ENVIRONMENTAL PHYSICS COMPUTER LAB #2
The Hubbert Model

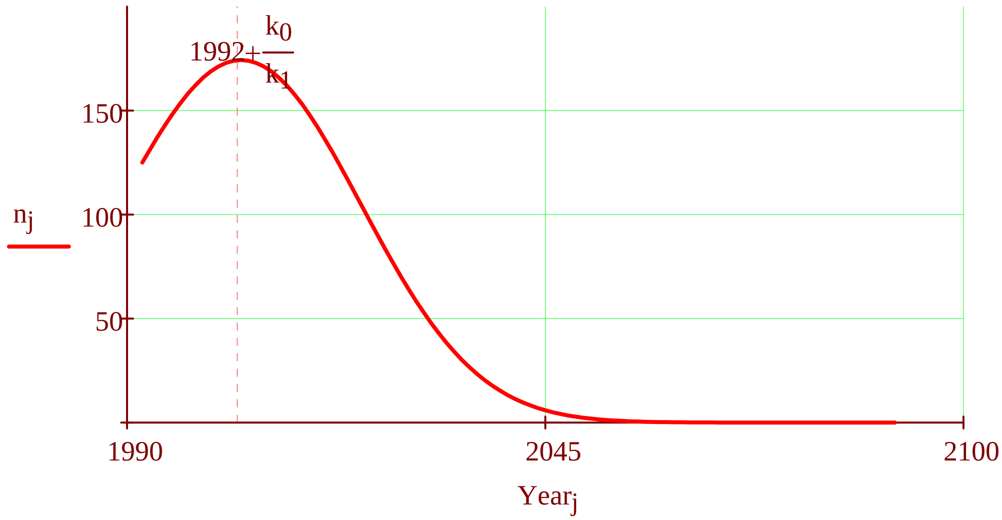
We continue the work done in Lab #1, where we modeled the annual consumption as growing at a constant rate. If it is perceived that the resources are approaching depletion, they are detrimental to the environment, the rate of growth is decreased as other resources are used instead. We will denote n_j the yearly consumption during year j, and q_j the cumulative consumption until year j (not included). In this program we use the "step function" also known as the Phi function or Heaviside function: $\Phi(x) = 1$ if $x > 0$ and $\Phi(x) = 0$ if $x < 0$. We use it in the model to insure that the annual consumption is zero when the cumulative consumption reaches the value of the total resources available q_{tot}.

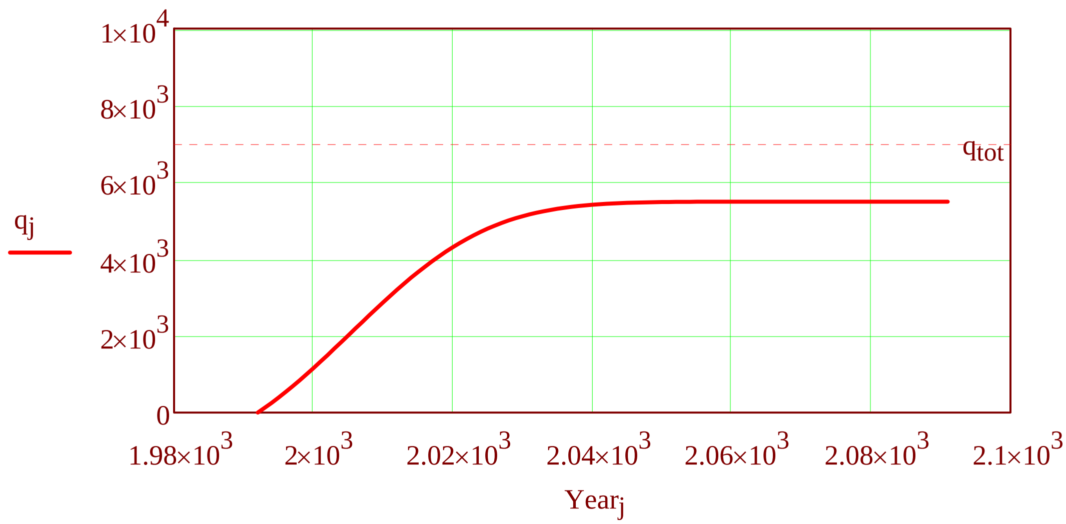
The continuum version of this model was invented by the American geophysicist Hubbert (see Ch.3 of McFadden, Hunt and Campbell), and predicted correctly a maximum in the US oil production. We are going to use the discret version of the Hubbert model to simulate the world oil consumption. In 1992 the annual world oil consumption was 125 quads/yr. The oil resources remaining that year were 7000 quads.

