

# Online Appendix (Not For Publication)

## Risk Premia at the ZLB: A Macroeconomic Interpretation

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The first section of this online appendix provides additional empirical results that serve as robustness checks. The second section describes the numerical method used to solve the model. The last section presents additional numerical results from the quantitative model.

### 1 Additional empirical results

Table 1 in the main text report measures of association of inflation compensation and stock returns using the break date of 2008:7. Table 1 in this online appendix presents results that exclude the core of the financial crisis (2008:7-2009:6), during which some financial markets, in particular the TIPS market, were disrupted. Specifically, we estimate the same regression:

$$R_t^s = \beta_0 + \beta_1 D_{t \geq 2008:7} + \beta_2 Surp_t + \beta_3 D_{t \geq 2008:7} \times Surp_t + \varepsilon_t,$$

but drop all observations between July 1, 2008 and June 30, 2009. Similarly, figure 2 in the main text presents binned scatter plots of daily changes, and figure 1 below presents the results excluding the financial crisis. The results shown are quite similar to those that include the financial crisis - if anything, they may be a bit stronger, i.e. the change in association is larger or more statistically significant.

Going back to the baseline sample, figure 2 is an alternative to figure 2 in the main text where we present binned scatter plots using 20-day changes instead of daily changes. The results here are also somewhat stronger than those that use daily data. The corresponding regression is including in the main text in table 1.

Figures 3, 4, 5, below present rolling regressions estimates of the slope coefficient in the association of inflation compensation and stock returns, inflation and consumption, and inflation

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and stock returns, respectively. (We use the same variables defined in the main text in sections 3.1-3.3, 3.5, and 3.4, respectively, using 240 days of rolling window for daily data and 120 months for monthly data.) The goal of these figures is to illustrate the timing of the changes in these associations. Figure 3 shows a sharp rise in the association of inflation compensation and stock returns in 2008. Figures 4 and 5 both show a rise in the association of inflation and consumption (resp. stock returns) that starts around 2000, but also a further increase around 2008. These figures support the results of the formal statistical break tests, which find a break in 2008 but sometimes also one earlier break in the late 1990s or 2000.

Finally, we also report some additional results for the structural break dates. First, to illustrate the results of the Andrews test reported in table 6 of the main body of the paper, figure 6 below plots the F-statistic of the QLR test for all possible break dates for a few of the variables we study. This provides a graphical illustration of the “identification” (in a loose sense) of the break date.

Further, table 2 reports the results of the break tests over the longer sample 1959m1-2015m12 instead of 1984m1-2015m12 in the main text. (We only report the results for the series for which we have data prior to 1980m1, since otherwise extending the sample is irrelevant.) Similar to the shorter sample, we find that 2008 is a key break date for the association of inflation and stock returns or of consumption and stock returns. Also similar to the shorter sample, there is no statistical evidence of a break for core inflation and stock returns. One difference is in the association of manufacturing industrial production and inflation or core inflation. Once the sample is extended, 2008 is not selected as a break any more. In particular, 1980 and 2000 are break dates (1999 or 2000 was already chosen by the Bai-Perron test.) This likely reflects that the longer sample naturally focuses more on secular changes.

## 2 Numerical method

Our solution is divided into two stages. In the first stage we use a numerical method to approximate time-invariant policy function for macroeconomic variables, the dividend, bond and stock prices, and expected inflation. In the second stage, we compute the rest of the variables using the policy functions found in the first stage. In this section we explain the first stage, since the second stage is mechanical. Our solution method is close to, but slightly different from, the one used in Fernandez-Villaverde et al. (2015). Similar to their method, we do not approximate the policy

