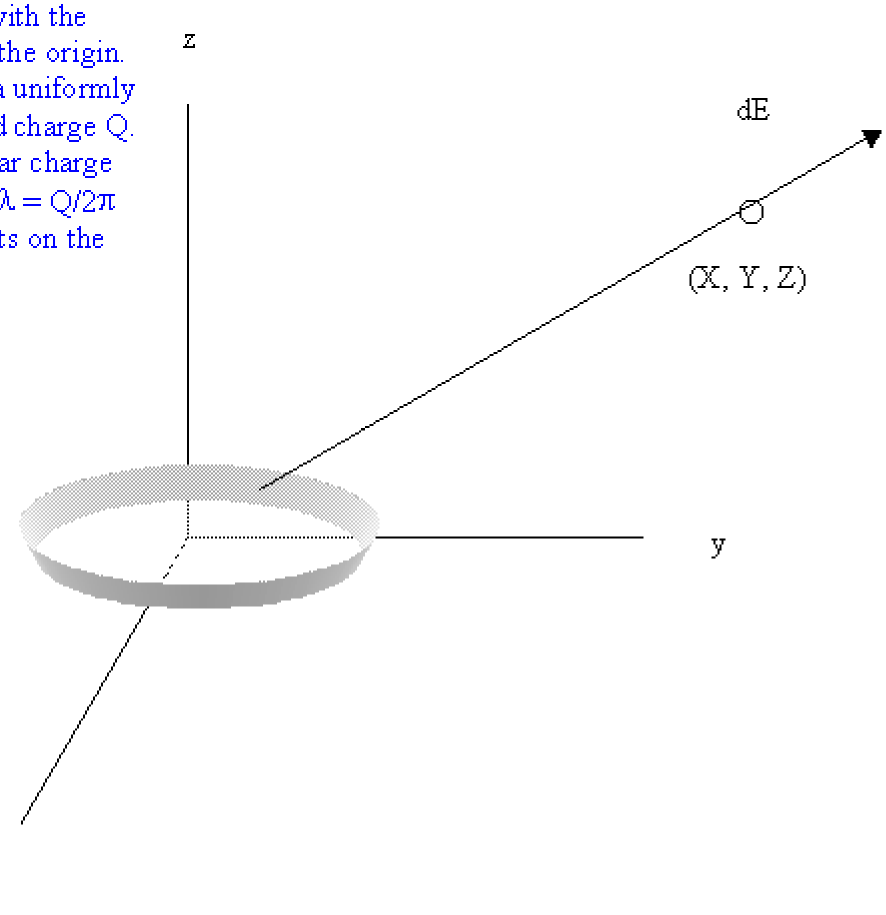
Electricity and Magnetism
Computer Lab #2b: Electrostatics, Electric Field of Charged Ring

We calculate the electric field at a pont (X,Y,Z) created by a circular insulator of radius R carying a charge Q, lying in the xy plane and centered on the origin.

We divide the insulator in small pieces of length $Rd\theta$ carying a charge $dQ=(Q/2\pi)d\theta$.
The electric field created by the charge dQ is:
$dE=dQ/\{4\pi\varepsilon_0[(X - R\cos(\theta))^2 + (Y - R\sin(\theta))^2 + Z^2]\}$.
The x component of the electric field is:
$dE_x = dE(X - R\cos(\theta))/[(X - R\cos(\theta))^2 + (Y - R\sin(\theta))^2 + Z^2]^{1/2}$.
The y component of the electric field is:
$dE_y = dE(Y - R\sin(\theta))/[(X - R\cos(\theta))^2 + (Y - R\sin(\theta))^2 + Z^2]^{1/2}$.
The z component of the electric field is:
$dE_z = dEZ/[(X - R\cos(\theta))^2 + (Y - R\sin(\theta))^2 + Z^2]^{1/2}$.

We get the electric field components by integrating over θ from 0 to 2π.

An insulating ring of radius R lies in the xy plane with the center on the origin. It carries a uniformly distributed charge Q. The angular charge density is $\lambda = Q/2\pi$ at all points on the ring.



$q := 5 \cdot 10^{-8}$ $\quad r := .5$ $\quad \varepsilon_0 := 8.85 \cdot 10^{-12}$ $\quad \frac{1}{4 \cdot \pi \cdot \varepsilon_0} = 8.992 \times 10^{9}$

