

Optimal Discretionary Monetary Policy in a Micro-Founded Model with a Zero Lower Bound on Nominal Interest Rate

Phuong V. Ngo^{a,*}

^a*Department of Economics, Cleveland State University, 2121 Euclid Avenue, Cleveland, OH 44115*

Abstract

This paper investigates optimal discretionary monetary policy under the zero lower bound on the nominal interest rate (ZLB) in the case of a distorted steady state due to monopoly and taxation. Solving a fully nonlinear micro-founded (FNL) model using a global method, I find that the central bank in a more distorted economy would cut the interest rate less aggressively under a particularly adverse demand shock. This occurs because inflation and nominal interest rates are higher on average, making the ZLB less likely to bind and causing the economy to escape from the ZLB sooner. However, the social planner would choose the optimal inflation rate of approximately zero. The result emerges because the unconditional benefit of avoiding the ZLB is not big enough to offset the cost of higher relative price dispersion when inflation is significantly positive. In addition, I show that the conventional linear-quadratic (LQ) method is inaccurate in the case of a sufficiently distorted steady state.

JEL classification: C61, E31, E32, E52.

Keywords: optimal discretionary monetary policy, ZLB, distorted steady state, optimal inflation rate, Calvo price adjustments, nonlinear method

*Corresponding author. Tel. +1 617 347 2706
Email address: p.ngo@csuohio.edu (Phuong V. Ngo)

1. Introduction

The focus of researchers concerned with optimal monetary policy under the ZLB has been the case of a non-distorted steady state, where the overall economic distortion due to monopoly and taxation is assumed to be zero. Specifically, government subsidies exist to fully offset monopolistic distortion so that the steady state output is not distorted from its socially efficient level. Hence, we can simplify a fully nonlinear micro-founded problem of optimal discretionary monetary policy using the LQ approach developed by Woodford (2001, 2003) and can avoid computational difficulty.

This paper aims at filling the hole in the ZLB literature by investigating optimal discretionary monetary policy under the ZLB in the case of a distorted steady state due to positive overall economic distortion. To this end, I solve a FNL micro-founded model using a global method. Also, I use the LQ method to simplify the FNL model, which I solve using the same method. I then provide a comparison between the FNL and LQ models.

Studying the case of a distorted steady state brings the ZLB literature closer to reality. McGrattan (1994) reports that labor income taxes range from 10 – 40%, while Diewert and Fox (2008) estimate that monopolistic markups in some main industries range from 11 – 44%. As a result, the overall economic distortion ranges from 20 – 60%. This information might influence private expectations, which, in turn, would affect optimal policy before, during and after the ZLB period.¹

¹It is well-known in the literature of inflation bias that the greater the overall economic distortion, the higher the average inflation under discretion and, as a result, the higher the nominal interest rate, see Woodford (2003). Whether in reality we observe the kind of inflation bias that emerges in the model under discretionary policy in the case of a distorted steady state is still debatable and beyond the scope of this paper.

Solving FNL models also helps us to answer the question stated in Adam and Billi (2007): to what extent does the full nonlinearity affect the optimal monetary policy under discretion in the presence of the ZLB? In addition, we can study the role of relative price dispersion as an endogenous state variable, which is eliminated in the LQ framework due to the linear approximation.²

I obtain four sets of main findings. First, under a particularly adverse shock driving the economy near the ZLB, the central bank in an economy with a larger economic distortion would cut the interest rate less aggressively. The intuition is simply that, in this economy, inflation and nominal interest rate are higher on average. When the nominal interest rate is near the ZLB, a particularly adverse demand shock might have occurred. Given the mean-reverting nature of shocks, the conditional probability that another adverse shock occurs and pushes the economy into the liquidity trap with binding ZLB is very small. Furthermore, even when this shock occurs and the ZLB binds, the output losses and reduction in inflation are smaller than they would be in an economy with a smaller economic distortion. Therefore, downward pressure on the conditional expected inflation is smaller and the central bank cuts the nominal interest rate less aggressively.

