Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences

Second Edition

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2.9 STATISTICAL POWER

2.9.1 Introduction

In the last section we presented methods of appraising sample data in regard to α, the risk of mistakenly rejecting the null hypothesis when it is true, that is, drawing a spuriously positive conclusion (Type I error). We now turn our attention to methods of determining β, the probability of failing to reject the null hypothesis when it is false (Type II error), and ways in which β can be controlled in planning research.

Any given statistical test of a null hypothesis can be viewed as a complex relationship among the following four parameters:

1. The *power* of the test, defined as $1 - \beta$ (the probability of rejecting the null hypothesis).
2. The region of rejection of the null hypothesis as determined by the α level and whether the test is one-tailed or two-tailed. As α increases, power increases.
3. The sample size n. As n increases, power increases.
4. The magnitude of the effect in the population, or the degree of departure from the null hypothesis. The larger this is, the greater the power.

These four parameters are so related that when any three of them are fixed, the fourth is completely determined. Thus, when an investigator decides for a given research plan his significance criterion α and the sample size he will use, the power of his test is determined. However, the investigator does not in general know what this power is, since he does not know the magnitude of the effect size (ES) in the population.

There are three general strategies for determining the size of the population effect that a research is trying to detect:

1. To the extent that studies that have been carried out by the current investigator or others are closely related to the present investigation, the ESs found in these studies reflect the magnitude that can be expected. Thus, if a review of the relevant literature reveals r's ranging from .32 to .43, the population ES in the current study may be expected to be somewhere in the vicinity of these values. If the investigator wishes to be conservative he may wish to determine the power to detect a population r of .25 or .30.

[5]Some investigators follow the practice of counting the rate of .05 - level significant r's, here 2 out of 10, or .20 and, finding it greater than .05, conclude that there is at least *some* nonzero correlation in the population. Although fancier, this approach is also invalid, because the rate of significance must then be demonstrably *significantly* greater than .05, and that is not the case here.

2. In some research areas an investigator may posit some minimum population effect that would have either practical or theoretical significance. Investigator A determines that unless the population $r = .5$, the importance of the relationship is insufficient to warrant a change in the policy or operations of the relevant institution. Another investigator may decide that a population $r = .10$ would have a material import for the adequacy of the theory within which the experiment has been designed, and thus would wish to plan the experiment so as to detect such an ES

3. A third strategy in deciding what ES values to use in determining the power of a study is to use certain suggested conventional definitions of *small*, *medium*, and *large* effects (Cohen, 1977). These conventional ES values may be used either by choosing one of these values (for example, the conventional medium effect size is a population r of .30) or by determining power for all three population ESs. If the latter strategy is taken, the investigator would then make a revision in his research plan according to his estimation of the relevance of the various ESs to his problem.

The point in doing an analysis of the power of a given research plan is that when the power turns out to be insufficient the investigator may decide to revise these plans, or even drop the investigation entirely if such revision is impossible. Obviously, because little or nothing can be done after the investigation is completed, determination of statistical power is of primary value as a preinvestigation procedure. If power is found to be insufficient, the research plan may be revised in ways that will increase it, primarily by increasing n (or possibly, by increasing α). A more complete general discussion of the concepts and strategy in statistical power analysis may be found in Cohen (1965, 1977), and further discussion of power analysis in MRC in Sections 3.8 and 4.5.

2.9.2 Power of Tests of the Significance of *r* and *B*

When the null hypothesis to be tested is that the population $r = 0$, r itself is the appropriate measure of ES. Thus the investigator may proceed by determining the population r that he expects (or wishes to allow for), the α criterion he plans to use in the determination of significance, and the sample size included in his plan. Having made such determinations, he may then enter tables with these values and look up the resulting power, $1 - \beta$, or probability that his statistical test will be significant if he has correctly assessed the population value of r. For example, an investigation is planned for which a population $r = .30$ is posited, either on the basis of previous work in the area or because .30 is a conventional definition of a medium effect size. It is planned to test the null hypothesis with α (two-tailed) set at .05 and to gather data on 50 subjects. The investigator enters the r power table for $\alpha = .05$ (Appendix Table F.2) with $r = .30$ and $n = 50$ and reads off power = .57. Thus, if the population $r = .30$, the odds are only a little better than 50–50 that the statistical test will be significant and the null hypothesis rejected. If these odds sufficiently distress the investigator (as they should) he may determine that if he increases n to 80 the power will be increased to .78, and that with $n = 84$, power = .80. It has been proposed (Cohen, 1965; 1977) that much as $\alpha = .05$ is used as a convention for significance, power $= 1 - \beta = .80$ be used as a convention for power (see Section 4.5.5). Thus a decision may be made to increase the sample size to 84 to effect the necessary increase in power to .80.