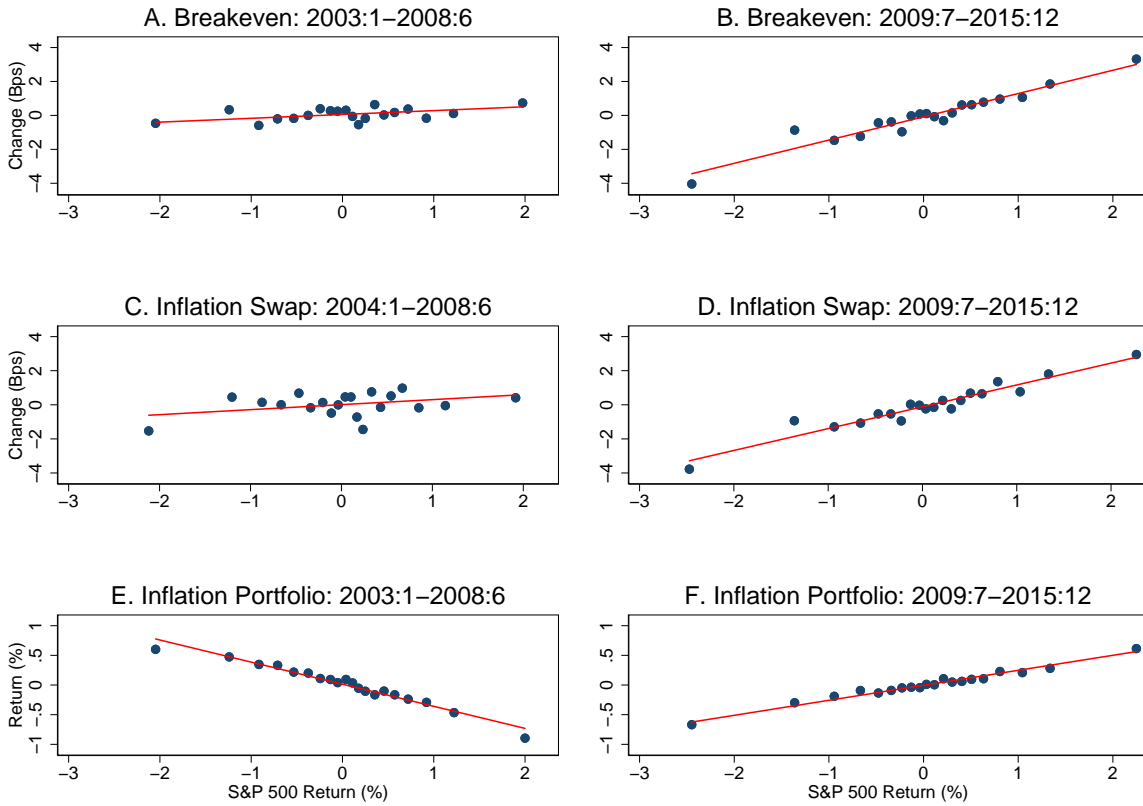


Figure 1: Binned scatter plot of daily changes in S&P 500 (x-axis) vs. daily changes in inflation compensation (y-axis) for two subsamples (left column: before July 2008; right column: July 2009 to December 2015), with regression lines superimposed. The inflation compensation is measured by 10-year breakevens in the top row, 10-year inflation swaps in the center row, and the inflation portfolio in the bottom row. See section 3.3 for details on the construction of the inflation portfolio.

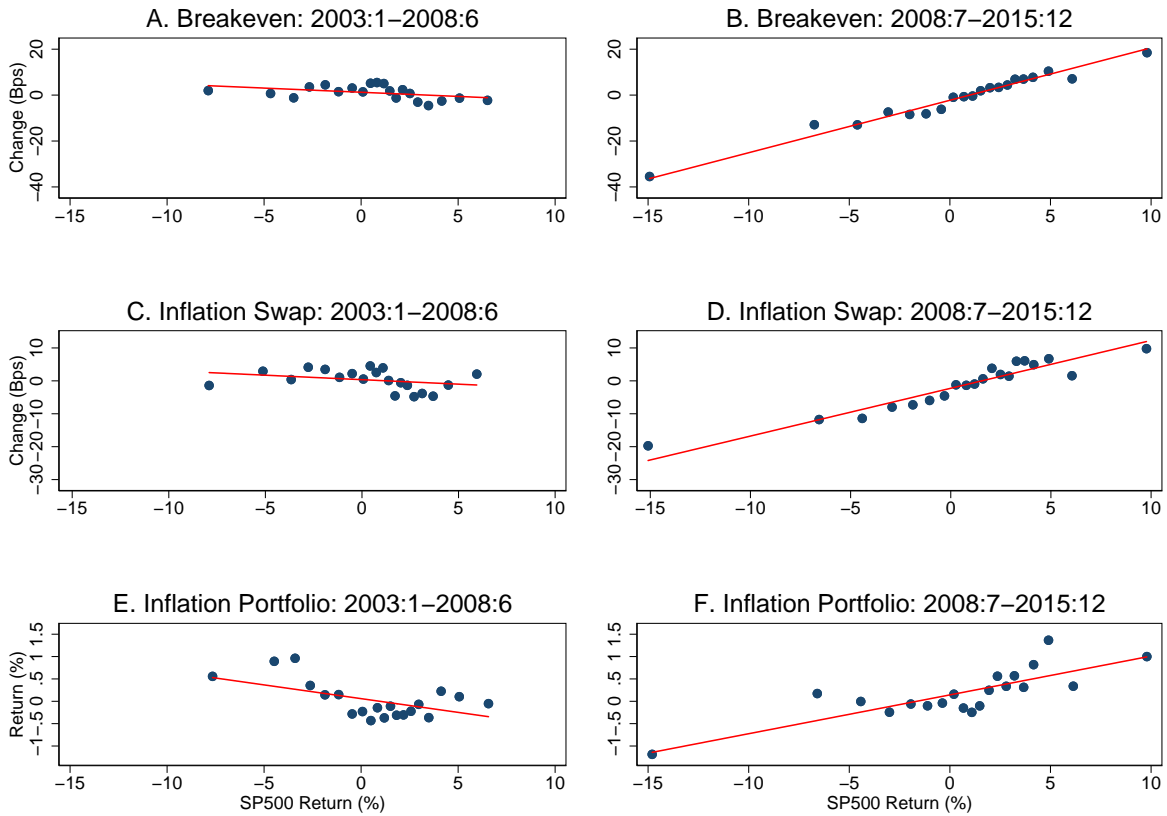


Figure 2: Binned scatter plot of 20-day changes in S&P 500 (x-axis) vs. 20-day changes in inflation compensation (y-axis) for two subsamples (left column: before July 2008; right column: July 2008 to December 2015), with regression lines superimposed. The inflation compensation is measured by 10-year breakevens in the top row, 10-year inflation swaps in the center row, and the inflation portfolio in the bottom row. See section 3.3 for details on the construction of the inflation portfolio.