$n_0 := 125$ $\quad q_0 := 0$ $\quad Year_0 := 1992$ $\quad j := 0..99$

$k_0 := 0.05$ $\quad k_1 := 0.004$ $\quad q_{tot} := 7000$

$$\begin{pmatrix} Year_{j+1} \\ n_{j+1} \\ q_{j+1} \end{pmatrix} := \begin{bmatrix} Year_j + 1 \\ n_j \cdot (1 + k_0 - k_1 \cdot j) \cdot \Phi(q_{tot} - q_j) \\ (q_j + n_j) \cdot \Phi(q_{tot} - q_j) + q_{tot} \cdot (1 - \Phi(q_{tot} - q_j)) \end{bmatrix}$$





Note a maximum in the annual rate n_j. It occurs at $j = k_0/k_1$. Can you explain this? By experimenting with the values of k_0 and k_1 you can get qualitatively different outcomes. Run the simulation for: (a) $k_0 = 0.05$ and $k_1 = 0.004$; (b) $k_0 = 0.05$ and $k_1 = 0.003$; (c) $k_0 = 0.05$ and $k_1 = 0.002$. Describe qualitatively what happens in each case by looking at the plots of n versus time and q versus time.

You can read about applications of the Hubbert model to predicting oil production in: R. A. Kerr "The Next Oil Crisis Looms Large- and Perhaps Close" published in the Science Magazine 281, 1128-1131, 1998. The author notes that in a report of the International Energy Agency the following is stated: "...sometime between 2010 and 2020 the gush of oil from wells around the world will peak at 80 million barrels per day, then begin a steady inevitable decline..." Our model case (b) predicts that the maximum ocurs in the year 2005 and is 174 quads/year. Transform from quads/year to barrels/day to compare to the report prediction.

1year = 365day; 1quad = 10^{15}Btu; 1barrel crude oil = 5.5*10^6Btu.

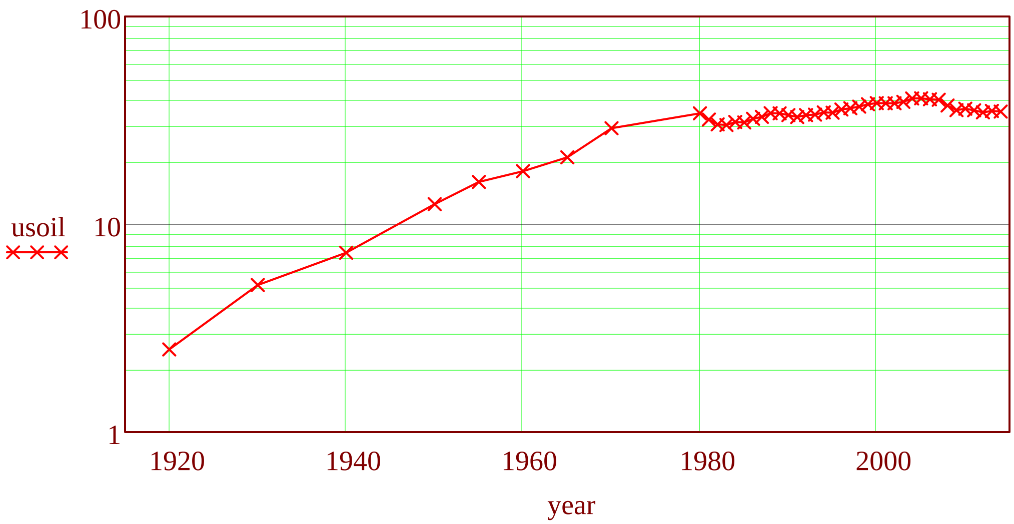
$$\frac{174\cdot 10^{15}}{365\cdot 5.5\cdot 10^{6}} = 8.6675\times 10^{7}$$ barrels/day