Second, with a larger overall economic distortion, inflation and interest rates are higher on average, resulting in a smaller probability of reaching the ZLB. However, the social planner would choose the optimal inflation rate of approximately zero, corresponding to very small overall economic distortion. This occurs because the unconditional benefit from avoiding the ZLB is not big enough to offset the cost of higher relative price dispersion when inflation is high. In sum, the unconditional

²Alvarez et al. (2011) find that relative price variance is significantly positive when inflation is high, while Zandweghe and Wolman (2010) show that initial relative price dispersion could affect monetary policy. So studying the role of relative price dispersion is interesting.

expected welfare is maximal when the average long-run inflation is around zero.

Third, when the initial relative price distortion is greater than the steady state value, the central bank tends to pursue a higher nominal interest rate, making the ZLB less likely to bind. The intuition is that the relative price dispersion is an inefficiency wedge: when the relative price dispersion is high, the central bank would like to reduce it by tightening the monetary policy and, as a result, lowers the front-loading behavior by firms in setting their prices, leading to a smaller current relative price dispersion. This result is interesting and can not be found using the LQ method because the change in relative price dispersion is always zero.

Finally, the FNL model and the LQ model produce different results if there is a particularly adverse shock that makes the ZLB binding. When the ZLB binds, the central bank cannot stabilize output and the price level, making the relative price dispersion distant from the steady state. While the impact of the relative price dispersion as an endogenous state variable in the FNL model is significant, it is always zero and has no role in the LQ model due to the first order approximation. However, the difference between the FNL and LQ model is not significant in the case of a non-distorted steady state.

When the overall economic distortion is large, the two methods produce very different results, especially when the ZLB binds. The approximated inflation and interest rate in the LQ model are substantially smaller than the true values derived using the FNL model. Consequently, given the ZLB binds in both models, the output losses in the FNL model are significantly smaller than those in the LQ model. In addition, the interest rate cut in the FNL model is less aggressive under a shock driving the interest rate near the ZLB.

The related literature on optimal monetary policy under the ZLB was inspired by seminal work by Krugman (1998), which extensively discusses causes and conse-

quences of the ZLB in a series of simple two-period perfect-foresight models. Since then, extensive research related to the ZLB has been implemented, including Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006), Nakov (2008), Levin et al. (2010), Bodenstein et al. (2010), Eggertsson and Krugman (2010), and Werning (2011). The common feature of these papers is that they focus on the case of non-distorted steady state and use the LQ method.

The papers closest to mine are Adam and Billi (2007), and Anderson et al. (2010). Like my paper, Adam and Billi (2007) use a global method to solve an optimal discretionary monetary policy problem that allows for an occasionally-binding ZLB. However, because they use the LQ approach, the only nonlinearity in their paper is the ZLB. This paper extends the work of Adam and Billi (2007) by considering a fully nonlinear model. In addition, this paper studies the implications of positive overall economic distortion on discretionary policy and optimal inflation rate in the presence of the ZLB.

Anderson et al. (2010) investigate the size of inflationary biases under discretion in the presence of overall economic distortion using nonlinear methods. However, in their model, the nominal interest rate can be adjusted freely because the ZLB is not imposed. Hence, the average long-run inflation is the same as the deterministic steady state inflation. This paper extends their work by considering the ZLB, a very important constraint faced by policymakers.

There are three recent working papers studying the ZLB using fully nonlinear methods. Nakata (2011) studies optimal fiscal and monetary policy in a nonlinear sticky price model of the Rotemberg-type instead of the Calvo-type as in my model. I choose to use the Calvo-type price adjustments so that I can examine relative price dispersion as an endogenous variable and compare my results directly to the results in the previous literature. I study the role of economic distortion, while Nakata

(2011) focuses on fiscal policy. Fernandez-Villaverde et al. (2012) study the ZLB in a fully nonlinear model using the collocation method associated with Smolyak nodes. Judd et al. (2011) solve a fully nonlinear New Keynesian model with the ZLB using a cluster-grid algorithm. The monetary policy in these two papers is a Taylor rule. They do not consider optimal discretionary monetary policy.

The remainder of this paper is organized as follows. Section 2 presents the structure of the economy, and Section 3 describes the discretionary monetary policy problem faced by a central bank and explains briefly solution methods. I calibrate key parameters and report main results in Section 4, and conclude in Section 5. Numerical algorithms and mathematical derivations are presented in Appendices.

