Multiplicity in New Keynesian Models*

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Abstract

The common practice in monetary economics is to linearize a model around its deterministic equilibrium. However, in this paper we show analytically that when central banks try to stabilize both output and inflation, the standard dynamic New Keynesian model actually has three deterministic equilibria under a realistic parameterization. One is associated with targeted inflation as as commonly found in the literature; the other two are associated with deflation and high inflation. Our findings suggest that empirical research should allow for multiple equilibria or regimes, including both the one with high inflation and the one with deflation, in modeling inflation dynamics.

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Keywords: Multiple steady state, Multiple equilibria, New Keynesian model, deflation, high inflation.

1 Introduction

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Macroeconomics has the long tradition of solving a dynamic stochastic general equilibrium (DSGE) model by first finding the deterministic steady state of the model then (log) linearing the model around the stead state value to obtain a linear stochastic difference system; for example Kydland and Prescott (1982), King et al. (1987), and Woodford (2003). Nowadays, global solution methods are used to solve more complicated nonlinear DSGE models; for example Ngo (2014) and Fernandez-Villaverde et al. (2015). However, finding deterministic steady state values is still an important first step to initialize policy functions.[1]

More importantly, the number of deterministic steady states can be a crucial reference for us to determine the number of regimes in our model. For example, Aruoba et al. (2018) study macroeconomic dynamics in a dynamic New Keynesian (DNK) model that allows for two regimes associated with two deterministic equilibria: one with deflation and the other with targeted inflation. The idea of utilizing a two-regime model comes from the seminal work by Benhabib et al. (2001), where they found that there are two steady states in the standard DNK model where the central bank tries to stabilize inflation around the target.

The remaining question is how many regimes we should actually include in our analysis. One potential approach to answer this question is to find out all possible deterministic steady states.[2] In this paper, we show analytically that when central banks try to stabilize both inflation and output, the conventional DNK model has up to three deterministic equilibria under a realistic parameterization. One is associated with targeted inflation as commonly found in the literature. The others are associated with deflation and high

[1] In this paper, we use deterministic steady states and deterministic equilibria interchangeably.

[2] Another approach is to let data to speak out the number of regimes directly. For example, Bianchi and Melosi (2017) estimate a MS-VAR model to determine that the number of regimes in a DSGE model should be three. Isakin and Ngo (2019) use a bootstrap to identify three as the number of regimes in an unobserved component stochastic volatility model.

inflation. As a result, there may be three stochastic regimes associated with different deterministic equilibria.

We also use calibrated parameter values to illustrate the multiplicity of deterministic equilibria. In addition we explore the sensitivity of the inflation rate in different deterministic equilibria to changes in important parameter values.

Our findings suggest that allowing for three regimes, including the one with high inflation, in modeling inflation dynamics can improve likelihood of a model in matching U.S. data, especially for the Great Inflation period (late 1970s to early 1980s). It can also improve the forecasting performance of the model. In fact, the triplicity of deterministic equilibria in this paper is consistent with the findings by Isakin and Ngo (2019). They show that a model with three regimes and regime-dependent constant volatilities has superior performance. Note that regimes with different volatilities are typically associated with different mean inflation, e.g. Ball (1992). Our findings are also consistent with Bianchi and Melosi (2017), where they estimate a MS-VAR model and find that the number of regimes is three: one associated with high inflation, one with targeted inflation, and one with low inflation.

The multiplicity of equilibria and regime switching in the DNK framework has become prominent in the macroeconomics literature. Our paper is closely related to Benhabib et al. (2001) and Aruoba et al. (2018).[3] Benhabib et al. (2001) show that there are two deterministic equilibria in a DNK model when the central bank sets the interest rate based on inflation. One is associated with the targeted inflation and the other with

[3]Another branch of the regime switching DSGE literature is associated with a single deterministic steady state. Bianchi (2012) models structural changes in monetary and fiscal policies and estimates a model in which the monetary/fiscal policy mix follows a six-regime Markov chain. Liu et al. (2009) study the expectation effect of regime switches using a model where monetary policy switches between dovish and hawkish regimes. Baele et al. (2015) estimate a DNK model with regime switches in monetary policy and macro-shocks using survey-based expectations for inflation and output. In these papers, there is only one deterministic steady state because the switching in monetary and fiscal policies are relevant at steady state.

deflation. Aruoba et al. (2018) study inflation dynamics in a DNK model that allows for two regimes associated with these two deterministic equilibria: a deflationary regime and a targeted-inflation regime. The paper finds that a sunspot shock is important to explain economic behavior at the zero lower bound.