Next we are going to fit the Hubbert formula: $N(t)= N_{max}exp[-(t-T_{max})^2/2\sigma^2]$ to the US annual oil consumption expressed in quads/year versus time. The data from 1980 to 2014 was obtained from the DOE web site: http://www.eia.doe.gov/

$$\mathrm{usoil} := \begin{pmatrix} 2.5 \\ 5.1 \\ 7.3 \\ 12.5 \\ 16.0 \\ 18.0 \\ 21.0 \\ 29.0 \\ 34.20 \\ 31.93 \\ 30.23 \\ 30.05 \\ 31.05 \\ 30.925 \\ 32.20 \\ 32.86 \\ 34.22 \\ 34.21 \\ 33.55 \\ 32.85 \\ 33.525 \\ 33.69 \\ 34.56 \\ 34.44 \\ 35.675 \\ 36.16 \\ 36.82 \\ 37.84 \\ 38.26 \end{pmatrix} \qquad \mathrm{year} := \begin{pmatrix} 1920 \\ 1930 \\ 1940 \\ 1950 \\ 1955 \\ 1960 \\ 1965 \\ 1970 \\ 1980 \\ 1981 \\ 1982 \\ 1983 \\ 1984 \\ 1985 \\ 1986 \\ 1987 \\ 1988 \\ 1989 \\ 1990 \\ 1991 \\ 1992 \\ 1993 \\ 1994 \\ 1995 \\ 1996 \\ 1997 \\ 1998 \\ 1999 \\ 2000 \end{pmatrix}$$

38.19	2001
38.22	2002
38.81	2003
40.29	2004
40.39	2005
39.955	2006
39.77	2007
37.28	2008
35.40	2009
36.01	2010
35.37	2011
34.58	2012
35.10	2013
34.88	2014

When using a semi-logarithmic graph we see that the linear dependence predicted by the exponential model does not describe well the data from 1970 to 2014.



We do the fitting by using the minerr(a,b,c) function. This returns the values of the parameters a, b, c that come closest to satisfying the equations and inequalities in a solve block. A solve block must start with the statement **given**. Mathcad evaluates the minerr function using the Levenberg-Marquardt method which requires a guess value for each unknown to begin the search for solutions. So the guess values must appear in the worksheet before **given**. The minerr function returns a vector whose first element is a, second element is b and third is c.

Guessing values: $N_{max} := 40$ $T_{max} := 2020$ $\sigma := 40$

This is the function that we fit to the data:

$$\mathrm{hubbert}(T, N_{max}, T_{max}, \sigma) := N_{max} \cdot e^{\frac{-(T - T_{max})^2}{2 \cdot \sigma^2}}$$

We start the solve block with: Given

$$\sum_{j=0}^{42} \left(\mathrm{usoil}_j - \mathrm{hubbert}(\mathrm{year}_j, N_{max}, T_{max}, \sigma)\right)^2 = 0$$

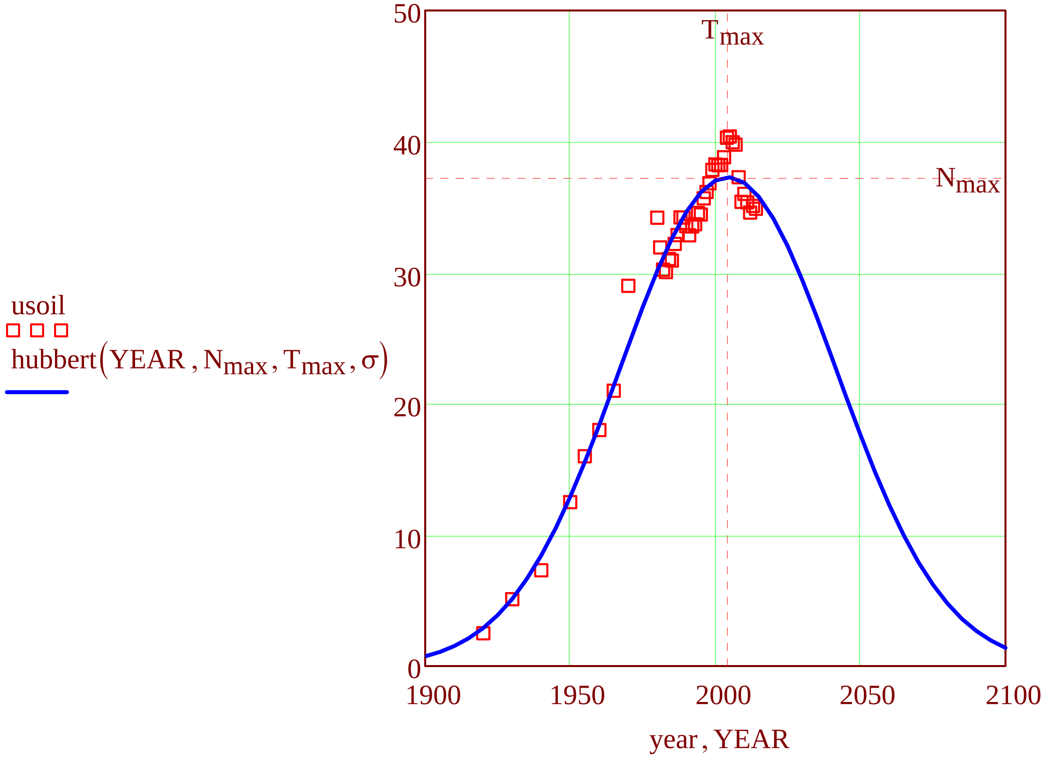
We use the **boolean equals** which is obtained from the *evaluation palette* or by simultaneously hitting control and = keys. The constraint is the least squares criterion: the sum of squared distances from the data to the curve is as small as possible.

