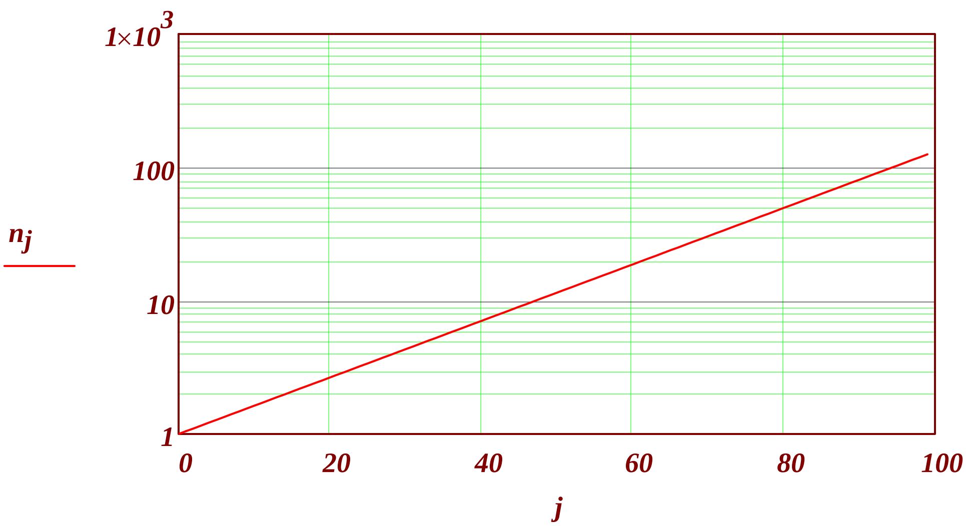
This program illustrates the constant rate of growth (exponential) model. We will also demonstrate how to use semilogarithmic plot to analyze exponential dependence. Here we use a vector. j is the range variable- corresponding to time. Type n[j+1:n[j*C. Here C is the constant multiplying factor and its logarithm is the growth constant k = ln(C).

$C := 1.05$ $\qquad$ $k := \ln(C)$ $\qquad$ $k = 0.04879$

$j := 0 .. 99$ $\qquad$ $n_0 := 1$

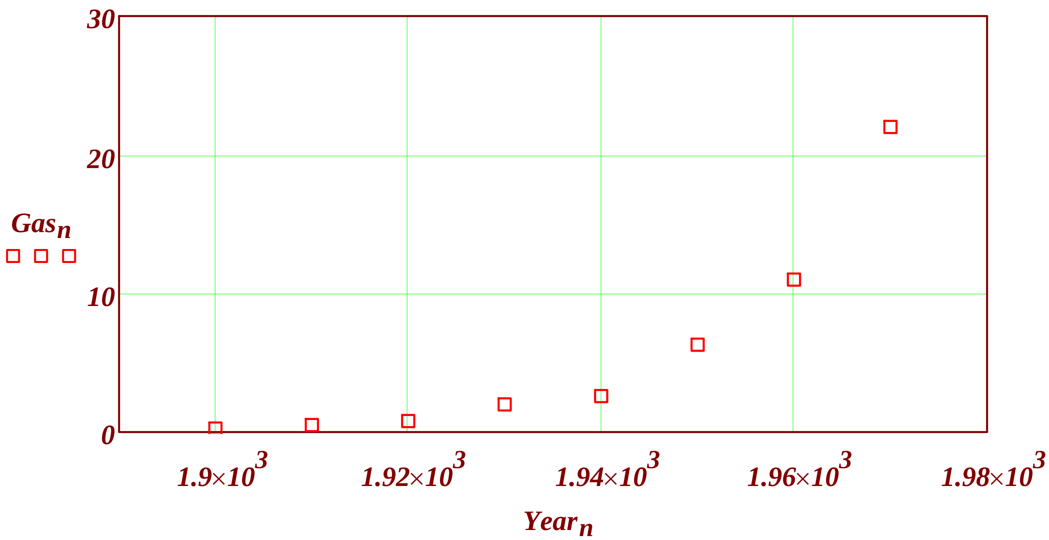
$n_{j+1} := n_j \cdot C$





We now use real data to show the applicability of the constant growth rate model.
First we look at US annual natural gas consumption (in quads).

