ENVIRONMENTAL PHYSICS
COMPUTER LAB#6: RADIOACTIVITY

In this workshhet we simulate radioactive decay. We also learn to use the data analysis functions provided by MathCad. Start with N_0 nuclei of $^{14}C_6$. The probability that such a nucleus will decay in one year is: $\lambda = 1.21*10^{-4} year^{-1}$. Carbon 14 undergoes β decay: $^{14}C_6 \rightarrow {}^{14}N_7 + {}^{0}e_{-1} + n$. This decay is used in carbon dating of objects 1000 years to 25000 years old.

$N_0 := 5000 \quad T := 100 \quad \lambda := 1.21 \cdot 10^{-4}$

$j := 0 .. N_0 - 1 \quad n := 0 .. T - 1$

$\Delta t := 200 \quad \lambda\Delta t := \lambda \cdot \Delta t$

The time interval Δt should be small enough so: $\lambda\Delta t << 1$. Since λ is expressed in $year^{-1}$, Δt is expressed in years.

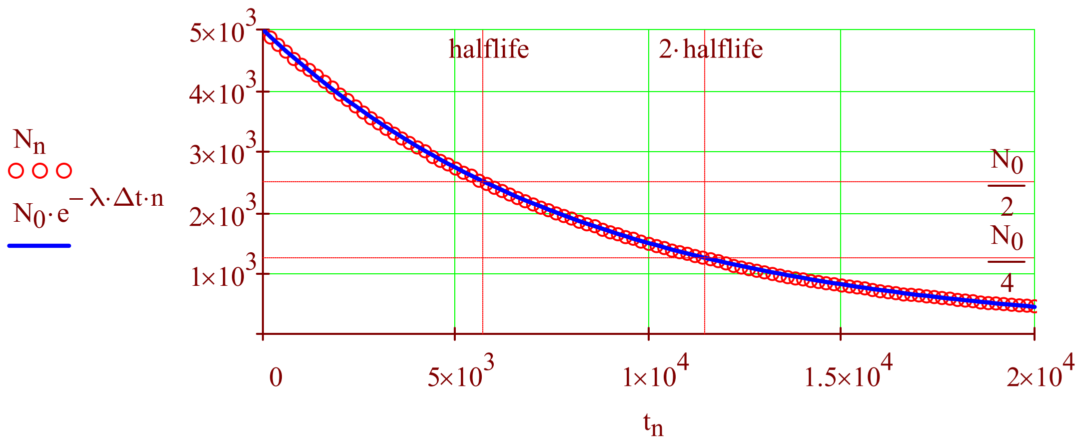
$nuclei_{0,j} := 1$

$nuclei_{n+1,j} := (rnd(1) > \lambda\Delta t) \cdot nuclei_{n,j}$

$n := 0 .. T \quad t_n := n \cdot \Delta t$

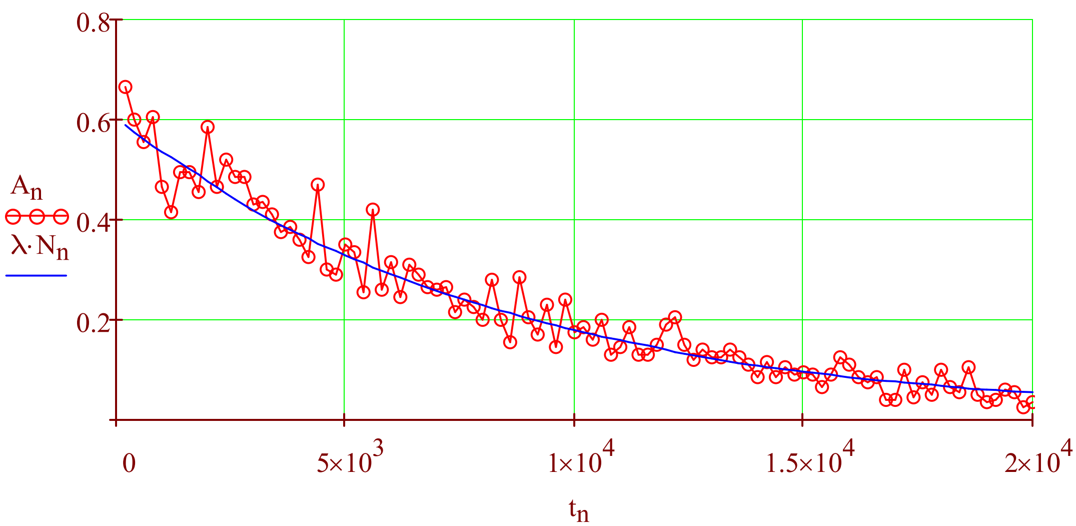
The variable $nuclei_{n,j}$ is 1 if the nucleus #j is alive at time $n\Delta t$ or 0 if the nucleus #j is dead (i.e. it decayed) at time $n\Delta t$. The quantity N_n gives the number of nuclei alive at time $n\Delta t$. The half-life time is: $\ln(2)/\lambda$.

