Neuendorf

Moments about the Mean

The mean and standard deviation are closely related to a family of descriptive statistics known as *moments*. The first four moments about the arithmetic mean are:

Moment 1 $= \frac{\Sigma(X - \text{Mean})}{n} = 0$

Moment 2 $= \frac{\Sigma(X - \text{Mean})^2}{n} = \frac{n-1}{n} \times s^2$ (variance)

Moment 3 $= \frac{\Sigma(X - \text{Mean})^3}{n}$ is related to the calculation of the skewness

Moment 4 $= \frac{\Sigma(X - \text{Mean})^4}{n}$ is related to the calculation of the kurtosis

The term "moment" comes from mechanics. When applied to frequency distributions, the origin is analogous to the fulcrum on a lever, and the frequencies in the various intervals are analogous to forces operating at various distances from the origin.

The first two moments give us some understanding of the **mean** and **standard deviation** (*s*). The third and fourth moments can be used to construct measures of **skewness** and **kurtosis**.

A common measure of skewness is:

$$g1 = \frac{\text{moment 3}}{\text{moment 2} \sqrt{\text{moment 2}}}$$

The rationale for this statistic is based on the observation that when a distribution is symmetrical, the sum of deviations above the mean, when raised to the third power, will balance the sum of deviations below the mean, when raised to the third power. Thus, for a symmetrical distribution, moment 3 = 0 and g1 = 0.

A common measure of kurtosis is:

$$g2 = \frac{\text{moment 4}}{(\text{moment 2})^2} - 3$$

Large deviations from the mean, when raised to the fourth power, will contribute substantially to the fourth moment. (Kurtosis actually is more about the thickness of a distribution's tails than the peakedness.)

Source: Ferguson, G. A., & Takane, Y. (1989). *Statistical analysis in psychology and education* (6th ed.). New York: McGraw-Hill.

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