In addition, Arifovic et al. (2018) show that in an economy with two states the liquidity trap equilibrium is stable under certain assumptions on the learning process. Mertens and Ravn (2014) show that self-fulfilling expectations can result in a non-fundamental liquidity trap equilibrium.

In contrast to these papers, we assume that the central bank targets both inflation and output gap. This setup is more realistic because the Fed actually has the dual mandate: price stability and maximum employment. Targeting both inflation and output gap gives rise to the third deterministic equilibrium associated with high inflation.

The paper is organized as follows. In Section 2, we develop a standard DNK model. Section 3 defines three deterministic equilibria and formulates sufficient conditions for their existence. In Section 4, we illustrate the existence of the three deterministic equilibria using calibrated parameter values and study their robustness. Section 5 concludes.

2 The model

We use a standard DNK model with Rotemberg pricing mechanism. The model consists of a continuum of identical households, a continuum of identical competitive final good producers, a measure one of monopolistically competitive intermediate goods producers, and a government (monetary authority).

2.1 Households

The representative household maximizes his expected discounted utility

$$E_t \left\{ \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right) + \sum_{\tau=1}^{\infty} \left\{ \left(\Pi_{j=0}^{t-1} \beta_{t+j} \right) \left(\frac{C_{t+\tau}^{1-\gamma}}{1-\gamma} - \chi \frac{N_{t+\tau}^{1+\eta}}{1+\eta} \right) \right\} \right\} \tag{1}$$

subject to the budget constraint

$$P_{t+\tau} C_{t+\tau} + \left(1 + i_{t+\tau}\right)^{-1} B_{t+\tau} = W_{t+\tau} N_{t+\tau} + B_{t+\tau-1} + F_{t+\tau} + T_{t+\tau}, \tag{2}$$

where C_t is consumption of final goods, i_t is the nominal interest rate, B_t denotes one-period bond holdings, N_t is labor, W_t is the nominal wage rate, F_t is the profit income, T_t is the lump-sum tax, and β_t denotes the preference shock. We assume that log of β_t follows an AR(1) process:

$$\ln \left(\beta_t \right) = \left(1 - \rho_\beta\right) \ln \beta + \rho_\beta \ln \left(\beta_{t-1} \right) + \varepsilon_{\beta t}, \tag{3}$$

where $\rho_\beta \in (0, 1)$ is the persistence of the preference shock and $\varepsilon_{\beta t}$ is the innovation of the preference shock with mean 0 and variance σ_β^2. The preference shock is a reduced form of more realistic forces that can drive the nominal interest rate to the ZLB.

The first-order conditions for the household optimization problem are given by

$$\chi N_t^\eta C_t^\gamma = w_t, \tag{4}$$

and

$$E_t \left[M_{t,t+1} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right] = 1, \tag{5}$$

where $w_t = W_t / P_t$ is the real wage, $\pi_t = P_t / P_{t-1} - 1$ is the inflation rate, and the stochastic discount factor is given by

$$M_{t,t+1} = \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{6}$$

2.2 Final good producers

To produce the final good, the final good producers buy and aggregate a variety of intermediate goods indexed by $i \in [0, 1]$ using a CES technology

$$Y_t = \left(\int_0^1 Y_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ε is the elasticity of substitution among intermediate goods. The profit maximization problem is given by

$$\max \; P_t Y_t - \int_0^1 P_t\left(i\right) Y_t\left(i\right) di,$$

where $P_t\left(i\right)$ and $Y_t\left(i\right)$ are the price and quantity of intermediate good i. Profit maximization and the zero-profit condition give the demand for intermediate good i

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t} \right)^{-\epsilon} Y_t, \tag{7}$$

and the aggregate price level

$$P_t = \left(\int P_t\left(i\right)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{8}$$

2.3 Intermediate goods producers

There is a unit mass of intermediate goods producers on $[0, 1]$ that are monopolistic competitors. Suppose that each intermediate good $i \in [0, 1]$ is produced by one producer using the linear technology

$$Y_t(i) = Z_t N_t(i), \tag{9}$$

where $N_t(i)$ is labor input. We normalize the steady state level of technology to unity, i.e. $Z = 1$, and assume that log of Z_t follows an AR(1) process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt}, \tag{10}$$

where $\rho_Z \in (0, 1)$ is the persistence of the preference shock and ε_{Zt} is the innovation of the preference shock with mean 0 and variance σ_Z^2.

