Degrees of Freedom

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The Freedom To Vary

- First, forget about statistics. Imagine you're a person who loves to wear hats. You couldn't care less what a degree of freedom is. You believe that variety is the spice of life.
- Unfortunately, you have constraints. You have only 7 hats. Yet you want to wear a different hat every day of the week.

- On the first day, you can wear any of the 7 hats. On the second day, you can choose from the 6 remaining hats, on day 3 you can choose from 5 hats, and so on.

- When day 6 rolls around, you still have a choice between 2 hats that you haven't worn yet that week. But after you choose your hat for day 6, you have no choice for the hat that you wear on Day 7. You must wear the one remaining hat. You had 7-1 = 6 days of "hat" freedom—in which the hat you wore could vary!

- That's kind of the idea behind degrees of freedom in statistics. Degrees of freedom are often broadly defined as the number of "observations" (pieces of information) in the data that are free to vary when estimating statistical parameters.

Degrees of Freedom for a Mean

- Imagine you have measured number of cats a person owns. You have done this with 5 people.
- What are your degrees of freedom?

Case ID	Number of Cats
Person 1	
Person 2	
Person 3	
Person 4	
Person 5	
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- We know the total, and the *n*, so we know the mean (2.0).
- If we know the total, do we have "freedom" for case 1?

Case ID	Number of Cats
Person 1	2
Person 2	
Person 3	
Person 4	
Person 5	
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- Yes. Let‘s say person 1 has 2 cats.
- Now, do we have freedom for case 2?

Case ID	Number of Cats
Person 1	2
Person 2	0
Person 3	
Person 4	
Person 5	
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- Yes. Let‘s say person 2 has no cats.
- Now, do we have freedom for case 3?

Case ID	Number of Cats
Person 1	2
Person 2	0
Person 3	1
Person 4	
Person 5	
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- Yes. Let‘s say person 3 has 1 cat.
- Now, do we have freedom for case 4?

Case ID	Number of Cats
Person 1	2
Person 2	0
Person 3	1
Person 4	4
Person 5	
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- Yes. Let‘s say person 4 has 4 cats.
- Now, do we have freedom for case 5?

Case ID	Number of Cats
Person 1	2
Person 2	0
Person 3	1
Person 4	4
Person 5	3 (NO FREEDOM)
TOTAL CATS	10
MEAN CATS PER PERSON	10/5 = 2

- No. Person 5 must have 3 cats.
- Thus, for n = 5, we had 4 degrees of freedom.
- **For the calculation of a mean, df is n – 1.**

Degrees of Freedom for a Chi-Square

	Nuclear family	Single parent	Blended family	TOTAL
Permissive				20
Authoritarian				65
Authoritative				15
TOTAL	30	45	25	100

- Imagine we have two nominal variables, parenting style and family type. And imagine that the marginal frequencies are as shown above.

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	???			20
Authoritarian				60
Authoritative				20
TOTAL	30	40	30	100

- Do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	???		20
Authoritarian				65
Authoritative				15
TOTAL	30	45	25	100

- Yes. Let's say there are 10 people who have a Nuclear family and practice Permissive parenting.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	???	20
Authoritarian				65
Authoritative				15
TOTAL	30	45	25	100

- Yes. Let's say there are 5 people who are a Single parent and practice Permissive parenting.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	???			65
Authoritative				15
TOTAL	30	45	25	100

- No. That cell must have 5 cases, because of the marginal total of 20.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	???		65
Authoritative				15
TOTAL	30	45	25	100

- Yes. Let's say there are 10 cases that have a Nuclear family and practice Authoritarian parenting.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	???	65
Authoritative				15
TOTAL	30	45	25	100

- Yes. Let's say there are 40 cases that are a Single parent and practice Authoritarian parenting.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	15 (NO FREEDOM)	65
Authoritative	???			15
TOTAL	30	45	25	100

- No. That cell must have 15 cases, because of the marginal total of 65.
- Now, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	15 (NO FREEDOM)	65
Authoritative	10 (NO FREEDOM)	???		15
TOTAL	30	45	25	100

- No. That cell must have 10 cases because of the marginal total of 30.
- Do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	15 (NO FREEDOM)	65
Authoritative	10 (NO FREEDOM)	0 (NO FREEDOM)	???	15
TOTAL	30	45	25	100

- No. That cell must have 0 cases because of the marginal total of 45.
- Finally, do we have freedom for the cell marked ??? above?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	15 (NO FREEDOM)	65
Authoritative	10 (NO FREEDOM)	0 (NO FREEDOM	5 (NO FREEDOM)	15
TOTAL	30	45	25	100

- No. That cell must have 5 cases because of the marginal total of 25, and also because of the marginal total of 15.
- So how many degrees of freedom does this chi-square table have?

	Nuclear family	Single parent	Blended family	TOTAL
Permissive	10	5	5 (NO FREEDOM)	20
Authoritarian	10	40	15 (NO FREEDOM)	65
Authoritative	10 (NO FREEDOM)	0 (NO FREEDOM	5 (NO FREEDOM)	15
TOTAL	30	45	25	100

- The yellow cells were "free"...thus, we have 4 degrees of freedom.
- **For a chi-square test, degrees of freedom are (r – 1) x (c – 1)**

  where r = number of rows and c = number of columns

	A	B	C	D	TOTAL
E					
F					
G					
H					
I					
TOTAL					

- So for a 5 by 4 chi-square table, the df is (5 – 1) x (4 – 1) = 4 x 3 = 12.

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