ENVIRONMENTAL PHYSICS COMPUTER LAB #4: BLACKBODY RADIATION

EARTH RADIATION

We will use SI units throughout this worksheet. To speed up the computations we will not use the MathCad built-in units.

$\lambda := 10^{-6}, 2\cdot 10^{-6} .. 30\cdot 10^{-6}$ Wavelength range

$k_B := 1.381\cdot 10^{-23}$ The Boltzmann constant

$c := 3.0\cdot 10^{8}$ Speed of light

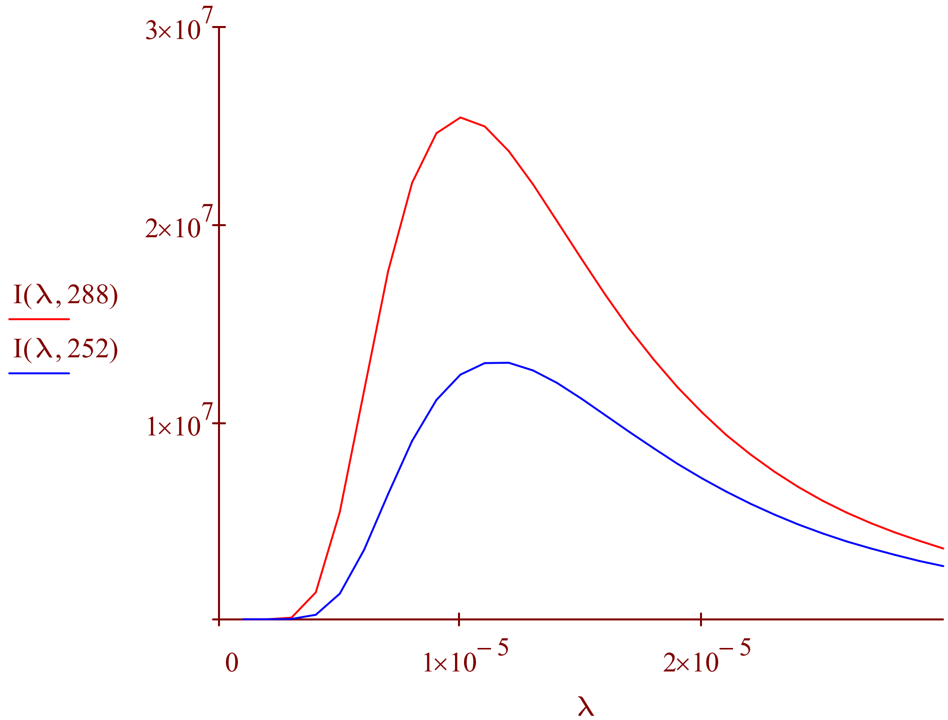
$h := 6.63\cdot 10^{-34}$ The Planck constant

$$I(\lambda, T) := \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5\cdot\left(e^{\frac{h\cdot c}{\lambda\cdot k_B\cdot T}} - 1\right)}$$

$I(\lambda,T)$ is the spectral intensity density per unit wavelength at a temperature T.

$$\lambda_{max}(T) := \frac{2.898\cdot 10^{-3}}{T}$$

According to the Wien Law the maximum in the curve occurs for a wavelength which is inversely proportional to the temperature.



Earth's average temperature is 288K. The maximum in the spectral intensity occurs in the infrared part of the spectrum. The graphs for 288K and 312K show: (a) when T increase the maximum shifts to lower λ; (b) when T increases the area under the curve (intensity) increases. To demonstrate the second statement mathematically we are going to compute the area under the curve by integrating numerically the spectral intensity.

$$\int_{10^{-7}}^{10^{-1}} I(\lambda, 288)\, d\lambda = 389$$

We compare this to the theoretical result: σT^4 where σ is the Stefan constant. The integral should be evaluated from 0 to infinity. To avoid overflow and underflow we chose an appropriate integration range.

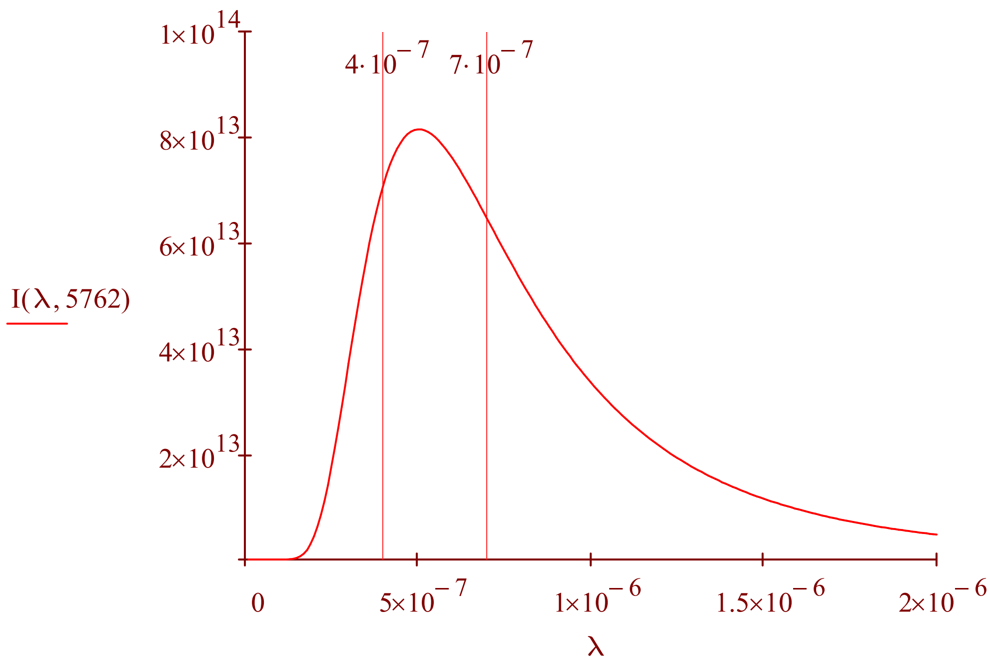
$$\sigma := 5.67 \cdot 10^{-8}$$

$$\sigma \cdot 288^4 = 390$$

SUN'S BLACK-BODY RADIATION

$\lambda := 10^{-8}, 2\cdot 10^{-8} .. 200\cdot 10^{-8}$ The wavelength range

The Sun's surface temperature is 5762K [Source: M.P.Thekaekara et al., Applied Optics, 8, 1713-1732(1969)].



The graph shows the visible range of the spectrum: 400nm to 700nm. The wavelength giving the maximum spectral intensity is in the visible part of the spectrum: $\lambda_{max}(5762) = 5\times 10^{-7}$

The fraction of all the Sun radiation in the visible part of the spectrum is:

$$\frac{\int_{4\cdot 10^{-7}}^{7\cdot 10^{-7}} I(\lambda, 5762)\, d\lambda}{\sigma\cdot 5762^4} = 0.365$$