Cost minimization implies that each firm faces the same real marginal cost

$$mc_t = mc_t(i) = \frac{w_t}{Z_t}. \tag{11}$$

2.4 Price setting mechanism

Rotemberg (1982) assumes that each intermediate goods firm i faces costs of adjusting prices in terms of final goods. In this paper, we use a quadratic adjustment cost function, which is proposed by Ireland (1997) and which is one of the most common functions used in the ZLB literature:

$$\frac{\phi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \overline{\Pi}\right)^2 Y_t,$$

where ϕ is the adjustment cost parameter which determines the degree of nominal price rigidity.

The problem of firm i is given by

$$\max_{\{P_{i,t}\}} E_t \sum_{j=0}^{\infty} \left\{ M_{t,t+j} \left[\left(\frac{P_{t+j}(i)}{P_{t+j}} - mc_t \right) Y_{t+j}(i) - \frac{\phi}{2} \left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - \overline{\Pi} \right)^2 Y_{t+j} \right] \right\} \tag{12}$$

subject to its demand (7). In a symmetric equilibrium, all firms will choose the same price and produce the same quantity, i.e., $P_t(i) = P_t$ and $Y_t(i) = Y_t$. The optimal pricing rule then implies that

$$\left(1 - \varepsilon + \varepsilon \frac{w_t}{Z_t} - \phi\left(\Pi_t - \overline{\Pi}\right)\Pi_t\right) Y_t + \phi E_t\left[M_{t,t+1}\left(\Pi_{t+1} - \overline{\Pi}\right)\Pi_{t+1} Y_{t+1}\right] = 0. \tag{13}$$

2.5 Monetary and fiscal policies

The central bank conducts monetary policy by setting the interest rate using a simple Taylor rule subject to the ZLB condition:

$$R_t = \max\left\{1, R^{TR}\left(\frac{\Pi_t}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{GDP_t}{GDP_t^n}\right)^{\phi_y}\right\} \tag{14}$$

where $GDP_t \equiv C_t + G_t$ denotes the gross domestic product (GDP); GDP^n denotes the natural rate of GDP where prices are completely flexible; Π^{TR} and R^{TR} denote the target inflation and the intercept of the Taylor rule, respectively.

2.6 Aggregate resources and equilibrium

Aggregate output satisfies:

$$Y_t = Z_t N_t, \tag{15}$$

and the resource constraint is given by

$$C_t + G_t + \frac{\phi}{2}\left(\Pi_t - \overline{\Pi}\right)^2 Y_t = Y_t. \tag{16}$$

The system governing the equilibrium for the Rotemberg model consists of seven nonlinear difference equations (4), (5), (6), (13), (14), (15), (16) for seven variables

w_t, C_t, M_t, i_t, π_t, N_t, and Y_t; they represent the real wage, consumption, the stochastic discount factor, the gross nominal interest rate, the gross inflation rate, labor, and output respectively. Collecting these nonlinear equations we have:

1. Real discount factor:

$$M_{t,t+1} = \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{17}$$

2. Euler equation:

$$1 = E_t \left[\frac{R_t M_{t+1}}{\Pi_{t+1}} \right] \tag{18}$$

3. MRS:

$$w_t = \chi C_t^{\sigma} N_t^{v} \tag{19}$$

4. Phillips curve:

$$0 = \left(1 - \varepsilon + \varepsilon \frac{w_t}{Z_t} - \phi(\Pi_t - \overline{\Pi})\Pi_t \right) Y_t + \phi E_t \left(M_{t+1}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1} Y_{t+1} \right) \tag{20}$$

5. Resource constraint:

$$C_t = \left(1 - \frac{\phi}{2}(\Pi_t - \overline{\Pi})^2 \right) Y_t \tag{21}$$

6. Production function:

$$Y_t = Z_t N_t \tag{22}$$

7. Truncated Taylor rule:

$$R_t = \max \left\{ 1, R^{TR} \left(\frac{\Pi_t}{\Pi^{TR}} \right)^{\phi_\pi} \left(\frac{GDP_t}{GDP_t^n} \right)^{\phi_y} \right\} \tag{23}$$

where GDP and GDP^n are actual GDP and its counterpart when prices are completely flexible; in this framework $GDP = C$; Π^{TR} and R^{TR} denote the target inflation and the intercept of the Taylor rule, respectively; $\overline{\Pi}$ denotes the inflation used to index prices; β_t and Z_t follow AR(1) processes.