<b>Inflation compensation</b>	$\widehat{\beta}_2$	$se(\beta_2)$	$\widehat{\beta}_3$	$se(\beta_3)$	$N$	$R^2$
<i>A. Daily data</i>						
10-year breakeven (H15)	0.23**	(0.09)	1.14***	(0.14)	2,997	0.120
10-year breakeven (GSW)	0.22**	(0.09)	1.27***	(0.14)	2,984	0.140
10-year inflation swaps	0.29*	(0.16)	0.99***	(0.19)	2,532	0.090
Inflation portfolio (ISP)	-0.37***	(0.02)	0.63***	(0.02)	2,998	0.340
Inflation portfolio, value-weighted	-0.43***	(0.02)	0.83***	(0.03)	2,998	0.300
Inflation portfolio, using CPI	-0.09***	(0.02)	0.29***	(0.02)	2,998	0.170
Inflation portfolio, excl. financials	-0.35***	(0.02)	0.54***	(0.02)	2,998	0.260
Inflation portfolio, with market control	-0.14***	(0.01)	0.30***	(0.02)	2,998	0.190
Inflation portfolio, no rebalancing	-0.39***	(0.01)	0.16***	(0.02)	2,998	0.410
<i>B. 20-day change</i>						
10-year breakeven (H15)	-0.36***	(0.09)	2.64***	(0.13)	2,997	0.230
10-year inflation swaps	-0.27**	(0.12)	2.06***	(0.14)	2,504	0.180
Inflation portfolio (ISP)	-0.06***	(0.02)	0.06**	(0.02)	2,347	0.010

Table 1: Changing association of inflation compensation and stock returns. The table reports the estimated coefficients from the model  $\Delta IC_t = \beta_0 + \beta_1 D_{t \geq 2008:7} + \beta_2 R_t^s + \beta_3 D_{t \geq 2008:7} R_t^s + \varepsilon_t$ . Each row corresponds to a different measure of inflation compensation  $\Delta IC_t$ . See the text for variable descriptions. The sample is 2003:1-2015:12 but excludes 2008:7 through 2009:6 (for inflation swaps, the sample starts in 2004:1). White standard errors are presented in parentheses. \*, \*\*, \*\*\* denote the 10%, 5%, and 1% levels of significance.

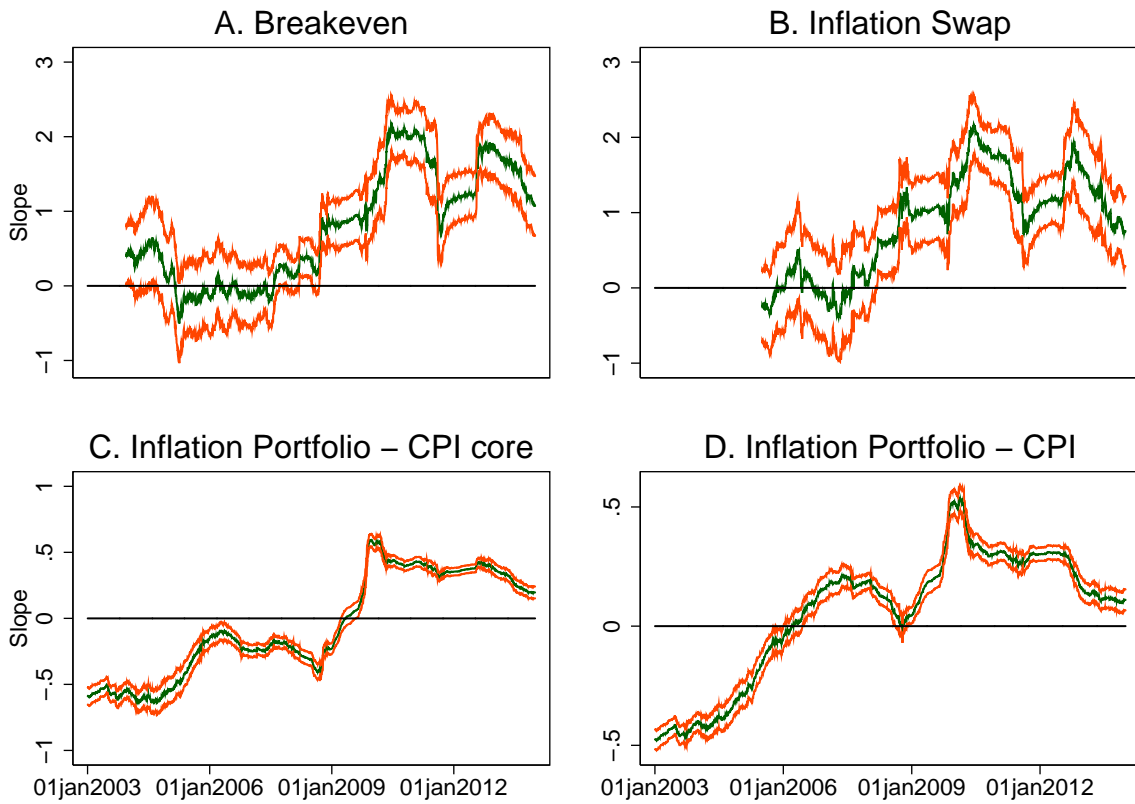


Figure 3: Rolling regression slope coefficient (and plus or minus two-standard deviation band) of stock return on inflation compensation, where inflation compensation is measured by (A) 10-year breakevens, (B) 10-year inflation swaps, (C) inflation portfolio constructed using CPI surprises, and (D) using CPI core surprises. See text for variable descriptions. The regression window is 240 days.