The form of power analysis that the preceding suggests and that is frequently very useful is the direct determination of the necessary sample size (n^*) to attain a given desired power ($1 - \beta$) to detect a specified population r for a specified α. Appendix Tables G.1 and G.2 are designed for this purpose. For example, with (as before) a population $r = .30$, at $\alpha = .05$, what sample size do we need (n^*) for power to be .80? Entering Table G.2 (for $\alpha = .05$) for column $r = .30$ and (row) desired power = .80, we read out $n^* = 84$ (as before). For power under these conditions to be .90, $n^* = 112$, that is, an increase of 28 cases (or 28/84 = 33%) over the sample size necessary for power to be .80 (for these conditions of r and α). If the more stringent $\alpha = .01$ is used in the above problem, for power = .80, $n^* = 124$ and for power = .90, $n^* = 157$ (Appendix Table G.1).

The importance of power analysis as part of research planning cannot be stressed too heavily. The thoughtful use of Appendix Tables F and G will greatly facilitate the related decisions about sample size, α, and the power to which one can aspire during the planning of a research, and to some degree, the interpretation of its results afterwards (Cohen, 1973b).

Conventional magnitudes of r corresponding to small, medium, and large ES that have been suggested as appropriate at least for many areas of psychological investigation are $r = .10$, .30, and .50, respectively. When there is reason to believe that the population ES is small (i.e., $r = .10$), rather large values of n are required—for $\alpha = .05$ and $1 - \beta = .80$, n must be $n^* = 783$ (Appendix Table G.2), and if more stringent (lower) α or β risk levels are desired, for example, $\alpha = .01$ and power = .95, even a sample of 1000 is insufficient to accomplish the goal ($n^* = 1790$, Appendix Table G.1).

Because the statistical test of the H_0 (null hypothesis): $r = 0$ is simultaneously a test of H_0: $B = 0$ (see Section 2.8.2), the power analysis for B may be carried out by means of its associated r.

2.9.3 Power Analysis for Other Statistical Tests Involving *r*

We have described, among other statistical tests involving r, the test that a population r has some specified (nonzero) value, and the test of the hypothesis that two population r's are equal. Both of these tests involve z' and require different definitions of ES and power and n^* tables from those used above. Because neither of these tests arise with much frequency, we conserve the space necessary for their exposition and instead refer the reader to Cohen (1977), which is a handbook of power analysis that fully treats these and most other cases of power analysis encountered in practice.

3.7 POWER ANALYSIS

3.7.1 Introduction

Section 2.9.1 explained the purpose and desirability of determining the power of a given research plan to reject at the α significance level a false null hypothesis that the population r equals zero. Thus, given a plan for determining the existence of nonzero correlation between two variables, including n and α, the investigator may enter the table for the selected α with n and the expected (alternate-hypothetical) value of the population r, and read off the power—the probability of finding the sample r to be significant. Alternatively, one may proceed in planning a research by deciding on the significance criterion α and the desired power. Then, having specified the expected population r, a table for the given α is entered with this r and the desired power. The tabled values provide the number of cases necessary (n^*) to have the specified probability of rejecting the null hypothesis (the desired power) at the α level of significance when the population r is as posited. In this section, we extend power analysis beyond simple correlation to the more general MRC for k IVs.

3.7.2 Power Analysis for R^2

Power and n^* can be conveniently tabled for the single IV case. However, the several different coefficients that may be tested in MRC analysis as well as provision for many possible values of k makes the direct tabling of power and n^* unwieldly. Instead, we provide a table of constants with which one can perform

power analysis of tests for the different null hypotheses in MRC with k IVs. These constants are then employed in a simple formula to determine the necessary number of cases (n^*).