3 Multiple deterministic equilibria

We drop the subscript t for deterministic (steady state) equilibrium. From equations (17) and (18) we have

$$R = \frac{\Pi}{\beta} \tag{24}$$

Combining (19), (20), (22), (23) results in

$$R = \max\left\{1, R^{TR}\left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y}\right\} \tag{25}$$

where

$$C = \left(\frac{w}{\chi}X^{\upsilon}\right)^{\frac{1}{\sigma+\upsilon}} \tag{26}$$

$$w = \frac{\varepsilon - 1}{\varepsilon} + \frac{(1-\beta)\,\phi}{\varepsilon}\left(\Pi - \overline{\Pi}\right)\Pi \tag{27}$$

$$X \triangleq \frac{C}{Y} = 1 - \frac{\phi}{2}(\Pi - \overline{\Pi})^2 \tag{28}$$

So, the deterministic equilibrium is characterized by two equations, (24) and (25), with two unknowns, Π and R.

3.1 Deterministic equilibrium with deflation and zero interest rate

From equation (24), $\Pi = \beta$. Using equation (26), we can compute consumption:

$$C = \left(\frac{w}{\chi} X^{\upsilon}\right)^{\frac{1}{\sigma+\upsilon}}$$

where

$$w = \frac{\varepsilon - 1}{\varepsilon} + \frac{(1-\beta)\,\phi}{\varepsilon}\left(\beta - \overline{\Pi}\right)\beta$$

$$X = 1 - \frac{\phi}{2}\left(\beta - \overline{\Pi}\right)^2$$

If the condition $R^{TR}\left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y} \leq 1$ is satisfied, there exists a deterministic equilibrium with zero interest rate.

Proposition 1 *If $\phi_\pi > 1, \phi_y > 0, \beta \in (0,1), \Pi^{TR} > 1$, and $R^{TR} \leq \frac{\Pi^{TR}}{\beta}$, there is a deterministic equilibrium associated with deflation and zero interest rate.*

Proof: Without loss of generality, we set $\chi = \frac{\varepsilon-1}{\varepsilon}(1/3)^{-\sigma-\upsilon}$ such that the deterministic equilibrium labor is one third in the economy with flexible prices. In this case the gross inflation $\Pi = \beta < 1$, which implies deflation. Now, let us consider if the condition $R^{TR}\left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y} \leq 1$ is satisfied. Under the above assumptions,

$$\begin{aligned} R^{TR}\left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y} &\leq \frac{\Pi^{TR}}{\beta}\left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y} \\ &= \frac{\Pi^{TR}}{\beta}\left(\frac{\beta}{\Pi^{TR}}\right)^{\phi_\pi}\left(\frac{C}{GDP^n}\right)^{\phi_y} \\ &= \left(\frac{\beta}{\Pi^{TR}}\right)^{\phi_\pi - 1}\left(\frac{C}{GDP^n}\right)^{\phi_y}. \end{aligned}$$

In this case, because the deterministic equilibrium consumption $GDP \leq GDP^n = \frac{1}{3}, \phi_\pi > 1, \phi_y > 0$, then $\left(\frac{\beta}{\Pi^{TR}}\right)^{\phi_\pi - 1} \left(\frac{GDP}{GDP^n}\right)^{\phi_y} < 1$. According to the truncated Taylor rule, the deterministic equilibrium gross interest rate is one. Hence, there exists a deterministic equilibrium with zero interest rate and deflation. ■

3.2 Deterministic equilibria with positive interest and inflation

In this case, combining equations (24) and (25) results in:

$$\frac{\Pi}{\beta} = R^{TR} \left(\frac{\Pi}{\Pi^{TR}}\right)^{\phi_\pi} \left(\frac{C}{GDP^n}\right)^{\phi_y}$$

or

$$C = \Pi^{\frac{1-\phi_\pi}{\phi_y}} \left(\frac{1}{\beta R^{TR}}\right)^{\frac{1}{\phi_y}} \left(\frac{1}{\Pi^{TR}}\right)^{-\frac{\phi_\pi}{\phi_y}} GDP^n. \tag{29}$$

Together with equation (26) , we have two equations and two unknowns. The solution of these equations will give us deterministic equilibria if the condition $\frac{\Pi}{\beta} \geq 1$ is satisfied.