$$Year := \begin{pmatrix} 1900 \\ 1910 \\ 1920 \\ 1930 \\ 1940 \\ 1950 \\ 1960 \\ 1970 \end{pmatrix} \qquad Gas := \begin{pmatrix} 0.25 \\ 0.51 \\ 0.80 \\ 2.0 \\ 2.6 \\ 6.3 \\ 11.0 \\ 22.0 \end{pmatrix} \qquad n := 0 .. 7$$

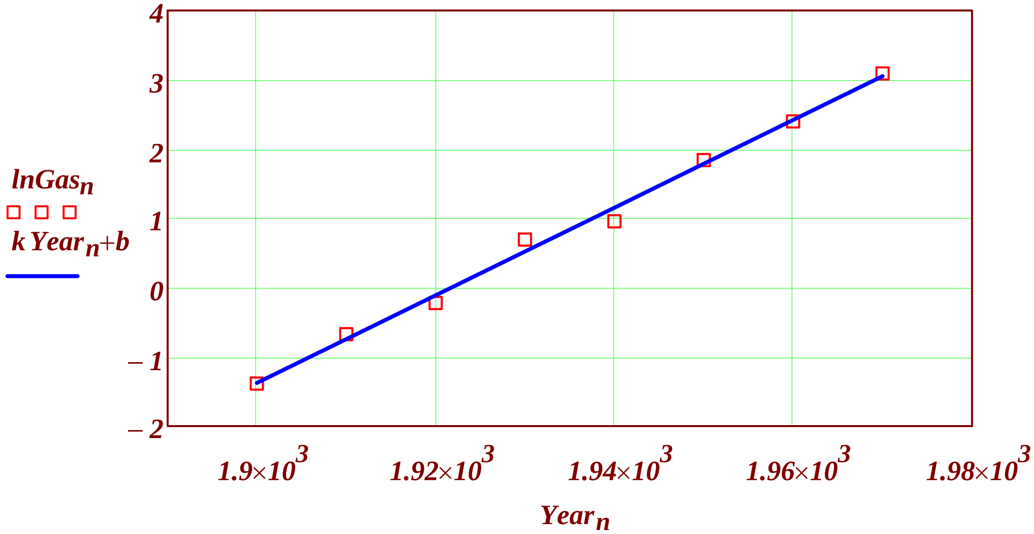


We fit the exponential model to the data.

$$lnGas_n := ln(Gas_n)$$

$$k := slope(Year, lnGas) \qquad k = 0.06328$$

$$b := intercept(Year, lnGas) \qquad b = -121.60021$$



The growth constant k is the slope of the semilogarithmic graph: $k = 0.06328$.

Then the constant multiplying factor is $C := e^{k}$ $\quad C = 1.06532$ So the annual growth rate is 6.5%. The doubling time is: $T_2 := \frac{ln(2)}{k}$ $\quad T_2 = 10.95452$ i.e. it takes about 11 years for the US annual gas consumption to double.

We estimate that the year 2000 US gas consumption was:

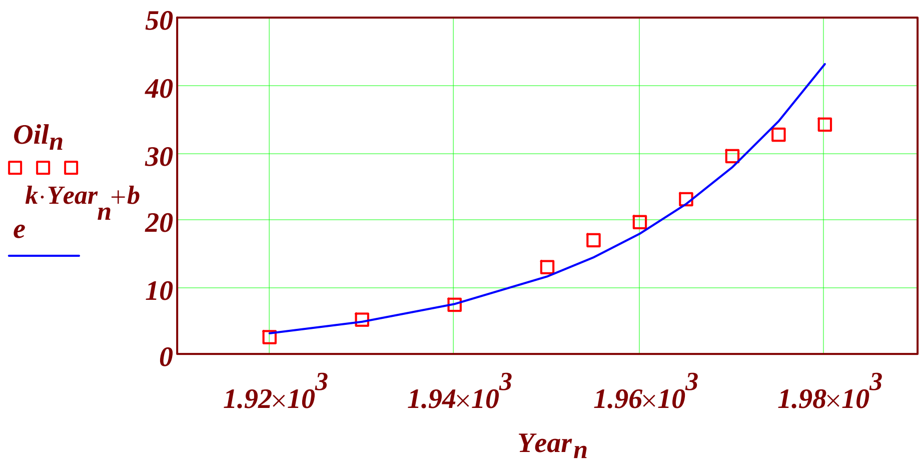
$e^{k \cdot 2000+b} = 141.1461$ quads. The actual 2000 US natural gas consumption was 23.824 quads. This discrepancy points to the fact that the exponential growth model has limited validity. In the second computer lab the graduate students will explore the Hubbert model.