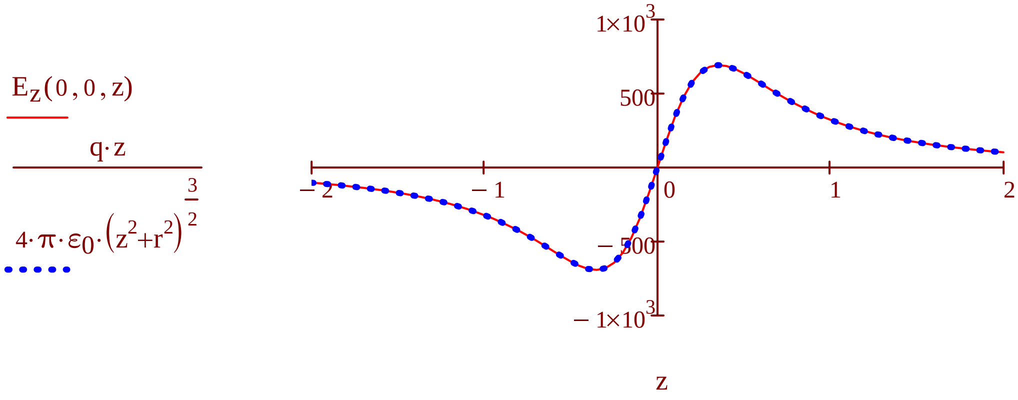
$$E_x(x,y,z) := \int_0^{2\cdot\pi} \frac{q}{8\cdot\pi^2\cdot\varepsilon_0}\cdot\frac{1}{\left[(x-r\cdot\cos(\theta))^2+(y-r\cdot\sin(\theta))^2+z^2\right]^{\frac{3}{2}}}\cdot(x-r\cdot\cos(\theta))\,d\theta$$

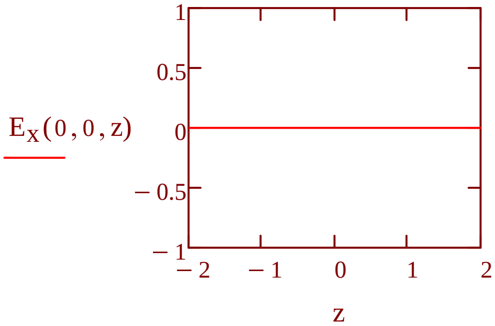
$$E_y(x,y,z) := \int_0^{2\cdot\pi} \frac{q}{8\cdot\pi^2\cdot\varepsilon_0}\cdot\frac{1}{\left[(x-r\cdot\cos(\theta))^2+(y-r\cdot\sin(\theta))^2+z^2\right]^{\frac{3}{2}}}\cdot(y-r\cdot\sin(\theta))\,d\theta$$

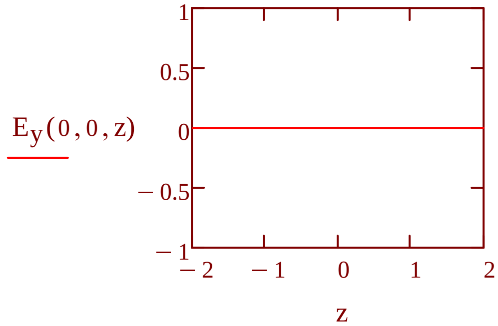
$$E_z(x,y,z) := \int_0^{2\cdot\pi} \frac{q}{8\cdot\pi^2\cdot\varepsilon_0}\cdot\frac{1}{\left[(x-r\cdot\cos(\theta))^2+(y-r\cdot\sin(\theta))^2+z^2\right]^{\frac{3}{2}}}\cdot z\,d\theta$$

We compare the numerically computed electric field to the analytical result of Griffiths problem 2.5.

$z := -2, -1.95 .. 2$







We get next the lines of electric field in the plane of the ring.

$i := 0..20$ $j := 0..20$

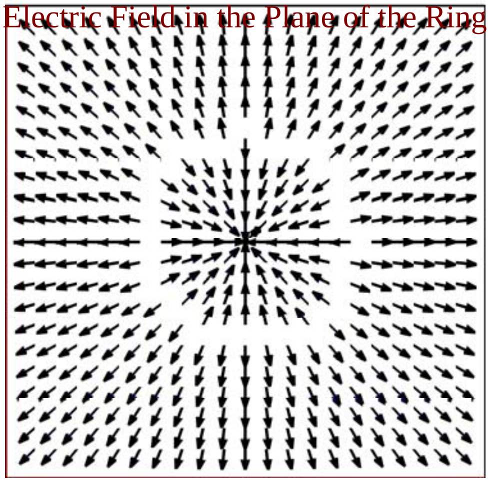
$x_{i,j} := \frac{i - 10.01}{10.01}$ $y_{i,j} := \frac{j - 10.01}{10.01}$

We now show the electric field in the plane of the ring:

$$e_{x_{i,j}} := \frac{E_x(x_{i,j}, y_{i,j}, 0)}{\sqrt{E_x(x_{i,j}, y_{i,j}, 0)^2 + E_y(x_{i,j}, y_{i,j}, 0)^2}}$$

$$e_{y_{i,j}} := \frac{E_y(x_{i,j}, y_{i,j}, 0)}{\sqrt{E_x(x_{i,j}, y_{i,j}, 0)^2 + E_y(x_{i,j}, y_{i,j}, 0)^2}}$$

Electric Field in the Plane of the Ring



(e_x, e_y)

Note the electric field points along the radial directions and away from the ring.