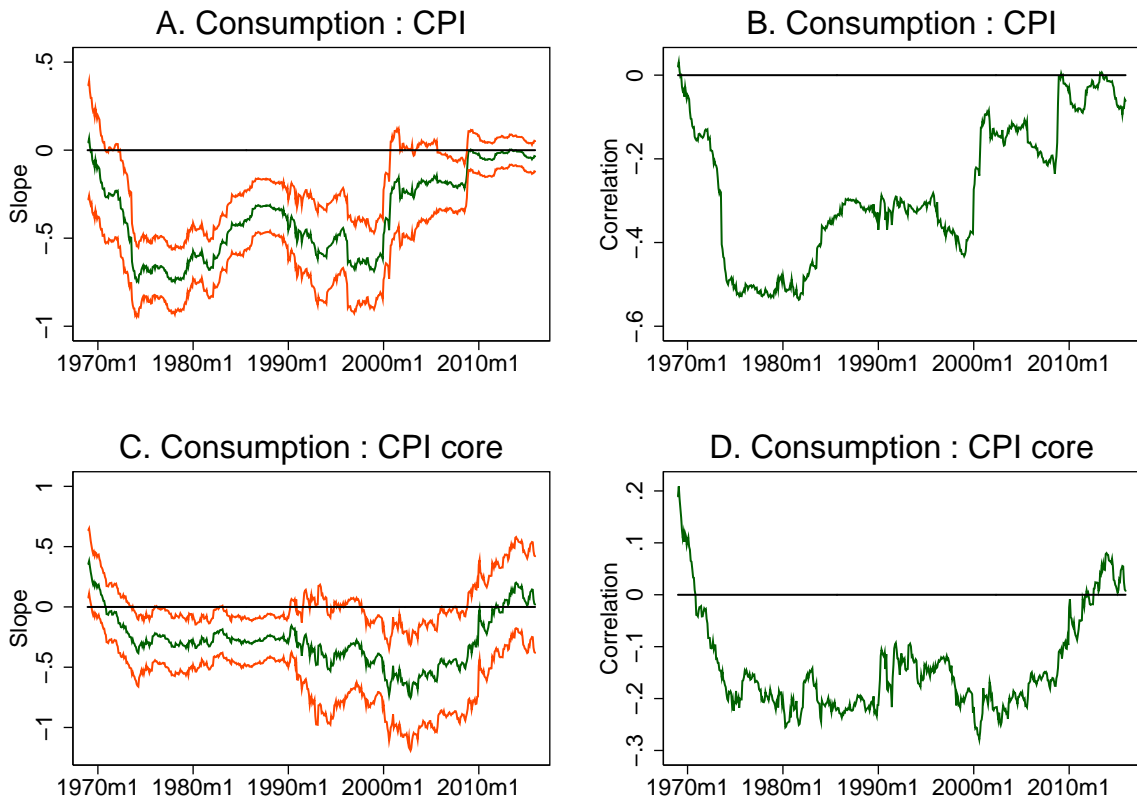


Figure 4: Rolling regression slope coefficient (left column; with plus or minus two-standard deviation band) and rolling correlation (right column) of real nondurable and services consumption growth on inflation (top row) or core inflation (bottom row). See text for variable descriptions. The regression window is 120 months.

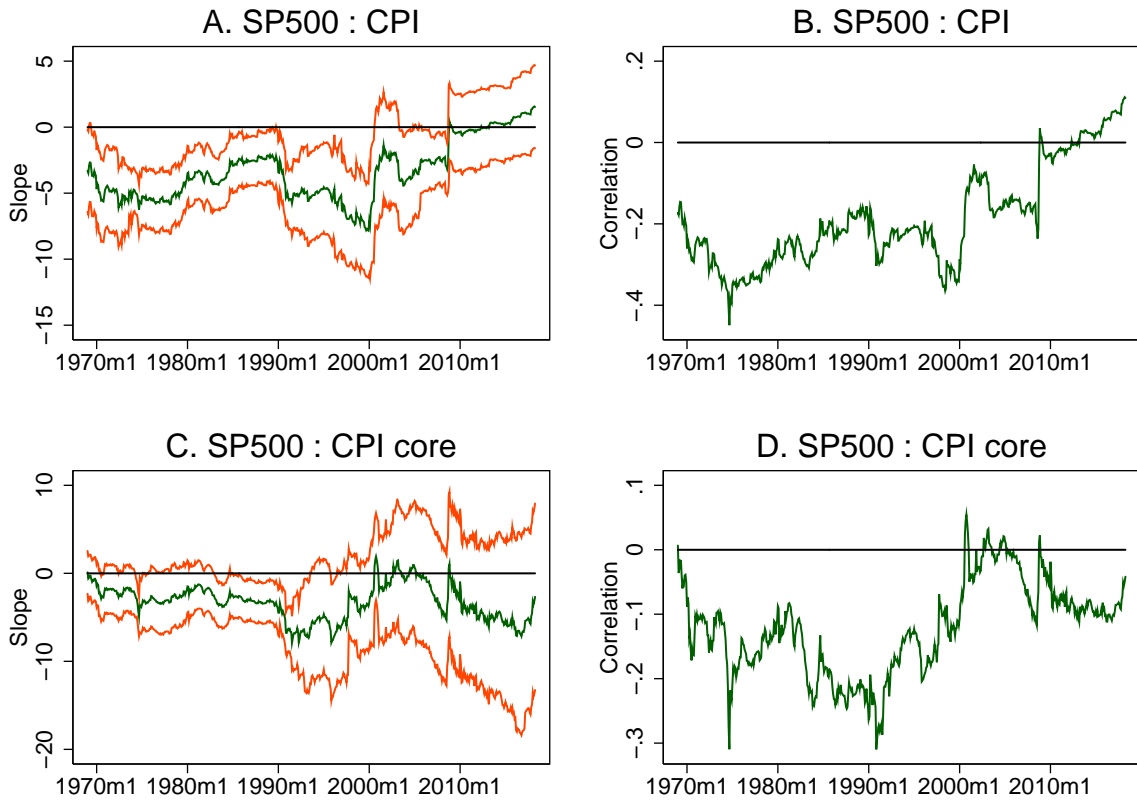


Figure 5: Rolling regression slope coefficient (left column; with plus or minus two-standard deviation band) and rolling correlation (right column) of real stock return with inflation (top row) or core inflation (bottom row). See text for variable descriptions. The regression window is 120 months.