2. Model

The economic structure in this paper presents two key New Keynesian features, such as in Rotemberg and Woodford (1997) and Yun (1996). Specifically, intermediate goods producers are monopolistic competitors. In addition, they reset their prices infrequently à la Calvo (1983).

2.1. Household

The representative household maximizes his total expected discounted flow utilities:

$$\max E_t \left\{ \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right) + \sum_{j=1}^{\infty} \left\{ \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) \left(\frac{C_{t+j}^{1-\gamma}}{1-\gamma} - \chi \frac{N_{t+j}^{1+\eta}}{1+\eta} \right) \right\} \right\}$$

subject to the budget constraint:

$$C_t + B_t = (1 - \tau_w) w_t N_t + B_{t-1} \left(\frac{1 + i_{t-1}}{1 + \pi_t} \right) + \int_0^1 D_t(i) di + T_t \quad (1)$$

where C, N are composite consumption and total labor; B, D, T denote real bonds, dividends, and lump sum transfers; i, π are the net nominal interest rate and the inflation rate, respectively; w is the real wage; τ_w is the labor income tax rate; γ, η, χ are the risk aversion parameter, the inverse elasticity of labor with respect to wages, and the steady state labor determining parameter; β is the stochastic subjective discount factor or preference that follows an AR(1) process with a steady state value $\bar{\beta}$:

$$\ln(\beta_{t+1}) = (1 - \rho_\beta) \ln(\bar{\beta}) + \rho_\beta \ln(\beta_t) + \varepsilon_{\beta,t+1}, \text{ where } \beta_t \text{ is given.} \quad (2)$$

The optimal choices of the household must satisfy the following conditions:

$$E_t \left[\beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \right] = 1 \quad (3)$$

$$\frac{\chi N_t^\eta}{C_t^{-\gamma}} = (1 - \tau_w) w_t \quad (4)$$

The first condition shows the marginal inter-temporal trade-off between today's and tomorrow's consumption. The second condition is the marginal trade-off between working and consuming.

The stochastic preference is a reduced form of more realistic forces that can drive the nominal interest rate to the ZLB. From the Euler equation, an increase in the discount factor causes the nominal interest rate to fall, given private expectations and households' desire to smooth their consumption. ³

³The conventional technology shock is not able to cause the ZLB to bind realistically. The reason is that we need a very big positive technology shock to generate massive savings that can drive the nominal interest rate to the ZLB. We did not observe this type of shock before the onset of the last

To produce the composite consumption goods, C_t , the household buys and aggregates a variety of intermediate goods using a CES technology. His cost-minimization problem is given below.

$$\min \int_0^1 P_t(i) C_t(i) di \quad s.t. \quad C_t = \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5)$$

where $C_t(i)$ is the amount of intermediate goods $i \in [0, 1]$ and ϵ is the elasticity of substitution among intermediate goods.

The optimal condition gives rise to the demand for the intermediate goods i , $C_t(i)$, and the aggregate price level, P_t , below :

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (6)$$

$$P_t = \left(\int P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \quad (7)$$

crisis, see Amano and Shukayev (2012).

Guerrieri and Lorenzoni (2011) model debt limit and household heterogeneity in labor productivity. They show that an exogenous decline in the debt limit acts as an increase in the subjective discount factor in my representative agent model. The decline in the debt limit causes future consumption to be more volatile because with a lower debt limit households will be less able to insure their consumption against risks. Therefore, the savers will save more and the borrowers will borrow less due to precautionary savings.

Eggertsson and Krugman (2010) also model household debt limit and deleveraging as a key factor to drive the nominal interest rate to the ZLB. In their model, an initial shock to the debt limit causes borrowers to deleverage by cutting back their consumption, resulting in a decrease in the price level. This deflation puts more pressure on the real debt the borrowers have to pay back now, leading to further deleveraging and a sharper decline in the nominal interest rate.

Ngo (2013) extends Eggertsson and Krugman (2010) by endogenizing the debt limit. He studies the interaction between the ZLB and the endogenous debt limit in explaining the collapse of the housing market and the Great Recession. Hall (2011) models excessive capital stock and a sharp decline in capital utilization as the reason for the nominal interest rate to be pinned at the ZLB. Curdia and Woodford (2009) model a shock to the wedge between deposit and lending rates as a driving force.