Second we will look at the annual oil consumption in quads in US. Source for 1950 and later data: Energy Information Administration, Department of Energy http://www.eia.doe.gov/emeu/aer/pdf/pages/sec5 _30.pdf

$$Year := \begin{pmatrix} 1920 \\ 1930 \\ 1940 \\ 1950 \\ 1955 \\ 1960 \\ 1965 \\ 1970 \\ 1975 \\ 1980 \end{pmatrix} \qquad Oil := \begin{pmatrix} 2.5 \\ 5.1 \\ 7.3 \\ 12.9 \\ 16.9 \\ 19.6 \\ 23.0 \\ 29.4 \\ 32.6 \\ 34.1 \end{pmatrix} \qquad n := 0..9$$

$$lnOil_n := ln(Oil_n) \qquad k := slope(Year, lnOil) \qquad k = 0.043966$$

$$b := intercept(Year, lnOil) \qquad b = -83.28953$$



The growth constant k is the slope of the semilogarithmic graph: $k = 0.04397$.

Then the constant multiplying factor is: $C := e^{k}$ $C = 1.04495$ So the annual growth rate is 4.4%. The doubling time is: $T_2 := \frac{ln(2)}{k}$; $T_2 = 15.76547$ i.e. it takes about 16 years for the US annual oil consumption to double. We estimate that the year 2000 US oil consumption was: $e^{k \cdot 2000 + b} = 103.83658$ quads. The actual annual consumption in 2000 was 38.262quads. This discrepancy points to the fact that the exponential growth model has limited validity. In the next computer lab the graduate students will explore the Hubbert model.

The table below shows the annual world energy consumption in quads for the 1980-2014 period. The data is available at the web site of the Energy Information Administration, Department of Energy. The data is contained in the file International_data_3.csv available at the course website. Download the file. Click Insert; Data; File Input; Text; start at row 15 and column 4.

$data :=$

nternational_data_3.csv

$data =$

	31	32	33	34	35	36
0	542.07618	554.25212	565.05125	570.84421	573.43376	...

$worldenergy := data^{T}$

$worldenergy =$

	0
0	293.43884
1	291.18482
2	291.63284
3	295.3691
4	309.13013
5	317.50468
6	325.38487
7	335.45738
8	348.33145
9	353.15298
10	359.86809
11	353.28148
12	352.0946
13	355.1955
14	359.16428
15	...