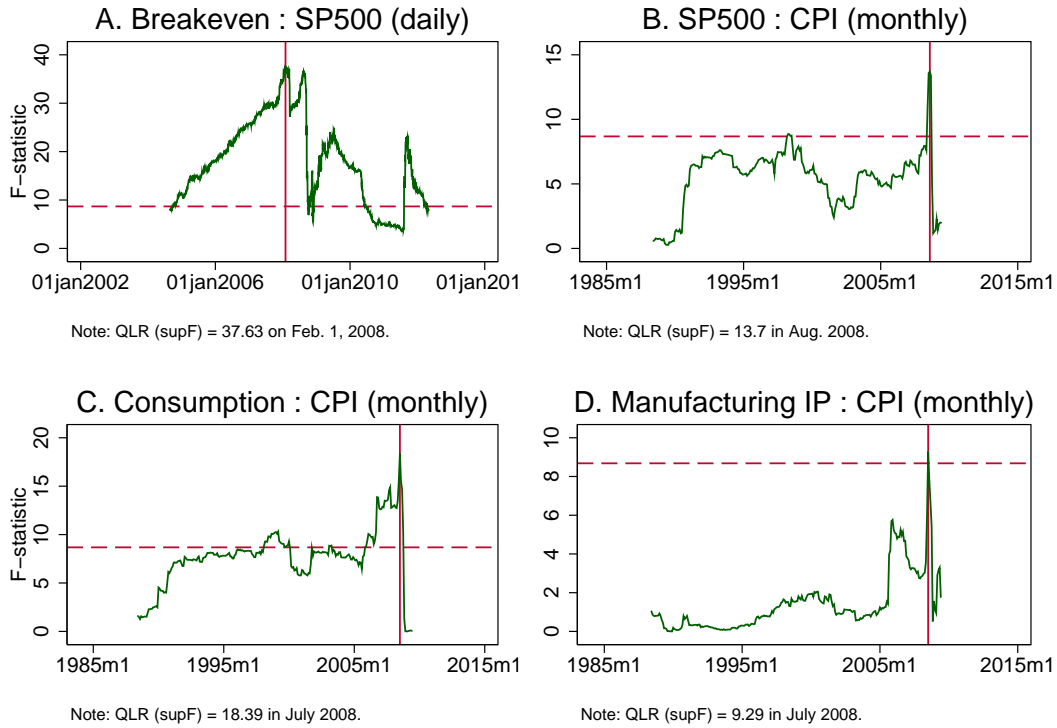


Figure 6: The figure plots the F-statistic for the QLR test of a break in the slope of the association. Top left panel presents the results for the association of inflation compensation (breakeven) and S&P 500 return using daily data; top right panel for the association of monthly stock return and inflation (CPI); bottom left and bottom right panels for the association of nondurable and services consumption (resp. manufacturing industrial production) and inflation (CPI). The F-statistic is computed for all potential break dates in the 70% central observations. The critical values are 7.12, 8.68, and 12.16 at the significance levels of 10%, 5%, and 1% respectively.

	Andrews (1993)		Bai and Perron (1998, 2003)	
	QLR	Break date	UDmax	Break dates
<b>A. Stock return</b>				
Inflation	19.25***	Oct. 2008	10.92**	Sep. 2008**
Core inflation	2.36	Sep. 1974	5.51	No break
<b>B. Consumption</b>				
Inflation	13.79***	Jul. 2008	13.19***	Jul. 1968, Dec. 2008*
Core inflation	7.21*	Feb. 1969	55.24***	Feb. 1969, Dec. 2000**
<b>C. Manuf. Ind. Prod.</b>				
Inflation	17.56***	Jun. 1980	4.49	No break
Core inflation	11.24**	Jun. 1980	17.26***	Apr. 2000***

Table 2: Test for unknown structural breaks in the association of measures of inflation and stock returns or economic activity. The sample is January 1959 to December 2015. For Andrews' QLR test, the F-statistic is computed for all potential break dates in the central 70% of the sample. For the Bai and Perron tests, we set the maximum number of breaks  $M=5$ , the trimming parameter  $\epsilon=0.15$ , the minimum length of subsample partitioned by break dates  $h = T \times \epsilon$ . \*, \*\*, \*\*\* denote the 10%, 5%, and 1% levels of significance.

function for the nominal interest rate. Instead, the nominal interest rate is always determined by the truncated Taylor rule, equation (9), at every state, in or out of the set of collocation nodes. The main advantage of this approach is that it enforces accurately the ZLB. However, different from this paper, we approximate the expectations as function of state using a finite element method called the cubic spline interpolation; see Judd (1998) and Miranda and Fackler (2002) for more details.