$$N_n := \sum_{j=0}^{N_0-1} nuclei_{n,j} \qquad halflife := \frac{\ln(2)}{\lambda} \qquad halflife = 5.728 \times 10^3$$

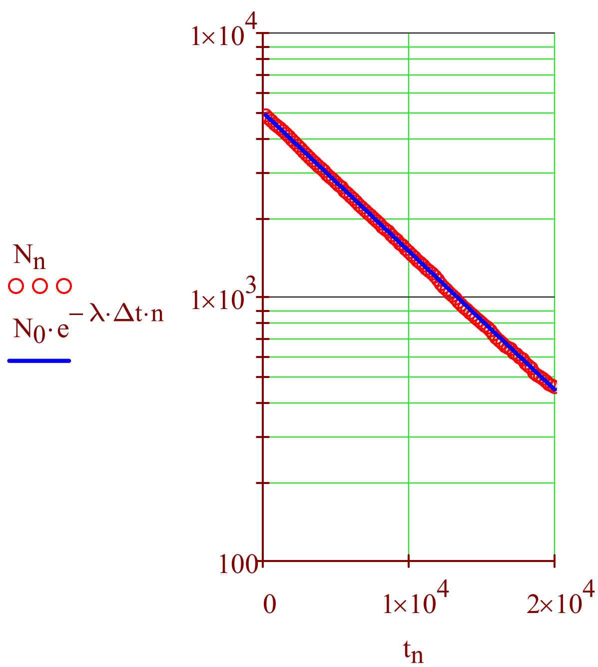


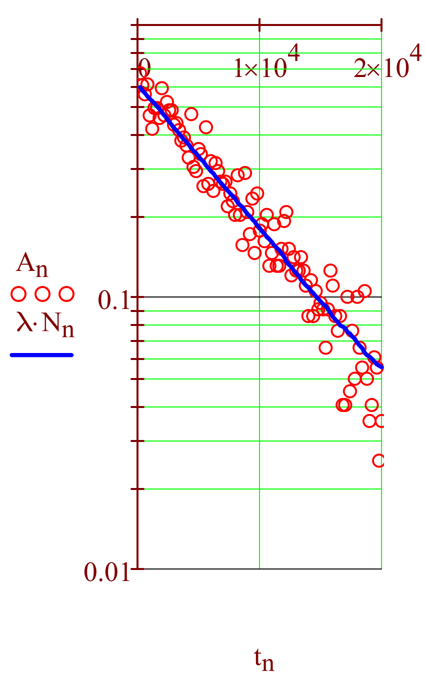
Activity is the number of decays per time interval: $A = |\Delta N/\Delta t|$. The activity is equal to: $A = \lambda N$. In the following the activity is expressed in decays /year. We are going to check this with our simulation.

$$n := 1 .. T \qquad A_n := \frac{|N_n - N_{n-1}|}{\Delta t}$$



When graphed on a semilog plot the data follows a linear dependence.





Data Analysis: We use the least-squares method to determine the line:
$\ln N = -\lambda t + \ln N_0$. The estimate of $-\lambda$ is given by the Mathcad function *slope;* the estimate of of N_0 is obtained by using the function *intercept*. The quality of the fit is measured by the Pearson correlation coefficient called *corr* in Mathcad. If the coefficient is closed to +1 or -1 then the data is consistent with the linear relation. The opposite holds if the coefficient is close to zero. Read more about *slope*, *intercept* and *corr* under Help.

$n := 0 .. T$

$\ln N_n := \ln(N_n)$

$\text{estimate}\lambda := -\text{slope}(t, \ln N)$

$\text{estimate}\lambda = 1.218 \times 10^{-4}$

$$\frac{\text{estimate}\lambda - \lambda}{\lambda} = 6.276 \times 10^{-3}$$

$\text{estimate}N_0 := e^{\text{intercept}(t, \ln N)}$

$\text{estimate}N_0 = 4.988 \times 10^{3}$

$$\frac{\text{estimate}N_0 - N_0}{N_0} = -2.419 \times 10^{-3}$$

$\text{corr}(t, \ln N) = -1$

Verify that the quality of the fit as measured by the Pearson correlation coefficient improves as N_0 increases.

We repeat the same procedure for the activity. You will note that the Pearson correlation coefficient is worse than in the N data. Can you explain this?

$n := 1 .. T$

$\ln A_n := \ln(A_n)$

$\text{estimate}\lambda := -\text{slope}(t, \ln A)$

$\text{estimate}\lambda = 1.318 \times 10^{-4}$

$$\frac{\text{estimate}\lambda - \lambda}{\lambda} = 0.089$$

$\text{estimate}A_0 := e^{\text{intercept}(t, \ln A)}$

$\text{estimate}A_0 = 0.656$

$$\frac{\text{estimate}A_0 - \lambda \cdot N_0}{\lambda \cdot N_0} = 0.085$$

$\text{corr}(t, \ln A) = -0.962$