To determine n^* for the F test of the significance of R^2, the researcher proceeds with the following steps:

1. Set the significance criterion to be used, α. Provision is made in the Appendix for $\alpha = .01$ and $\alpha = .05$ in the L tables (Appendix Tables E.1 and E.2).
2. Set desired power for the F test. The L tables provide for power values of .10, .30, .50, .60, .70, .75, .80, .85, .90, .95, and .99. (The use of the lower values is illustrated in Chapter 4.)
3. In the L tables (Appendix Tables E.1 and E.2), k_B is used to represent the number of df associated with the source of Y variance being tested. For $R^2_{Y\cdot 12\ldots k}$, k_B is simply k, the number of IVs. The L tables provide for k_B = 1 (1) 16 (2) 24 (4) 40 (10) 100, that is, for 30 values of k_B between 1 and 100.
4. Look up in the appropriate table ($\alpha = .01$ or .05) the value of L for the given k_B (row) and specified power (column).
5. Determine the population effect size, ES ($=f^2$, see following) of interest, and the expected or alternate-hypothetical value. As was the case for the single IV (where ES = r), the ES may represent a probable population value as indicated by previous work, a minimum value that would be of theoretical or practical significance, or some conventional value as discussed in Section 4.5.4.

The population ES for R^2 is given by

$$f^2 = \frac{R^2}{1 - R^2}. \tag{3.7.1}$$

6. Substitute L (from step 4) and f^2 in

$$n^* = \frac{L}{f^2} + k + 1. \tag{3.7.2}$$

The result is the number of cases necessary to have the specified probability of rejecting the null hypothesis (power) at the α level of significance when f^2 in the population is as posited.

For example, let us return to the research on academic salaries. As part of the planning preceding the research, the investigator performs a power analysis for R^2 in order to determine the n^* to be used. It is planned to use the $\alpha = .05$ significance criterion (Step 1), to have a .90 probability of rejecting the null hypothesis (Step 2) and to use four independent variables (Step 3). Checking Appendix Table E.2 ($\alpha = .05$) for $k_B = 4$ (row) and power = .90 (column), the L value is found to be 15.41. It is decided that a population R^2 as small as .10 would be of interest and thus the ES is determined to be

$$f^2 = \frac{R^2}{1 - R^2} = \frac{.10}{.90} = .1111$$

Substituting L and f^2 in Eq. (3.7.2)

$$n^* = \frac{15.41}{.1111} + 4 + 1 = 144$$

Thus, 144 cases are needed to detect (using $\alpha = .05$) a population R^2 as small as .10 with a 90% probability. If the researcher were content to be able to detect a population R^2 of .20 with the same power and α, only 67 cases would be necessary. Similarly, suppose another investigator feels that .40 is a more realistic value for the population R^2, is content with 80% power, but plans to use the more stringent .01 significance criterion. In this case, the Table E.1 value for L is 16.75 and $f^2 = .40/.60 = .6667$; thus, from Eq. (3.7.2), only [(16.75/.6667) + 4 + 1 =] 30 cases are needed. (Note how grossly insufficient the original n = 15 is in the light of the power analysis.)

3.7.3 Power Analysis for Partial Correlation and Regression Coefficients

A determination of the number of cases necessary for testing the null hypothesis that any partial correlation or regression coefficient for a given X_i (in a set of k IVs) is zero may proceed in the same manner as for R^2. Having set α and the desired power, the appropriate table is entered now for $k_B = 1$ (because the source of variance is a single X_i), and the L value is read off. The f^2 value for the partial coefficients of a single IV X_i, is determined by

$$f^2 = \frac{sr_i^2}{1 - R^2}. \qquad (3.7.3)$$

The L and f^2 values are substituted in Eq. (3.7.2) to determine n^*.

For example, in planning the study on academic salaries a researcher may expect the population R^2 to be about .40 and decide to determine n^* for the case in which each of the 4 IVs makes a unique contribution of $.04 = sr^2$. (Note that to the extent that IVs are expected to be somewhat redundant in accounting for Y, the sum of the sr^2, will typically be smaller than R^2.)

The researcher then proceeds by deciding that the significance criterion is to be $\alpha = .05$ and that the power desired is .80. Checking Appendix Table E.2 for $k_B = 1$ and power = .80, he finds the L value to be 7.85. Determining from Eq. (3.7.3) that $f^2 = .04/(1 - .40) = .0667$, the L and f^2 values are substituted in Eq. (3.7.2), and

$$n^* = \frac{7.85}{.0667} + 4 + 1 = 123.$$

Thus, according to this plan it will take a sample size of 123 to provide an 80% probability of rejecting the null hypothesis at $\alpha = .05$ if the population f^2 is as posited. It is useful to note here the substantial effects of redundancy among the IVs in reducing power (or increasing n^*). If the four IVs were each to account

uniquely for one-fourth of R^2, that is, if they were uncorrelated, the f^2 for each would equal .1667 (= .10/.60), and $n^* = 52$. (Again note the insufficiency of n = 15.)

