THERMAL PHYSICS PHY474 LAB. #0
TUTORIAL and ENTROPY MAXIMIZATION

First we learn how to do simple computations with MathCad.

(1) Type: 3*4= The answer is: 12.

$$3 \cdot 4 = 12$$

(2) Type 24-35= The answer is -11

$$24 - 35 = -11$$

(3) Type: 3\= (this is square root of 3). The answer is 1.73

$$\sqrt{3} = 1.732$$

(4) Type 2.3^4.5= (this is 2.3 to the power 4.5). The answer is 42.44

$$2.3^{4.5} = 42.44$$

(5) Type: (3-4)*4-6= The answer is -10

$$(3 - 4) \cdot 4 - 6 = -10$$

(6) Type: 7/(2.3-8.6)space bar+3=. The answer is: 1.889

$$\frac{7}{(2.3 - 8.6)} + 3 = 1.889$$

We learn next to do symbolic calculations and to graph with MathCad.
Let us consider two objects of temperatures T_1 and T_2 and heat capacities C_1 and C_2. The equilibrium temperature is obtained by equating the heat leaving the hot object 2 to the heat entering the cold object 1.
$Q = C_1(T-T_1)= C_2(T_2-T)$. Solve for: $T = (C_1T_1 + C_2T_2)/(C_1 + C_2)$. Note: $T_1 < T < T_2$
Since: dQ = CdT = TdS we can calculate the entropy change by integrating: dS = CdT/T. We use the "evaluate symbolically" operator from "evaluation and boolean palette".

$$\int_{T_1}^{T} \frac{C_1}{t}\,dt \rightarrow \begin{cases} C_1\cdot\left(\ln(T) - \ln\left(T_1\right)\right) & \text{if } 0 > T \vee T_1 > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

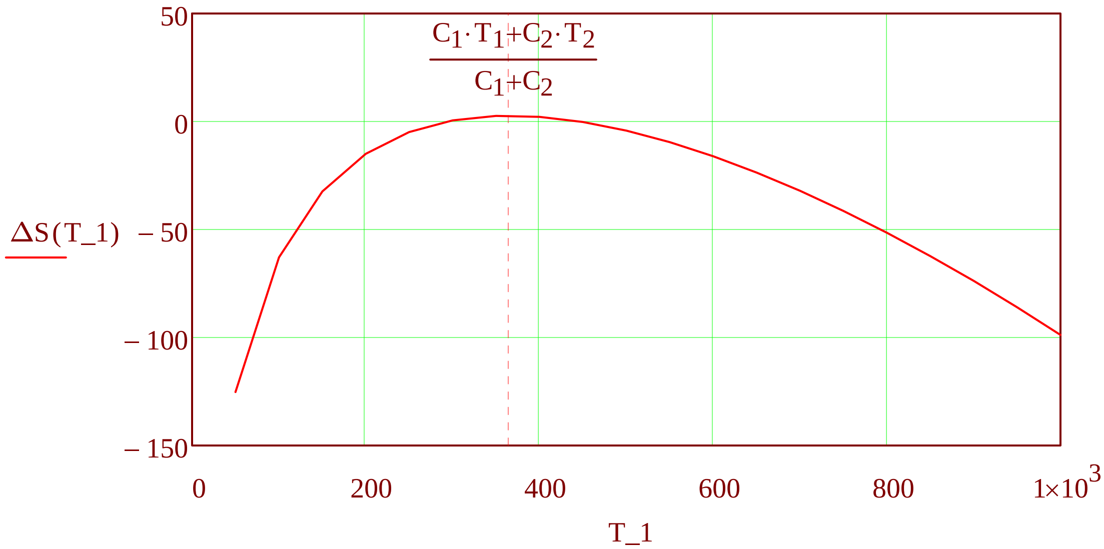
So the entropy change for object 1 is $\Delta S_1 = C_1\ln(T/T_1)$ and for object 2 is $\Delta S_2 = C_2\ln(T/T_2)$. Note: $\Delta S_1 > 0$, $\Delta S_2 < 0$. The total entropy change is positive acording to the 2'nd law: $\Delta S_1 + \Delta S_2 > 0$.
The fact that the two objects equilibrate at the same temperature is required by the 2'nd law of thermodynamics: entropy is at its largest possible value consistent with constraints. We are going to demonstrate this statement. Let us denote T_1 the final temperature of 1 and T_2 final temperature of 2. From: $Q = C_1(T\_1-T_1) = C_2(T_2-T\_2)$ we get: $T\_2 = T_2 + (C_1/C_2)(T_1-T\_1)$. We then graph $\Delta S(T\_1)$ and show that the entropy change is largest when: $T\_1 = T\_2 = (C_1T_1 + C_2T_2)/(C_1 + C_2)$.
Object #1 is a porcelain piece of mass 500g, specific heat 0.22cal/g*K and temperature 20C.
Object #2 is water of mass 1000g, specific heat 1cal/g*K and temperature 100C.

$C_1 := 500\cdot.22$ $\quad C_2 := 1000\cdot1$ $\quad T_1 := 20 + 273$ $\quad T_2 := 100 + 273$

$C_1 = 110$ $\quad C_2 = 1\times10^3$ $\quad T_1 = 293$ $\quad T_2 = 373$

$$\Delta S(T\_1) := C_1\cdot\ln\left(\frac{T\_1}{T_1}\right) + C_2\cdot\ln\left[\frac{\frac{C_1}{C_2}\cdot\left(T_1 - T\_1\right) + T_2}{T_2}\right]$$

$T\_1 := 50, 100 .. 1000$



Note that at the equilibrium temperature: $T := \frac{C_1 \cdot T_1 + C_2 \cdot T_2}{C_1 + C_2}$, $T = 365.072$ KelvinΔS the entropy increase is largest.
Note that the entropy change for the object #1 (porcelain) is
$C_1 \cdot \ln\left(\frac{T}{T_1}\right) = 24.191$ calories/Kelvin and for object #2 (water) is
$C_2 \cdot \ln\left(\frac{T}{T_2}\right) = -21.484$ calories/Kelvin. The total entropy change is positive:
$24.191 - 21.484 = 2.707$ calories/Kelvin.

We can show the fact that ΔS(T_1) is maximum when the temperatures are the same by using the symbolic processor. First copy the expression for ΔS(T_1).

$$C_1 \cdot \ln\left(\frac{T\_1}{T_1}\right) + C_2 \cdot \ln\left[\frac{\frac{C_1}{C_2} \cdot \left(T_1 - T\_1\right) + T_2}{T_2}\right]$$

Then set the blue marker on T_1 and use Differentiate under Variable under Symbolics.

$$\frac{C_1}{T\_1} - \frac{C_1}{\left[\frac{C_1}{C_2} \cdot \left(T_1 - T\_1\right) + T_2\right]}$$

Next set the above expression equal to zero by using the Boolean Equal from the Evaluation and Boolean palette.

$$\frac{C_1}{T\_1} - \frac{C_1}{\left[\frac{C_1}{C_2} \cdot \left(T_1 - T\_1\right) + T_2\right]} = 0$$

Then set the blue marker on T_1 and use Solve under Variable under Symbolics.

$$\frac{\left(C_1 \cdot T_1 + T_2 \cdot C_2\right)}{\left(C_1 + C_2\right)}$$