2.2. Intermediate goods producers

There is a mass one of intermediate goods producers that are monopolistic competitors. In each period, a firm keeps its previous price with probability θ and resets its price with probability $(1 - \theta)$.

Given its price $P_t(i)$ and demand $Y_t(i)$, the firm i chooses labor that

$$\min \{w_t N_t(i)\} \quad s.t. \quad Y_t(i) = N_t(i) \quad (8)$$

Let $\varphi_t(i)$ be the Lagrange multiplier with respect to the production. The first order condition gives the same marginal cost to all firms, φ_t :

$$\varphi_t = \varphi_t(i) = w_t \quad (9)$$

Whenever a firm has a chance to reset its price, it chooses the new price to solve:

$$\underset{P_t(i)}{Max} E_t^i \left\{ \left[\frac{P_t(i)}{P_t} - \varphi_t \right] Y_t(i) + \sum_{j=1}^{\infty} \left\{ \theta^j \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) \left(\frac{C_{t+j}}{C_t} \right)^{-\gamma} \left[\frac{P_t(i)}{P_{t+j}} - \varphi_{t+j} \right] Y_{t+j}(i) \right\} \right\} \quad (10)$$

subject to its demand in equation (6).

The optimal relative price, $P_t^*(i)/P_t$, is the same for all firms that are able to reset their prices today:

$$\frac{P_t^*(i)}{P_t} = p_t^* = \frac{\left(\frac{\varepsilon}{\varepsilon-1} \right) E_t \left\{ C_t^{-\gamma} Y_t \varphi_t + \sum_{j=1}^{\infty} \left\{ \theta^j \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) C_{t+j}^{-\gamma} \left(\frac{P_{t+j}}{P_t} \right)^{\varepsilon} Y_{t+j} \varphi_{t+j} \right\} \right\}}{E_t \left\{ C_t^{-\gamma} Y_t + \sum_{j=1}^{\infty} \left\{ \theta^j \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) C_{t+j}^{-\gamma} \left(\frac{P_{t+j}}{P_t} \right)^{\varepsilon-1} Y_{t+j} \right\} \right\}} \quad (11)$$

With some manipulation, we can rewrite the optimal pricing rule as below:

$$p_t^* = \frac{S_t}{F_t} \quad (12)$$

where S_t, F_t are written in the following recursive forms:

$$S_t = \left(\frac{\varepsilon}{\varepsilon - 1} \right) C_t^{-\gamma} Y_t \varphi_t + \theta E_t [\beta_t \Pi_{t+1}^\varepsilon S_{t+1}] \quad (13)$$

$$F_t = C_t^{-\gamma} Y_t + \theta E_t [\beta_t \Pi_{t+1}^{\varepsilon-1} F_{t+1}] \quad (14)$$

and $\Pi = (1 + \pi)$ is gross inflation.

Combining (13) with (4) and (9), we obtain:

$$S_t = \frac{\chi C_t N_t^\eta}{(1 - \Phi)} + \theta E_t [\beta_t \Pi_{t+1}^\varepsilon S_{t+1}] \quad (15)$$

where

$$\Phi = 1 - (1 - \tau_w) \cdot (1 - \varepsilon^{-1}) \quad (16)$$

and Φ is called overall economic distortion. I will discuss this metric in a section below.

2.3. Aggregate conditions

Aggregate output satisfies:

$$Y_t = \frac{N_t}{\Delta_t} \quad (17)$$

where Δ_t is called the relative price dispersion and is defined as:

$$\Delta_t = \int \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di \quad (18)$$

or in a recursive form:

$$\Delta_t = \theta \Pi_t^\varepsilon \Delta_{t-1} + (1 - \theta) (p_t^*)^{-\varepsilon} \quad (19)$$

I write the price level (7) in a recursive form and divide both sides by P_t to obtain the optimal relative price:

$$p_t^* = \left(\frac{1 - \theta \Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} \quad (20)$$

Plugging this optimal relative price in the relative price dispersion equation (19) we obtain:

$$\Delta_t = (1 - \theta) \left(\frac{1 - \theta \Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi_t^\varepsilon \Delta_{t-1} \quad (21)$$