Although the power analysis of partial coefficients proceeds most conveniently by determining f^2 by means of sr^2, the analysis provides the appropriate power to reject the null hypotheses that β, B, and pr are zero as well. Because, as we have seen, these coefficients for a given X_i must have identical significance test results, the power to reject the null hypothesis for any one of them will be the same as for any other for analogous alternative-hypothetical values.

It sometimes happens that an investigator finds it more convenient to think in terms of units of change in the dependent variable that would be significant than in terms of proportions of variance. For example, the fictitious research described as our running example may have been motivated by a desire to determine whether salary discrimination on the basis of sex existed in the population. It might be decided that any discrepancy as large as $1000 in annual salary (net of the effects attributable to other causes) would be material to the people involved. The researcher may know that about 30% of faculty members are women; thus the sd of sex will be about $\sqrt{.30(.70)} = .458$ in the sample. The sd of faculty salaries may be determined from administration records to be about $6000. Thus, if B = $1000, β = $1000 (.458/$6000) = .0763. Recognizing that correlation with other variables will reduce the sr relative to the β [see Eq. (A2.4), Appendix 2], the researcher decides that an appropriate sr^2 to use for the power analysis would be $.071^2 = .005$. By Eq. (3.7.3) and assuming $R^2 = .40$, $f^2 = .005/(1 - .40) = .00833$. Using the given L value of 7.85 for $\alpha = .05$, power = .80, $k = 1$, we find (to our dismay)

$$n^* = \frac{7.85}{.00833} + 4 + 1 = 947!$$

If this n seems very demanding, it is instructive to note that in the example as given, assuming $n = 50$, with a net difference of nearly $3000 ($B_{SEX}$ = $2946) the researcher is in the embarrassing position of concluding that what is surely a personally significant difference is not statistically significant—that is, does not reliably indicate a nonzero difference in the population.

The preceding procedure may be employed for power analysis whenever an ES expressed as a B or β can be specified more readily than a desired proportion of unique variance (sr^2).

Several other topics in power analysis are presented in Chapter 4, following the exposition of power analysis in the most general form of MRC, where multiple sets of IVs are used. Among the issues discussed there are determination of power for a given n (Section 4.5.8), reconciling different n^*s for different hypotheses in a single analysis (Section 4.5.6), and the considerations involved in setting f^2 and power values (Sections 4.5.4 and 4.5.5). Section 4.5.9 discusses some general tactical issues in power analyses.