$j := 0..36$ $\qquad$ $Year_j := j + 1980$

$lnworldenergy_j := \ln(worldenergy_j)$

$k := slope(Year, lnworldenergy)$ $\qquad$ $k = 0.019915$

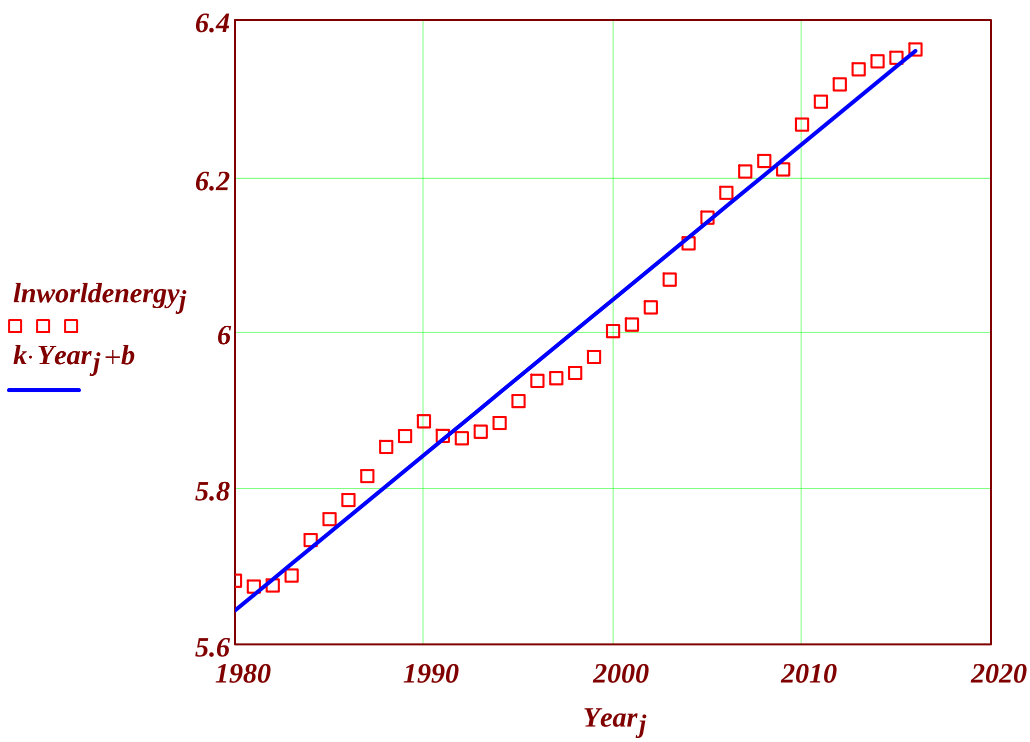
$b := intercept(Year, lnworldenergy)$ $\qquad$ $b = -33.78822$

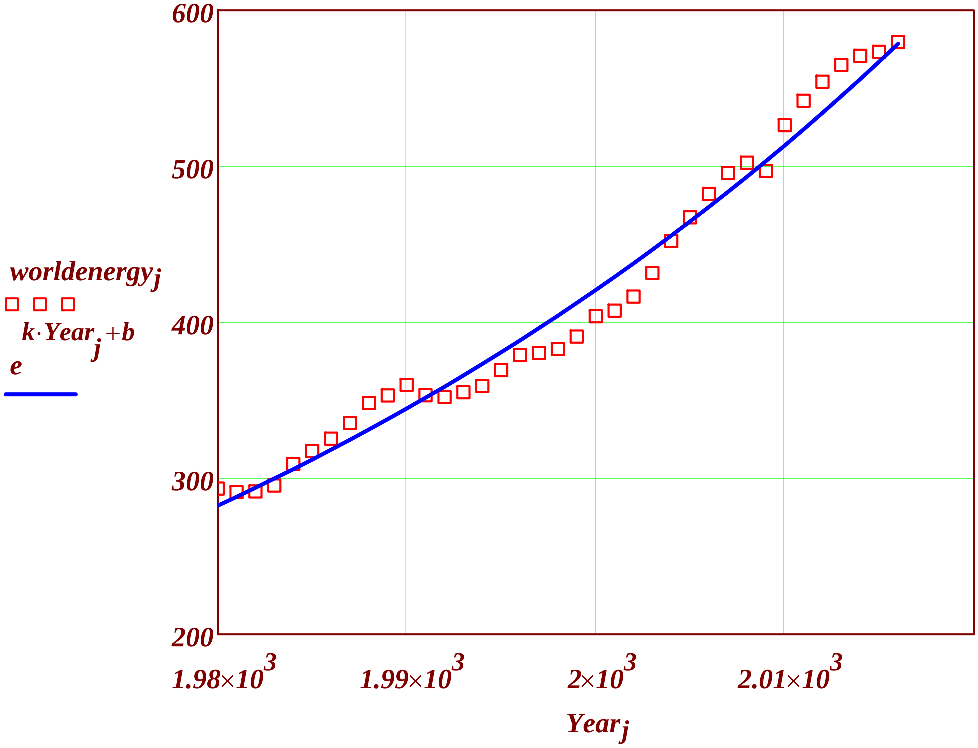
Estimate the world energy consumption in the year 2017:

$e^{k\cdot 2017+b} = 590.16919$ quads.

A measure of the goodness of the linear fit is the correlation coefficient r. If r is close to 1 or -1 the data show high linear correlation, i.e. the linear fit is good. The Mathcad function corr(X,Y) gives the correlation of the X and Y data values.

$r := corr(Year, lnworldenergy)$ $\qquad$ $r = 0.98921$





The doubling time (in years) for world energy comsumption is:

$$\frac{\ln(2)}{k} = 34.80521$$