

Neuendorf

Probability

The probability of an event is a ratio. It is equal to the expected frequency divided by the total frequency. The expected frequency is a number that is logically expected on the basis of some definition, such as the concept of a "fair" coin. The total frequency refers to the total number of possible outcomes. The value of the probability can range between 0 and 1. It reaches its lower limit when an event is never expected to occur, and reaches its upper limit when the event is always expected to occur.

An illustration of probability can be found in looking at the probability of getting a head when tossing a fair coin. The expected frequency of getting a head is 1, the total frequency is 2 (1 head and 1 tail), and the probability is $\frac{1}{2}$. The probability of rolling a six on one die is $\frac{1}{6}$. The probability of drawing the ace of spades from a deck of cards is $\frac{1}{52}$.

Probabilities for more than one event can be calculated. Two simple rules apply:

The additive rule applies to "either-or" cases. The probability of drawing an ace of spades or an ace of clubs is $\frac{1}{52} + \frac{1}{52}$, or $\frac{1}{26}$. The restriction is that the combined events must be mutually exclusive, that is they both cannot occur at the same time.

The multiplicative rule applies to "both-and" cases. The probability of drawing an ace of spades twice in a row is $\frac{1}{52} \times \frac{1}{52}$, or $\frac{1}{2704}$. The restriction is that the combined events must be independent, that is the occurrence of one event can have no effect on the probability of other combined events (they are causally unrelated).

The additive and multiplicative rules may be applied in a complex combination or chain.

Sample problems:

1. A single card is drawn from a standard deck of 52 playing cards. What is the probability that the card drawn is a spade?
2. A second card is drawn from the same deck after the first card drawn was replaced in the deck and the deck was reshuffled. What is the probability that the new card drawn is a spade?
3. A spade has been drawn from the deck and laid aside. Now still another card is drawn from those cards remaining in the deck. What is the probability that the new card drawn is a spade? What is the probability that it is a club?
4. What is the probability of getting either a spade or a club on a single draw from a standard deck?
5. What is the probability of getting an ace on both of two successive draws from a standard deck when the first card is replaced before the second draw and the deck is reshuffled?

6. A die is tossed three times, and each of the times it turns up one. What is the probability of obtaining this series of outcomes from an unloaded die?
7. A die is tossed three times. Two of the three times it turns up two, and once it turns up one. What is the probability of this result if the die is unbiased? What is the probability that the first two tosses will give both two (regardless of the third toss)?
8. Another die is tossed three times. It turns up one one, one two, and one three (any order). What is the probability of obtaining this outcome from an unbiased die? What is the probability that three events will occur in the orderly sequence one, two, three?
9. A college dormitory contains 500 women students and 1000 men students. If all 1500 names are printed alphabetically in a 30-page directory, what is the probability of finding the name of a woman at the bottom of page 9? What is the probability that the name in that position will be that of a man?
10. If a "random" sample of 60 names is drawn from the dorm directory, what is the expected frequency of men's names in the sample?

Answers:

1. $1/4$
2. $1/4$
3. $12/51$; $13/51$
4. $1/2$
5. $1/169$
6. $1/216$
7. $1/72$; $1/36$
8. $1/36$; $1/216$
9. $1/3$; $2/3$
10. 40