TABLE E.1
L Values for $\alpha = .01$

k_B	Power .10	.30	.50	.60	.70	.75	.80	.85	.90	.95	.99
1	1.67	4.21	6.64	8.00	9.61	10.57	11.68	13.05	14.88	17.81	24.03
2	2.30	5.37	8.19	9.75	11.57	12.64	13.88	15.40	17.43	20.65	27.42
3	2.76	6.22	9.31	11.01	12.97	14.12	15.46	17.09	19.25	22.67	29.83
4	3.15	6.92	10.23	12.04	14.12	15.34	16.75	18.47	20.74	24.33	31.80
5	3.49	7.52	11.03	12.94	15.12	16.40	17.87	19.66	22.03	25.76	33.50
6	3.79	8.07	11.75	13.74	16.01	17.34	18.87	20.73	23.18	27.04	35.02
7	4.08	8.57	12.41	14.47	16.83	18.20	19.79	21.71	24.24	28.21	36.41
8	4.34	9.03	13.02	15.15	17.59	19.00	20.64	22.61	25.21	29.29	37.69
9	4.58	9.47	13.59	15.79	18.30	19.75	21.43	23.46	26.12	30.31	38.89
10	4.82	9.88	14.13	16.39	18.97	20.46	22.18	24.25	26.98	31.26	40.02
11	5.04	10.27	14.64	16.96	19.60	21.13	22.89	25.01	27.80	32.16	41.09
12	5.25	10.64	15.13	17.51	20.21	21.77	23.56	25.73	28.58	33.02	42.11
13	5.45	11.00	15.59	18.03	20.78	22.38	24.21	26.42	29.32	33.85	43.09
14	5.65	11.35	16.04	18.53	21.34	22.97	24.83	27.09	30.03	34.64	44.03
15	5.84	11.67	16.48	19.01	21.88	23.53	25.43	27.72	30.72	35.40	44.93
16	6.02	12.00	16.90	19.48	22.40	24.08	26.01	28.34	31.39	36.14	45.80
18	6.37	12.61	17.70	20.37	23.39	25.12	27.12	29.52	32.66	37.54	47.46
20	6.70	13.19	18.45	21.21	24.32	26.11	28.16	30.63	33.85	38.87	49.03
22	7.02	13.74	19.17	22.01	25.21	27.05	29.15	31.69	34.99	40.12	50.51
24	7.32	14.27	19.86	22.78	26.06	27.94	30.10	32.69	36.07	41.32	51.93
28	7.89	15.26	21.15	24.21	27.65	29.62	31.88	34.59	38.11	43.58	54.60
32	8.42	16.19	22.35	25.55	29.13	31.19	33.53	36.35	40.01	45.67	57.07
36	8.92	17.06	23.48	26.80	30.52	32.65	35.09	38.00	41.78	47.63	59.39
40	9.39	17.88	24.54	27.99	31.84	34.04	36.55	39.56	43.46	49.49	61.57
50	10.48	19.77	27.00	30.72	34.86	37.23	39.92	43.14	47.31	53.74	66.59
60	11.46	21.48	29.21	33.18	37.59	40.10	42.96	46.38	50.79	57.58	71.12
70	12.37	23.05	31.25	35.45	40.10	42.75	45.76	49.35	53.99	61.11	75.27
80	13.22	24.51	33.15	37.55	42.43	45.21	48.36	52.11	56.96	64.39	79.13
90	14.01	25.89	34.93	39.53	44.62	47.52	50.80	54.71	59.75	67.47	82.76
100	14.76	27.19	36.62	41.41	46.70	49.70	53.11	57.16	62.38	70.37	86.18

TABLE E.2
L Values for $\alpha = .05$

k_B	Power .10	.30	.50	.60	.70	.75	.80	.85	.90	.95	.99
1	.43	2.06	3.84	4.90	6.17	6.94	7.85	8.98	10.51	13.00	18.37
2	.62	2.78	4.96	6.21	7.70	8.59	9.64	10.92	12.65	15.44	21.40
3	.78	3.30	5.76	7.15	8.79	9.77	10.90	12.30	14.17	17.17	23.52
4	.91	3.74	6.42	7.92	9.68	10.72	11.94	13.42	15.41	18.57	25.24
5	1.03	4.12	6.99	8.59	10.45	11.55	12.83	14.39	16.47	19.78	26.73
6	1.13	4.46	7.50	9.19	11.14	12.29	13.62	15.26	17.42	20.86	28.05
7	1.23	4.77	7.97	9.73	11.77	12.96	14.35	16.04	18.28	21.84	29.25
8	1.32	5.06	8.41	10.24	12.35	13.59	15.02	16.77	19.08	22.74	30.36
9	1.40	5.33	8.81	10.71	12.89	14.17	15.65	17.45	19.83	23.59	31.39
10	1.49	5.59	9.19	11.15	13.40	14.72	16.24	18.09	20.53	24.39	32.37
11	1.56	5.83	9.56	11.58	13.89	15.24	16.80	18.70	21.20	25.14	33.29
12	1.64	6.06	9.90	11.98	14.35	15.74	17.34	19.28	21.83	25.86	34.16
13	1.71	6.29	10.24	12.36	14.80	16.21	17.85	19.83	22.44	26.55	35.00
14	1.78	6.50	10.55	12.73	15.22	16.67	18.34	20.36	23.02	27.20	35.81
15	1.84	6.71	10.86	13.09	15.63	17.11	18.81	20.87	23.58	27.84	36.58
16	1.90	6.91	11.16	13.43	16.03	17.53	19.27	21.37	24.13	28.45	37.33
18	2.03	7.29	11.73	14.09	16.78	18.34	20.14	22.31	25.16	29.62	38.76
20	2.14	7.65	12.26	14.71	17.50	19.11	20.96	23.20	26.13	30.72	40.10
22	2.25	8.00	12.77	15.30	18.17	19.83	21.74	24.04	27.06	31.77	41.37
24	2.36	8.33	13.02	15.87	18.82	20.53	22.49	24.85	27.94	32.76	42.59
28	2.56	8.94	14.17	16.93	20.04	21.83	23.89	26.36	29.60	34.64	44.87
32	2.74	9.52	15.02	17.91	21.17	23.04	25.19	27.77	31.14	36.37	46.98
36	2.91	10.06	15.82	18.84	22.23	24.18	26.41	29.09	32.58	38.00	48.96
40	3.08	10.57	16.58	19.71	23.23	25.25	27.56	30.33	33.94	39.54	50.83
50	3.46	11.75	18.31	21.72	25.53	27.71	30.20	33.19	37.07	43.07	55.12
60	3.80	12.81	19.88	23.53	27.61	29.94	32.59	35.77	39.89	46.25	58.98
70	4.12	13.79	21.32	25.20	29.52	31.98	34.79	38.14	42.48	49.17	62.53
80	4.41	14.70	22.67	26.75	31.29	33.88	36.83	40.35	44.89	51.89	65.83
90	4.69	15.56	23.93	28.21	32.96	35.67	38.75	42.14	47.16	54.44	68.92
100	4.95	16.37	25.12	29.59	34.54	37.36	40.56	44.37	49.29	56.85	71.84