$$\begin{pmatrix} N_{max} \\ T_{max} \\ \sigma \end{pmatrix} := \mathrm{Minerr}(N_{max}, T_{max}, \sigma)$$

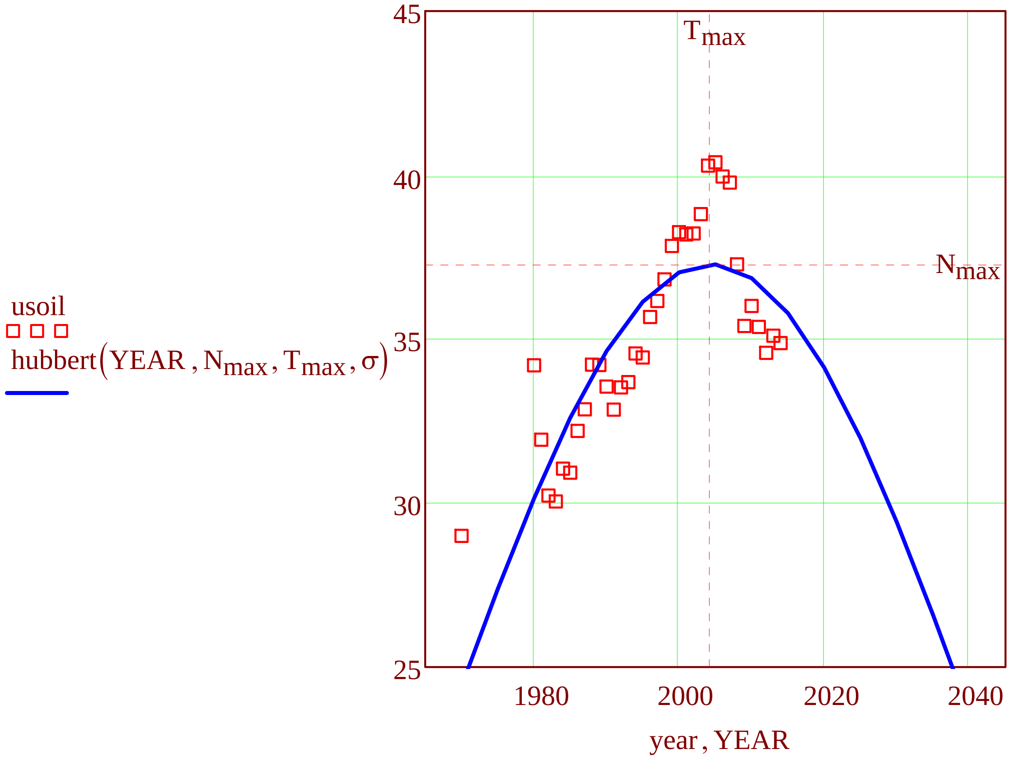
Those are the values of the parameters N_{max}, T_{max} and σ:

$N_{max} = 37.2849$ $T_{max} = 2.004 \times 10^3$ $\sigma = 37.3125$

$YEAR := 1900, 1905 .. 2100$



Here is a zoom close to the Hubbard's peak estimated to have occured in 2004. Note the increased volatility close to the peak.



The model predictions for the US oil consumption (quads/year) in the year 1950 is:

$$\text{hubbert}\left(1950, N_{max}, T_{max}, \sigma\right) = 12.9159$$

The 1950 US oil consumption was 12.5quad/yr.
The percentage error is:

$$100 \cdot \frac{12.916 - 12.5}{12.5} = 3.328 \ \%$$

The model prediction for the US oil consumption (quads/year) in the year 2005 is:

$$\text{hubbert}\left(2005, N_{max}, T_{max}, \sigma\right) = 37.279$$

The 2005 US oil consumption was: 40.39quads
The percentage error is:

$$100 \cdot \frac{40.39 - 37.28}{40.39} = 7.7 \ \%$$