Proposition 2 *If $\phi_\pi > 1$, $\phi_y > 0$, $\beta \in (0,1)$, $\Pi^{TR} \geq \beta$, $R^{TR} = \frac{\Pi^{TR}}{\beta}$, $\overline{\Pi} = \Pi^{TR}$, and $\varepsilon > \frac{2}{\nu}(1-\beta) + 1$ there are two steady states associated with positive interest rates. One has inflation that is the same as the target inflation in the Taylor rule. The other has higher inflation.*

Proof: Without loss of generality, we set $\chi = \frac{\varepsilon - 1}{\varepsilon} (1/3)^{-\sigma - \upsilon}$ such that deterministic equilibrium labor is one third in the economy with flexible prices. For consumption to be positive, X given by (X) must be positive and, therefore, inflation must be bounded as $\Pi^{TR} - \left(\frac{2}{\phi}\right)^{1/2} < \Pi < \Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}$. We define function $G(\Pi)$ as the difference of the right-hand sides of equations (26) and (29), i.e.

$$G(\Pi) = \left(\left(\frac{w}{\chi}\right)(XZ)^{\upsilon}\right)^{\frac{1}{\sigma+\upsilon}} - \Pi^{\frac{1-\phi_\pi}{\phi_y}} \left(\frac{1}{\beta R^{TR}}\right)^{\frac{1}{\phi_y}} \left(\frac{1}{\Pi^{TR}}\right)^{-\frac{\phi_\pi}{\phi_y}} GDP^n \tag{30}$$

$$w = \frac{\varepsilon - 1}{\varepsilon} Z + \frac{(1-\beta)\,\phi}{\varepsilon}\left(\Pi - \overline{\Pi}\right)\Pi Z \tag{31}$$

$$X = \left(1 - \frac{\phi}{2}(\Pi - \overline{\Pi})^2\right). \tag{32}$$

By construction, at any deterministic equilibrium inflation Π^*, $G(\Pi^*) = 0$. Since $R^{TR} = \frac{\Pi^{TR}}{\beta}$, target inflation $\Pi = \Pi^{TR}$ is a root of the function. From (26), at this deterministic equilibrium consumption $C = \frac{1}{3}$.

Another deterministic equilibrium inflation lies in the interval $\left(\Pi^{TR}, \Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}\right)$. To show this, we interpret w and X given by (31) and (32), respectively, as functions of inflation Π and take their derivatives:

$$\begin{aligned}
w' &= \frac{1-\beta}{\varepsilon}\phi\left(2\Pi - \overline{\Pi}\right) \\
w'' &= 2\frac{1-\beta}{\varepsilon}\phi \\
X' &= -\phi\left(\Pi - \overline{\Pi}\right) \\
X'' &= -\phi.
\end{aligned}$$

Further, we omit the argument of w and X and write the first derivative of $G(\Pi)$ as

$$\begin{aligned}
G'(\Pi) &= k\varphi w^{\varphi-1} w'^{\nu\varphi} + k w^{\varphi} \nu\varphi X^{\nu\varphi-1} X' \\
&= k w^{\varphi-1} X^{\nu\varphi-1}\left(\varphi w' X + \nu\varphi w X'\right) - q\xi\Pi^{\xi-1},
\end{aligned}$$

where $\varphi = \frac{1}{\sigma+\nu}$, $\xi = \frac{1-\phi_\pi}{\phi_y}$, $k = \left(\frac{Z^\nu}{\chi}\right)^\varphi$, and $q = \left(\frac{1}{\beta R^{TR}}\right)^{\frac{1}{\phi_y}} \left(\frac{1}{\Pi^{TR}}\right)^{-\frac{\phi_\pi}{\phi_y}} GDP^n$. In the right vicinity of Π^{TR}, $G(\Pi)$ is positive, i.e. $G\left(\Pi^{TR}+\delta\right) > 0$ for sufficiently small $\delta > 0$ because $G'^{TR}) > 0$. At $\Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}$ the function is negative, $G\left(\Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}\right) < 0$. Since $G(\Pi)$ is continuous, by the intermediate value theorem, there is a root in the interval $\left(\Pi^{TR}, \Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}\right)$.

To establish uniqueness of the root, we show that $G(\Pi)$ is concave in this interval. The second derivative of $G(\Pi)$ can be written as

$$
\begin{aligned}
G''(\Pi) &= k(\varphi-1)w^{\varphi-2}w'^{\nu\varphi-1}\left(\varphi w'X + \nu\varphi wX'\right) && (33)\\
&+ kw^{\varphi-1}(\nu\varphi-1)X^{\nu\varphi-2}x'\left(\varphi w'X + \nu\varphi wX'\right) && (34)\\
&+ kw^{\varphi-1}X^{\nu\varphi-1}\left(\varphi w''X + (\varphi+\nu\varphi)w'X' + \nu\varphi wX''\right) && (35)\\
&+ q\xi(1-\xi)\Pi^{\xi-2} && (36)\\
&= kw^{\varphi-2}X^{\nu\varphi-2}\big(2\nu\varphi^2 wXw'X' + (\varphi-1)\varphi\left(w'X\right)^2 + (\nu\varphi-1)\nu\varphi\left(wX'\right)^2 && (37)\\
&+ wX\left(\varphi w''X + \nu\varphi wX''\right)\big) && (38)\\
&+ q\xi(1-\xi)\Pi^{\xi-2} && (39)
\end{aligned}
$$