TABLE F.1

Power of Significance Test of r at $\alpha = .01$ (Two Tailed)[a]

n	Population r .10	.20	.30	.40	.50	.60	.70	.80	.90
15	01	03	06	13	25	44	68	90	*
16	01	03	07	14	28	48	73	93	
17	01	03	08	16	30	52	77	95	
18	01	04	08	17	33	56	80	96	
19	02	04	09	19	36	59	83	97	
20	02	04	09	20	38	62	85	98	
21	02	04	10	21	41	66	88	98	
22	02	04	11	23	43	68	90	99	
23	02	04	12	25	46	71	91	99	
24	02	05	12	26	49	74	93	99	
25	02	05	13	28	51	76	94	*	
26	02	05	14	30	53	78	95		
27	02	06	14	31	55	80	96		
28	02	06	15	33	57	82	96		
29	02	06	16	34	60	84	97		
30	02	06	17	36	62	85	98		
31	02	07	17	37	64	87	98		
32	02	07	18	39	66	88	98		
33	02	07	19	40	67	89	99		
34	02	07	20	42	69	90	99		
35	02	08	20	43	71	91	99		
36	02	08	21	45	72	92	99		
37	02	08	22	47	74	93	99		
38	02	08	23	48	76	94	*		
39	02	09	24	49	77	95			
40	02	09	25	50	78	95			
42	03	09	26	53	81	96			
44	03	10	28	56	83	97			
46	03	11	29	58	85	98			
48	03	11	31	61	87	98			

n	Population r .10	.20	.30	.40	.50	.60	.70	.80	.90
50	03	12	33	63	89	99	*	*	*
52	03	12	34	66	90	99			
54	03	13	36	68	91	99			
56	03	14	38	70	93	99			
58	03	14	39	72	94	*			
60	03	15	41	74	94				
64	04	16	44	77	96				
68	04	17	47	80	97				
72	04	19	50	83	98				
76	04	20	53	85	98				
80	04	21	56	87	99				
84	05	23	59	89	99				
88	05	24	61	91	99				
92	05	25	64	92	*				
96	05	27	66	94					
100	06	29	69	95					
120	07	35	78	98					
140	08	42	85	99					
160	09	49	90	*					
180	11	55	94						
200	12	61	96						
250	16	73	99						
300	20	82	*						
350	24	89							
400	28	93							
500	37	97							
600	45	99							
700	53	*							
800	60								
1000	72								

Note: Decimal points omitted in power values.

*Power values at and below this point exceed .995.

[a]Slightly abridged from Table 3.3.4 in Cohen (1977). Reproduced with the permission of the publisher.