2.4. Overall economic distortion

In this section, I discuss the overall economic distortion, which is defined as in equation (16). To understand more about the meaning of this notation, let us consider an economy with flexible price. In this economy, the marginal cost, φ , equals the inverse of markup (or $(1 - \varepsilon^{-1})$). From equation (4), (9), and (17) we compute the equilibrium flexible-price output (Y_t^f) and the equilibrium efficient output (Y_t^*) as follows:

$$Y_t^f = N^* \cdot (1 - \Phi)^{\frac{1}{\eta+\gamma}} \quad (22)$$

$$Y_t^* = N^* \quad (23)$$

where N^* is the long-run efficient output/labor. The percentage deviation of the flexible-price output from the efficient output equals:

$$\left(\frac{Y_t^f - Y_t^*}{Y_t^*} \right) \cdot 100 \simeq -\frac{1}{\eta + \gamma} \cdot \Phi \cdot 100$$

The larger the overall economic distortion, the smaller the flexible-price output relative to the efficient output. It is important to note that while the overall economic distortion is zero, it does not mean there is not any type of economic distortions. Instead, it means that we can attain the efficient output level by designating a labor income subsidy to fully offset the monopoly power, given no price stickiness.

It is also important to emphasize that when the overall economic distortion is large or the inverse labor elasticity and risk aversion are small, the flexible price output is far below the efficient output level. Under discretion, the central bank tends to create positive inflation to try to attain the efficient output. In equilibrium, the greater the overall economic distortion, the smaller the flexible-price output relative to the efficient output, and the greater the inflation the central bank tends to create.

3. Optimal discretionary policy problem under the ZLB

In the discretionary framework, the central bank is able to re-optimize its problem every period, and the economic agents take this information into account when they form their expectations. Given this common knowledge, the central bank's problem is to maximize the society's (or the representative household's) expected present-discounted lifetime utility subject to the optimality conditions of the economic agents, the aggregate conditions, the law of motion for the state variables, and the explicit ZLB on the nominal interest rate. The FNL model features an endogenous state variable - the relative price dispersion. As a result, the central bank takes into account how today's price dispersion may affect the agents' expectations of future inflation, consumption, etc.⁴

⁴The central bank can manipulate private expectations by committing to a path of current and future inflation, interest rates, etc. However, the time-inconsistency issue arises, and the commitment may not be credible.

The problem can be stated in the form of a Bellman equation:

$$V(\Delta_{t-1}, \beta_t) = \max_{\{i_t, C_t, N_t, S_t, F_t, \pi_t, \Delta_t\}} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + \beta_t E_t V(\Delta_t, \beta_{t+1}) \right\} \quad (24)$$

subject to

- (i) Households' and firms' optimality conditions, and aggregate conditions.
- (ii) Law of motion for state variables.
- (iii) ZLB on the nominal interest rate ($i_t \geq 0$).
- (iv) No commitment to future policy that is made in the past.⁵

The solution of the above nonlinear system is called the Markovian invariant policy function of the state, $s_t = (\Delta_{t-1}, \beta_t)$, where Δ_{t-1} is an endogenous state and β_t is an exogenous one. In the paper, I solve the above FNL model using a global method called the collocation method. First, I use equidistant collocation nodes to solve the model and find out policy function. Based on this information, I investigate potential kinks. I then redistribute the nodes by clustering them around these potential kinks and resolve the model. I employ the time-iteration method. At each collocation node, I solve a complementarity problem using the Newton method and the semi-smooth root-finding algorithm as described in Miranda and Fackler (2002). I also provide an "analytical" Jacobian matrix computed from the approximating functions.⁶ Moreover, I write my code using a parallel computing method that allows me to split up a large number of collocation nodes into smaller groups that then are assigned to different processors to solve simultaneously. These computational characteristics help to significantly increase the rate of convergence and make the

⁵See Appendix A for how to write down the problem in detail.