It follows that all terms in lines (37) and (39) are negative for $\Pi \in \left(\Pi^{TR}, \Pi^{TR} + \left(\frac{2}{\phi}\right)^{1/2}\right)$. For the sum in line (38), we have

$$\varphi w''X + \nu\varphi wX'' = \frac{\varphi\phi}{\varepsilon}\left((1-\beta)\left(2 - \phi\left(\Pi - \overline{\Pi}\right)^2 - \nu\phi\left(\Pi - \overline{\Pi}\right)\Pi\right) - \nu(\varepsilon-1)\right).$$

This expression is negative if $\varepsilon > \frac{2}{\nu}(1-\beta)+1$. This constraint naturally holds with parameter values standard in the literature. ■

Table 1: Parameter calibration

Symbol	Description	Value	Source
β	Quarterly discount factor	0.995	Christiano et al. (2011)
σ	Intertemporal elasticity of substitution	1	
υ	Frisch labor supply elasticity	1	Christiano et al. (2011)
χ	Chosen to normalize SS labor to 1/3	7.83	
ε	Monopoly power	7.66	Boneva et al. (2016)
ϕ	Adjustment cost parameter	238	Boneva et al. (2016)
$\overline{\Pi}$	Indexed inflation (quarterly)	$1.02^{1/4}$	
Z	Steady state technology (normalized)	1	
ϕ_π	Weight on inflation in the Taylor rule	1.5	Gali (2008)
ϕ_y	Weight on output in the Taylor rule	0.5	Gali (2008)
GDP^{TR}	Taylor rule GDP	1/3	
Π^{TR}	Taylor rule inflation target	$1.02^{1/4}$	
R^{TR}	Taylor rule intercept	$\frac{\Pi^{TR}}{\beta}$	

4 Numerical example

In this section, we illustrate the existence of the three deterministic equilibria using parameter values that are standard in the literature. Table 1 reports the parameter values and their sources.

Under this calibration, there are three deterministic equilibria. The first one is associated with zero interest rate and deflation of 2% annually. In this equilibrium, consumption is about 0.4% lower than the target. This deterministic equilibrium is called Regime 1 or the deflation (ZLB) regime hereafter.

In addition, there are two other deterministic equilibria associated with positive in-

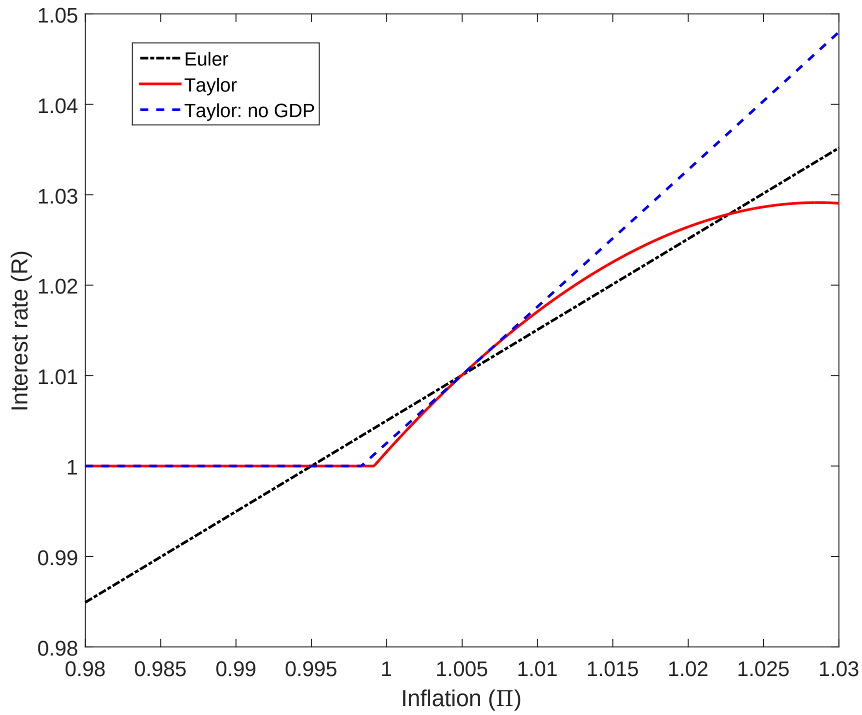

Figure 1: The relationship between the interest rate and inflation at deterministic equilibrium.

terest rates. The first equilibrium is associated with target inflation rate of 2% per year, which is the same as the target inflation. The second deterministic equilibrium has a higher inflation rate, about 9.08% per year (or 2.27% per quarter). The deterministic equilibria associated with target and higher inflation are called Regime 2 (or Targeted-inflation regime) and Regime 3 (or High-inflation regime), respectively.