TABLE F.2

Power of Significance Test of r at $\alpha = .05$ (Two Tailed)[a]

n	Population r .10	.20	.30	.40	.50	.60	.70	.80	.90
15	06	11	19	32	50	70	88	98	*
16	07	11	21	35	53	73	90	98	
17	07	12	22	37	56	76	92	99	
18	07	12	23	39	59	79	94	99	
19	07	13	24	41	62	81	95	99	
20	07	14	25	43	64	83	96	*	
21	07	14	27	45	66	85	96		
22	07	15	28	47	69	87	97		
23	07	15	29	49	71	89	98		
24	07	16	30	51	73	90	98		
25	08	16	31	53	75	91	99		
26	08	17	33	54	76	92	99		
27	08	17	34	56	78	93	99		
28	08	18	35	58	80	94	99		
29	08	18	36	59	81	95	99		
30	08	19	37	61	83	95	*		
31	08	19	38	62	84	96			
32	08	20	39	64	85	97			
33	09	20	40	65	86	97			
34	09	21	42	67	87	97			
35	09	21	43	68	88	98			
36	09	22	44	69	89	98			
37	09	22	45	70	90	98			
38	09	23	46	72	91	99			
39	09	23	47	73	91	99			
40	09	24	48	74	92	99			
42	10	25	50	76	93	99			
44	10	26	52	78	94	99			
46	10	27	54	80	95	*			
48	10	28	55	82	96				

n	Population r .10	.20	.30	.40	.50	.60	.70	.80
50	11	29	57	83	97	*	*	*
52	11	30	59	85	97			
54	11	31	61	86	98			
56	11	32	62	87	98			
58	12	33	64	89	98			
60	12	34	65	90	99			
64	12	36	68	91	99			
68	13	38	71	93	99			
72	13	39	73	94	*			
76	14	41	76	95				
80	14	43	78	96				
84	15	45	80	97				
88	15	47	82	98				
92	16	48	83	98				
96	16	50	85	98				
100	17	52	86	99				
120	19	59	92	*				
140	22	66	95					
160	24	72	97					
180	27	77	98					
200	29	81	99					
250	35	89	*					
300	41	94						
350	46	97						
400	52	98						
500	61	99						
600	69	*						
700	76							
800	81							
1000	89							

Note: Decimal points omitted in power values.

*Power values at and below this point exceed .995.

[a]Slightly abridged from Table 3.3.5 in Cohen (1977). Reproduced with the permission of the publisher.

TABLE F.1

Power of Significance Test of r at α = .01 (Two Tailed)[a]

n	.10	.20	.30	.40	.50	.60	.70	.80	.90
15	01	03	06	13	25	44	68	90	*
16	01	03	07	14	28	48	73	93	
17	01	03	08	16	30	52	77	95	
18	01	04	08	17	33	56	80	96	
19	02	04	09	19	36	59	83	97	
20	02	04	09	20	38	62	85	98	
21	02	04	10	21	41	66	88	98	
22	02	04	11	23	43	68	90	99	
23	02	04	12	25	46	71	91	99	
24	02	05	12	26	49	74	93	99	
25	02	05	13	28	51	76	94	*	
26	02	05	14	30	53	78	95		
27	02	06	14	31	55	80	96		
28	02	06	15	33	57	82	96		
29	02	06	16	34	60	84	97		
30	02	06	17	36	62	85	98		
31	02	07	17	37	64	87	98		
32	02	07	18	39	66	88	98		
33	02	07	19	40	67	89	99		
34	02	07	20	42	69	90	99		
35	02	08	20	43	71	91	99		
36	02	08	21	45	72	92	99		
37	02	08	22	47	74	93	99		
38	02	08	23	48	76	94	*		
39	02	09	24	49	77	95			
40	02	09	25	50	78	95			
42	03	09	26	53	81	96			
44	03	10	28	56	83	97			
46	03	11	29	58	85	98			
48	03	11	31	61	87	98			

n	.10	.20	.30	.40	.50	.60	.70	.80	.90
50	03	12	33	63	89	99	*	*	*
52	03	12	34	66	90	99			
54	03	13	36	68	91	99			
56	03	14	38	70	93	99			
58	03	14	39	72	94	*			
60	03	15	41	74	94				
64	04	16	44	77	96				
68	04	17	47	80	97				
72	04	19	50	83	98				
76	04	20	53	85	98				
80	04	21	56	87	99				
84	05	23	59	89	99				
88	05	24	61	91	99				
92	05	25	64	92	*				
96	05	27	66	94					
100	06	29	69	95					
120	07	35	78	98					
140	08	42	85	99					
160	09	49	90	*					
180	11	55	94						
200	12	61	96						
250	16	73	99						
300	20	82	*						
350	24	89							
400	28	93							
500	37	97							
600	45	99							
700	53	*							
800	60								
1000	72								

Note: Decimal points omitted in power values.
*Power values at and below this point exceed .995.
[a]Slightly abridged from Table 3.3.4 in Cohen (1977). Reproduced with the permission of the publisher.