Collecting the optimality conditions for households and firms, recursive utility, real and nominal SDF, aggregate resource constraint, production function, truncated monetary policy rule, dividends, stock price, real and nominal prices for risky and risk neutral bonds, and two shock processes we obtain a system of nonlinear equations governing equilibrium as following:

$$w_t = C_t^\sigma \chi N_t^v \quad (1)$$

$$1 = E_t [\xi_t^{-1} R_t M_{t,t+1}^n] \quad (2)$$

$$M_{t,t+1}^n = \frac{M_{t,t+1}^r}{\Pi_{t+1}} \quad (3)$$

$$M_{t,t+1}^r = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \quad (4)$$

$$V_t = (1 - \beta) \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi N_t^{1+\nu}}{1+\nu} \right) - \beta E_t \left( (-V_{t+1})^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \quad (5)$$

$$0 = \left( 1 - \varepsilon + \varepsilon \left( \frac{w_t}{Z_t} \right) - \phi(\Pi_t - \bar{\Pi})\Pi_t \right) Y_t + \phi E_t (M_{t+1}^r (\Pi_{t+1} - \bar{\Pi})\Pi_{t+1} Y_{t+1}) \quad (6)$$

$$Y_t = Z_t N_t \quad (7)$$

$$C_t = \left( 1 - \frac{\phi}{2} (\Pi_t - \bar{\Pi})^2 \right) Y_t, \quad (8)$$

$$R_t = \max \left\{ 1, R^* \left( \frac{\Pi_t}{\bar{\Pi}^*} \right)^{\phi_\pi} \left( \frac{C_t}{C^*} \right)^{\phi_y} \right\} \quad (9)$$

$$D_t = C_t^\zeta \quad (10)$$

$$P_t^s = E_t [M_{t+1}^r (P_{t+1}^s + D_{t+1})], \quad (11)$$

$$q_t^{i,\lambda} = E \left[ \frac{1}{\xi_t} M_{t+1}^i \left( 1 + \lambda q_{t+1}^{i,\lambda} \right) \right] \quad (12a)$$

$$q_t^{i,\lambda,RN} = E_t \left[ \left( 1 + \lambda q_{t+1}^{i,\lambda,RN} \right) \right] E_t \left[ \frac{1}{\xi_t} M_{t+1}^i \right], \quad (13)$$

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}, \quad (14)$$

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_{z,t}, \quad (15)$$

where  $i \in \{n, r\}$  corresponding to nominal and real;  $\lambda \in (\lambda_1, \lambda_{20}, \lambda_{40})$  corresponding to 1 quarter, 5 years, and 10 years respectively;  $RN$  denotes ‘‘risk neutral’’;  $\varepsilon_{\xi,t}$  i.i.d  $N(0, \sigma_\xi^2)$  and  $\varepsilon_{z,t}$  i.i.d  $N(0, \sigma_z^2)$ .

There are 23 equations and 23 unknowns:  $R_t, C_t, \Pi_t, V_t, N_t, Y_t, w_t, M_{t,t+1}^r, M_{t,t+1}^n, D_t, P_t^s, q_t^{i,\lambda}, q_t^{i,\lambda,RN}$ . In addition we have two shock processes for liquidity and TFP. Substituting out the real and nominal SDF ( $M_{t,t+1}^n, M_{t,t+1}^r$ ) and rearrange the above equations we have 21 equations and 21 unknowns, together with two shock processes:

$$w_t = C_t^\sigma \chi N_t^\nu \quad (16)$$

$$1 = \beta \xi_t^{-1} R_t C_t^\sigma E_t \left[ \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \right] \quad (17)$$

$$V_t = (1 - \beta) \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi N_t^{1+\nu}}{1+\nu} \right) - \beta E_t \left( (-V_{t+1})^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \quad (18)$$

$$0 = \left( 1 - \varepsilon + \varepsilon \left( \frac{w_t}{Z_t} \right) - \phi(\Pi_t - \bar{\Pi})\Pi_t \right) Y_t + \phi \beta C_t^\sigma E_t \left( C_{t+1}^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (\Pi_{t+1} - \bar{\Pi})\Pi_{t+1} Y_{t+1} \right) \quad (19)$$

$$Y_t = Z_t N_t \quad (20)$$

$$C_t = \left( 1 - \frac{\phi}{2} (\Pi_t - \bar{\Pi})^2 \right) Y_t, \quad (21)$$

$$R_t = \max \left\{ 1, R^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left( \frac{C_t}{C^*} \right)^{\phi_y} \right\} \quad (22)$$

$$D_t = C_t^\zeta \quad (23)$$

$$P_t^s = \beta C_t^\sigma E_t \left[ C_{t+1}^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (P_{t+1}^s + D_{t+1}) \right], \quad (24)$$

$$q_t^{r,\lambda} = \beta \frac{C_t^\sigma}{\xi_t} E_t \left[ C_{t+1}^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (1 + \lambda q_{t+1}^{r,\lambda}) \right] \quad (25a)$$

$$q_t^{n,\lambda} = \beta \frac{C_t^\sigma}{\xi_t} E_t \left[ \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (1 + \lambda q_{t+1}^{n,\lambda}) \right] \quad (26a)$$

$$q_t^{r,\lambda,RN} = \beta \frac{C_t^\sigma}{\xi_t} E_t \left[ (1 + \lambda q_{t+1}^{r,\lambda,RN}) \right] E_t \left[ C_{t+1}^{-\sigma} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \right] \quad (27)$$

$$q_t^{n,\lambda,RN} = \beta \frac{C_t^\sigma}{\xi_t} E_t \left[ (1 + \lambda q_{t+1}^{n,\lambda,RN}) \right] E_t \left[ \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} \left( \frac{V_{t+1}}{E_t (V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \right] \quad (28)$$