Figure 1 plots the deterministic equilibrium interest rate as a function of inflation using equations (24) and (25). It is obvious from this figure that the Euler curve (the dotted black line) using equation (24) intersects with the Taylor curve (the solid red line) based on equation (25) at three points. So we have three deterministic equilibria. When the central bank is not trying to stabilize output, the Taylor curve (now the dashed blue line) intersects the Euler curve (the dotted black line) only at two points, and we have only two deterministic equilibria.

The two deterministic equilibria case was pointed by Benhabib et al. (2001) and recent Aruoba et al. (2018). In this paper, our main contribution is we show that there are three deterministic equilibria instead of two when the central bank tries to stabilize both output and inflation.

Our findings suggest that allowing for multiple regimes, including the one with high inflation, in models of inflation dynamics can improve likelihood of the model and archive superior inflation forecasting. In fact, Isakin and Ngo (2019) show that a model with three regimes and regime-dependent constant volatilities has superior performance. It is well-known in the literature that different regimes with different mean inflation associate with different inflation volatilities. So, the multiple equilibria phenomenon in this paper is in line with their findings.

In addition, our findings are also consistent with Bianchi and Melosi (2017), where they estimate a MS-VAR model and find that the number of regimes is three: one associated with high inflation, one with targeted inflation, and one with low inflation.

4.1 Robustness

In this subsection, we illustrate how the deterministic equilibrium inflation changes in the three regimes when there is a change in primitive parameters, especially preference and price adjustment costs. Figures 2 and 3 show the relationship between deterministic equilibrium inflation with the discount factor and the price adjustment cost parameter in different regimes.[4]

[4]We also analyzed the role of deterministic technology on inflation in different regimes. However, we found that permanent technology shock does not affect relative inflation in the regimes. To save space,

As shown in Figure 2, an increase in the discount factor, β, causes the deterministic equilibrium inflation rate of Regime 2 to decline. It can be as small as approximately zero. The increase in the discount factor, β, can be explained as a permanent increase in the patience of households. It is interesting from this figure is that low inflation at the targeted inflation regime tends to associate with higher inflation in the other regimes. In addition, the deflation regime may become indistinguishable from the targeted inflation regime under a permanent adverse shock to the time discount factor.

On the contrary to the discount factor, Figure 3 shows that changes in the price adjustment cost parameter only affect the high inflation deterministic equilibrium, or regime 3. In particular, an increase in the price adjustment cost parameter, ϕ, causes the deterministic equilibrium inflation in Regime 3 to decline. An increase in the price adjustment cost parameter, ϕ, reflects more price rigidity and vice versa. Intuitively, deterministic equilibrium inflation does not depend on the price adjustment cost in regimes 1 and 2. It only depends on the time discount factor in these regimes.

5 Conclusion

We show that when the central bank tries to stabilized both inflation and output gap, the standard dynamic New Keynesian model has three deterministic equilibria. One is associated with zero nominal interest rate and deflation; the other two with positive interest rates. Our findings suggest that allowing for multiple regimes, including the one with high inflation, in models of inflation dynamics can improve likelihood of the model and archive superior inflation forecasting.

we do not report that result.

