Neuendorf

Combinations and Permutations

Combinations refer to collections of different events where order is not taken into account. Permutations are arrangements of a set of events in different orders. For example, the arrangements ABC and ACB represent two permutations, but only one combination. Below is an example of sets of combinations and permutations.

Combinations and permutations that can be derived from 4 events taken 3 at a time:

Combinations	Permutations
ABC	ABC, ACB, BAC, BCA, CAB, CBA
ABD	ABD, ADB, BAD, BDA, DAB, DBA
ACD	ACD, ADC, CAD, CDA, DAC, DCA
BCD	BCD, BDC, CBD, CDB, DBC, DCB

When all of the items in question are to be permuted, we write ${}_nP_n$. This is read as "n things permuted n at a time" and is equal to n! (n factorial). This is true only is repetition of an event (e.g., as in the case AAC) is not possible. When repetition <u>is</u> possible, ${}_nP_n$ is equal to n^n. When only r of n items are to be permuted, we write ${}_nP_r$, which is read as "n things permuted r at a time." We use ${}_nC_r$ to indicate the number of combinations of n things taken r at a time.

Two sets of equations may be used to find combinations and permutations. When events are independent, and therefore a given outcome may recur or be repeated (e.g., heads, tails, heads, etc.) the following equations are used:

$${}_nP_r = n^r \qquad {}_nC_r = \frac{n^r}{r!}$$

When a given outcome may <u>not</u> recur (e.g., horserace results), the following equations are used:

$${}_nP_r = \frac{n!}{n-r!} \qquad {}_nC_r = \frac{n!}{(n-r)! \times r!}$$

What, you may ask, does this have to do with probability? Well, we have seen that formal probability requires us to use some theoretical rule to calculate the number of different ways in which all possible outcomes may occur. Rules of combination and permutation help us compute the numbers that go into the fraction used to calculate a probability. For example, suppose we wish to find the probability of three horses, A, B, and C, finishing in the top 3 from a field of 6 horses. We need to know both the number of ways in which the three may finish in the top 3 (${}_3P_3 = 6$) and the number of different ways in which all outcomes may occur (${}_6P_3 = 120$). Probability = 6/120 = 1/20. Permutations and combinations are helpful as simple aids to logic when trying to calculate probabilities.