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \varepsilon_{\xi,t}, \quad (29)$$

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_{z,t}, \quad (30)$$

Following Miranda and Fackler (2002), we rewrite the functional equations governing the equilibrium in the Calvo model in a more compact form:

$$f(s, X(s), E_s[Z(X(s'))]) = 0. \quad (31)$$

where

- $f : R^{2+21+17} \rightarrow R^{21}$  is the equilibrium relationship;
- $s = (\xi, Z)$  is the current state of the economy;
- $X(s) = (R(s), C(s), \Pi(s), V(s), N(s), Y(s), w(s), D(s), P^s(s), q^{i,\lambda}(s), q^{i,\lambda,RN}(s))'$ , where  $i \in \{n, r\}$  corresponding to nominal and real;  $\lambda \in (\lambda_1, \lambda_{20}, \lambda_{40})$  corresponding to 1 quarter, 5 years, and 10 years respectively; and  $X : R^2 \rightarrow R^{21}$  is the policy function;
- $s'$  is the next period's state that evolves according to the following motion equation:

$$s' = g(s, X(s), \varepsilon) = \begin{bmatrix} \xi' = \xi^{\rho_\xi} \exp(\varepsilon_\xi) \\ Z' = Z^{\rho_z} \exp(\varepsilon_z) \end{bmatrix},$$

where  $\varepsilon_\xi$  and  $\varepsilon_Z$  are the innovations of the liquidity and TFP shocks;

$$Z(X(s')) = \begin{pmatrix} \frac{(C(s'))^{-\sigma}}{\Pi(s')} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \\ \left( (-V(s'))^{1-\alpha} \right)^{\frac{1}{1-\alpha}} \\ C(s')^{-\sigma} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (\Pi(s') - \bar{\Pi})\Pi(s')Y(s') \\ C(s')^{-\sigma} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (P(s')^{stock} + D(s')) \\ \frac{1}{\xi(s)} C(s')^{-\sigma} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (1 + \lambda q^{r,\lambda}(s')) \\ \frac{1}{\xi(s)} \frac{C(s')^{-\sigma}}{\Pi(s')} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right)^{-\alpha} (1 + \lambda q^{n,\lambda}(s')) \\ 1 + \lambda q^{r,\lambda,RN}(s') \\ 1 + \lambda q^{n,\lambda,RN}(s') \\ C(s')^{-\sigma} \left( \frac{V(s')}{E_s(V(s')^{1-\alpha})^{\frac{1}{1-\alpha}}} \right) \end{pmatrix}$$

Instead of solving for the policy function, we actually solve the expectations as functions of state using a finite element method called the cubic spline interpolation. Define  $h(s) = E[Z(X(s'))|s]$ , below is the simplified algorithm:

*Step 1: Define the space of the approximating functions and collocation nodes  $S = (S_1, \dots, S_N)$ , where  $N = N_\xi \times N_Z$ , and  $N_\xi, N_Z$  are the numbers of grid points along each dimension of the state space. In this paper, we approximate the expectations:*

$$h(s) = (\phi(s)\theta_{h_1}, \phi(s)\theta_{h_2}, \dots, \phi(s)\theta_{h_{17}})'$$

$$h(s) = \phi(s)\Theta,$$

where

-  $\phi(s)$  is a  $1 \times N$  matrix of cubic spline basis functions evaluated at state  $s \in S = (S_1, \dots, S_N)$ .

-  $\Theta = (\theta_{h_1}; \theta_{h_2}; \dots; \theta_{h_{17}})$  is a  $N \times 17$  coefficient matrix that we want to approximate.

*Step 2: Initialize the coefficient matrix  $\Theta^0$ , and set up stopping rules.*

*Step 3: At each iteration  $j$  given the corresponding  $\Theta^j$ , we implement the following sub-steps:*

1) At each collocation node  $s_i$ ,  $s_i \in \{S_1 \dots S_N\}$ , compute  $h(s_i)$  using the approximating functions for the expectations.

2) Solve for  $X(s_i)$  using the equilibrium relationship  $f(s_i, X(s_i), h(s_i)) = 0$ . We can solve this complementarity problem using the Newton method.

*Step 4: Update  $h$  using the following sub-steps:*

1) Approximate policy functions using the cubic spline interpolation,  $X(s)$ .

2) At each collocation node  $s_i$ ,  $s_i \in \{S_1..S_N\}$ , update  $h(s_i) = (h_1(s_i), h_2(s_i), \dots, h_{17}(s_i))$  using  $h(s_i) = \sum_j w_j Z(X(s_i))$ , where the innovations for the preference and government spending shocks are discretized using the Tauchen and Hussey (1991) method with 25 nodes.

*Step 5: Update  $\Theta^{j+1} = \Phi^{-1}\Theta^j$ , where  $\Phi = (\phi(s_1), \dots, \phi(s_N))'$ .*<sup>1</sup>

*Step 6: Check the stopping rules, if not satisfied go to Step 3, otherwise go to Step 7.*

*Step 7: Report results.* We use the approximated expectation functions to solve for the equilibrium value at any state. So, we are able to find almost exactly the kink for the nominal interest rate.