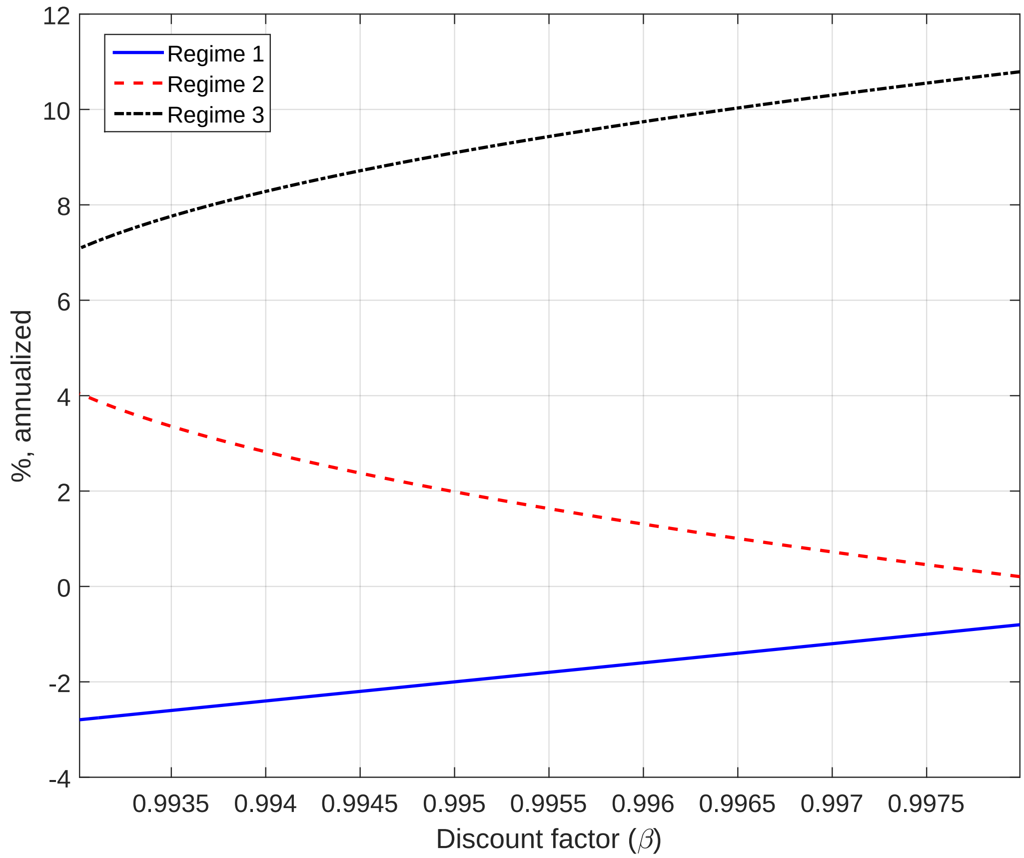

Figure 2: The relationship between deterministic equilibrium inflation and the discount factor, β, in different regimes.

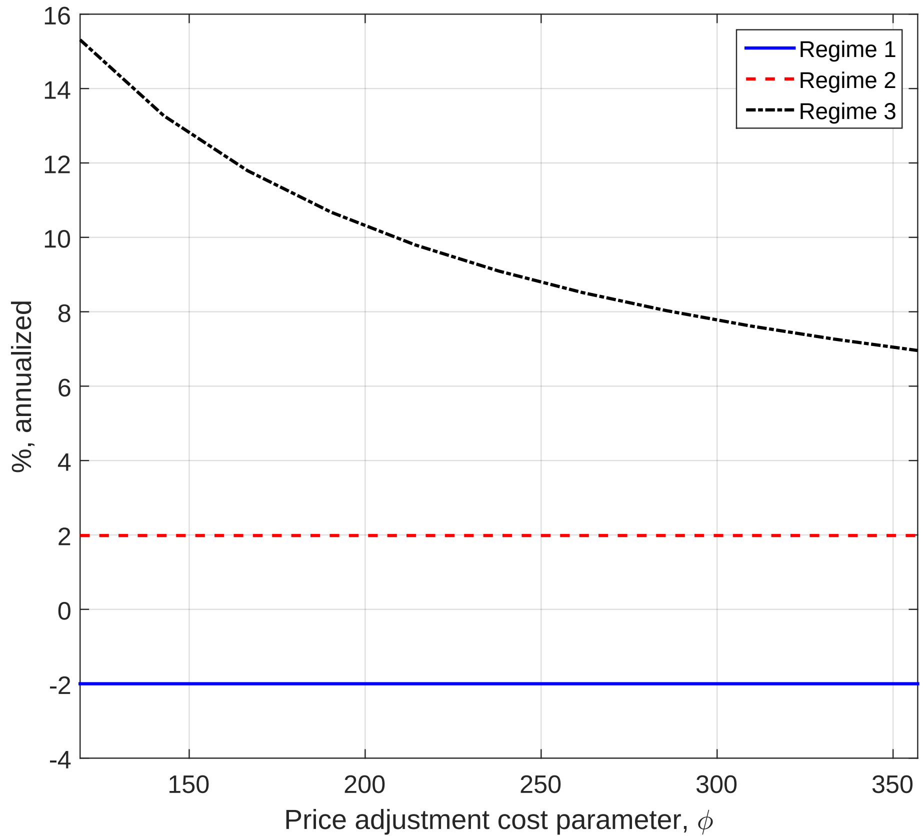

Figure 3: The relationship between deterministic equilibrium inflation and the price adjustment cost parameter, ϕ, in different regimes.

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