TABLE F.2

Power of Significance Test of r at α = .05 (Two Tailed)[a]

n	.10	.20	.30	.40	.50	.60	.70	.80	.90
15	06	11	19	32	50	70	88	98	*
16	07	11	21	35	53	73	90	98	
17	07	12	22	37	56	76	92	99	
18	07	12	23	39	59	79	94	99	
19	07	13	24	41	62	81	95	99	
20	07	14	25	43	64	83	96	*	
21	07	14	27	45	66	85	96		
22	07	15	28	47	69	87	97		
23	07	15	29	49	71	89	98		
24	07	16	30	51	73	90	98		
25	08	16	31	53	75	91	99		
26	08	17	33	54	76	92	99		
27	08	17	34	56	78	93	99		
28	08	18	35	58	80	94	99		
29	08	18	36	59	81	95	99		
30	08	19	37	61	83	95	*		
31	08	19	38	62	84	96			
32	08	20	39	64	85	97			
33	09	20	40	65	86	97			
34	09	21	42	67	87	97			
35	09	21	43	68	88	98			
36	09	22	44	69	89	98			
37	09	22	45	70	90	98			
38	09	23	46	72	91	99			
39	09	23	47	73	91	99			
40	09	24	48	74	92	99			
42	10	25	50	76	93	99			
44	10	26	52	78	94	99			
46	10	27	54	80	95	*			
48	10	28	55	82	96				

n	.10	.20	.30	.40	.50	.60	.70	.80
50	11	29	57	83	97	*	*	*
52	11	30	59	85	97			
54	11	31	61	86	98			
56	11	32	62	87	98			
58	12	33	64	89	98			
60	12	34	65	90	99			
64	12	36	68	91	99			
68	13	38	71	93	99			
72	13	39	73	94	*			
76	14	41	76	95				
80	14	43	78	96				
84	15	45	80	97				
88	15	47	82	98				
92	16	48	83	98				
96	16	50	85	98				
100	17	52	86	99				
120	19	59	92	*				
140	22	66	95					
160	24	72	97					
180	27	77	98					
200	29	81	99					
250	35	89	*					
300	41	94						
350	46	97						
400	52	98						
500	61	99						
600	69	*						
700	76							
800	81							
1000	89							

Note: Decimal points omitted in power values.
*Power values at and below this point exceed .995.
[a]Slightly abridged from Table 3.3.5 in Cohen (1977). Reproduced with the permission of the publisher.

TABLE G.1

n^* to Detect r by t Test at $\alpha = .01$ (Two Tailed)[a]

Desired power \ Population r	.10	.20	.30	.40	.50	.60	.70	.80	.90
.25	362	90	40	23	15	11	8	6	5
.50	662	164	71	39	24	16	12	8	6
.60	797	197	86	47	29	19	13	9	7
2/3	901	222	96	53	32	21	15	10	7
.70	957	236	102	56	34	23	15	11	7
.75	1052	259	112	61	37	25	17	11	8
.80	1163	286	124	67	41	27	18	12	8
.85	1299	320	138	75	45	30	20	13	9
.90	1480	364	157	85	51	34	22	15	9
.95	1790	440	190	102	62	40	26	17	11
.99	2390	587	253	136	82	52	34	23	13

[a]Reproduced from Table 3.4.1 in Cohen (1977) with permission of the publisher.

TABLE G.2

n^* to Detect r by t Test at $\alpha = .05$ (Two Tailed)[a]

Desired power \ Population r	.10	.20	.30	.40	.50	.60	.70	.80	.90
.25	166	42	20	12	8	6	5	4	3
.50	384	95	42	24	15	10	7	6	4
.60	489	121	53	29	18	12	9	6	5
2/3	570	141	62	34	21	14	10	7	5
.70	616	152	66	37	23	15	10	7	5
.75	692	171	74	41	25	17	11	8	6
.80	783	193	84	46	28	18	12	9	6
.85	895	221	96	52	32	21	14	10	6
.90	1046	258	112	61	37	24	16	11	7
.95	1308	322	139	75	46	30	19	13	8
.99	1828	449	194	104	63	40	27	18	11

[a]Reproduced from Table 3.4.1 in Cohen (1977) with permission of the publisher.