In addition, we write our code using a parallel computing method that allows us to split up a large number of collocation nodes into smaller groups assigned to different cores/processors to be solved simultaneously. This procedure reduces computation time significantly. We obtain the maximal absolute residual across the equilibrium conditions of the order of  $10^{-8}$  for almost all states off the collocation nodes. For a few states when the ZLB becomes binding, the maximal absolute residual is of the order of  $10^{-5}$ . This is quite standard given the kink in the interest rate policy function, see Miranda and Fackler (2002) and Judd et al. (2011) for more information.

### 3 Additional model Implication

In the paper, we illustrate the effect of risk aversion in Figure 7 (of the main text) by showing the impulse response to a productivity shock far from the ZLB or close to the ZLB for high or low risk aversion. Here in figure 7 we provide the same result for a liquidity shock rather than a productivity shock. The results are similar to those in the main text, namely far from the ZLB, the response to a liquidity shock is identical in the two models, but close to the ZLB, the high risk aversion model generates a larger drop in economic output and in inflation.

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<sup>1</sup>We also keep track of the convergence of the policy functions for  $C, \Pi, S, F, \Delta$ . They always converge to the fixed point much faster than the expectations functions do. Note that the expectations functions are quite smooth by nature, and so do the policy functions for  $C, \Pi, S, F, \Delta$ .

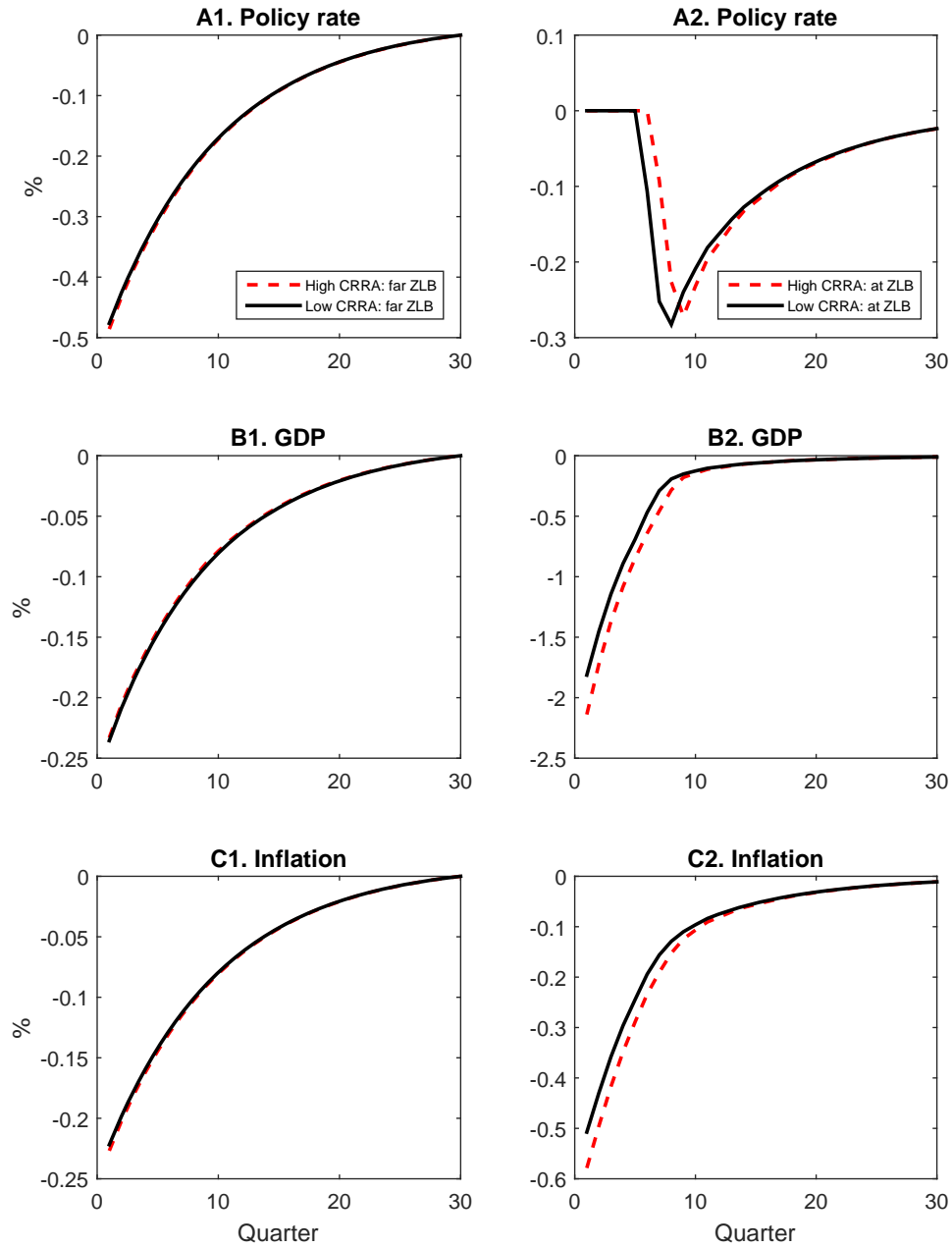


Figure 7: Impulse response function to a one-standard deviation liquidity shock in the benchmark model (high risk aversion) and in the same model but with low risk-aversion, when the economy is at the ZLB vs. in steady-state (far from the ZLB). The solid black lines present impulse responses for low risk aversion, while the dashed red lines show the results for high risk aversion. The left panels are for the case at steady state (far from the ZLB). The right panels present impulse responses at the ZLB, which is calculated as the difference between two paths: (i) a path with only large liquidity shock that brings the economy to the ZLB, and (ii) a path with the same shock, plus a one-standard-deviation shock.

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