⁶See Appendix C for the "analytical" Jacobian matrix.

solution method very reliable.⁷

I also use the LQ approach, as described in Woodford (2003), to simplify the FNL model which I then solve using the same method.⁸ Specifically, according to the LQ approach, the central bank's objective function is quadratically approximated and all the constraints and law of motion are (log)linearly approximated around the steady state values associated with zero inflation. The endogenous variables in the LQ framework are defined as below:

$$\widehat{\pi}_t = \log(1 + \pi_t) - \log(1) \quad (25)$$

$$\widehat{i}_t = \log(1 + i_t) - \log(1/\bar{\beta}) \quad (26)$$

$$x_t = (\log(Y_t) - \log(\bar{Y})) - (\log(Y_t^f) - \log(\bar{Y}^f)) \quad (27)$$

where $\bar{\beta}$ is the steady state discount factor; \bar{Y} and \bar{Y}^f are the steady state sticky-price and flexible-price outputs when the overall economic distortion presents; x is the output gap. First, I solve for the policy function in the LQ framework, including \widehat{i}_t , $\widehat{\pi}_t$, and x_t . Then, I back out the policy function for i_t , π_t , Y_t using equations (25) – (27), which I call LQ results.

⁷See Appendix B for how to solve the model in detail, including the error reported from checking the solution.

⁸See Appendix D for the simplified LQ model. Note that (i) the LQ model with the ZLB is also a nonlinear model and I have to use the global method to solve it; (ii) the LQ approach is actually not applicable when the overall economic distortion, Φ , is large.

4. Results

4.1. Parameter calibration

I calibrate the steady state quarterly time discount factor, $\bar{\beta}$, to be 0.993, corresponding to a real interest rate of 2.8% per year. The relative risk aversion (γ) is 4, as in Nakov (2008), and the inverse elasticity of labor with respect to wages (η) is 1. The monopoly power parameter (ε) is calibrated to be 10, corresponding to a 11% markup that is in the range found by Diewert and Fox (2008). The probability that a firm keeps its price unchanged each quarter, θ , is chosen to be 0.75 so that firms keep their prices for 4 quarters on average. This value is commonly used in the literature, such as Anderson et al. (2010).

TABLE 1. PARAMETERIZATION

<i>Symbol</i>	<i>Description</i>	<i>Values</i>
$\bar{\beta}$	Quarterly discount factor	0.993
γ	Constant relative risk aversion	4
η	Inverse elasticity of labor with respect to real wage	1
ε	Monopoly power	10
θ	Probability that a firm keeps its price unchanged each quarter	0.75
σ_{β}	Standard deviation of preference shocks (percent)	0.4
ρ_{β}	AR-coefficient of preference shocks	0.8
Φ	Overall economic distortion	0; 0.20
χ	Parameter associated with the disutility of labor	1

I calibrate the persistence of the preference shock to be 0.8, which is consistent with the persistence of the natural rate of interest rate as in Adam and Billi (2007). The difficulty is determining how to calibrate the variance of the preference shock. In

this paper, I calibrate this parameter to be 0.42% per quarter that enables the model to generate the unconditional probability of hitting the ZLB of around 6%. This value is a little small compared with the fact that the nominal interest in the U.S. has been at the ZLB since December 2008 and that it is projected to be at the bound until mid-2015. However, this value is still the upper value of the range 5% – 6% found in the empirical studies before the last financial crisis, as in Fernandez-Villaverde et al. (2012).

The overall economic distortion, Φ , is calibrated to be either 0 or 0.20. The first value corresponds to the well-known non-distorted steady state. The second value corresponds to the case where labor income tax is set to be 11%. Although, the tax rate is conservative, it is still in the range found by Diewert and Fox (2008). As I show below, a higher value of Φ only makes the LQ model more inaccurate.

4.2. Steady state

The steady state values depend on the overall economic distortion (Φ). With a labor income subsidy designed to fully offset the monopolistic distortion, the overall economic distortion is zero. In this case, the steady state inflation and gross interest rate are 0 and $1/\bar{\beta}$ respectively. However, in the case of positive overall economic distortion, it is difficult to compute the steady state values.

Figure 1 shows optimal gross inflation and relative price dispersion as a function of initial relative price dispersion, given the steady state preference of $\bar{\beta}$. The steady state relative price dispersion is the value that equals the initial relative price dispersion. In this example, they are 1.0029 (or about 1.2% annually). Using this value, we can compute the steady state gross inflation to be 1.006 (or 2.4% per year). See the Appendix D for the formula that can be used to compute the steady state inflation using the LQ method.

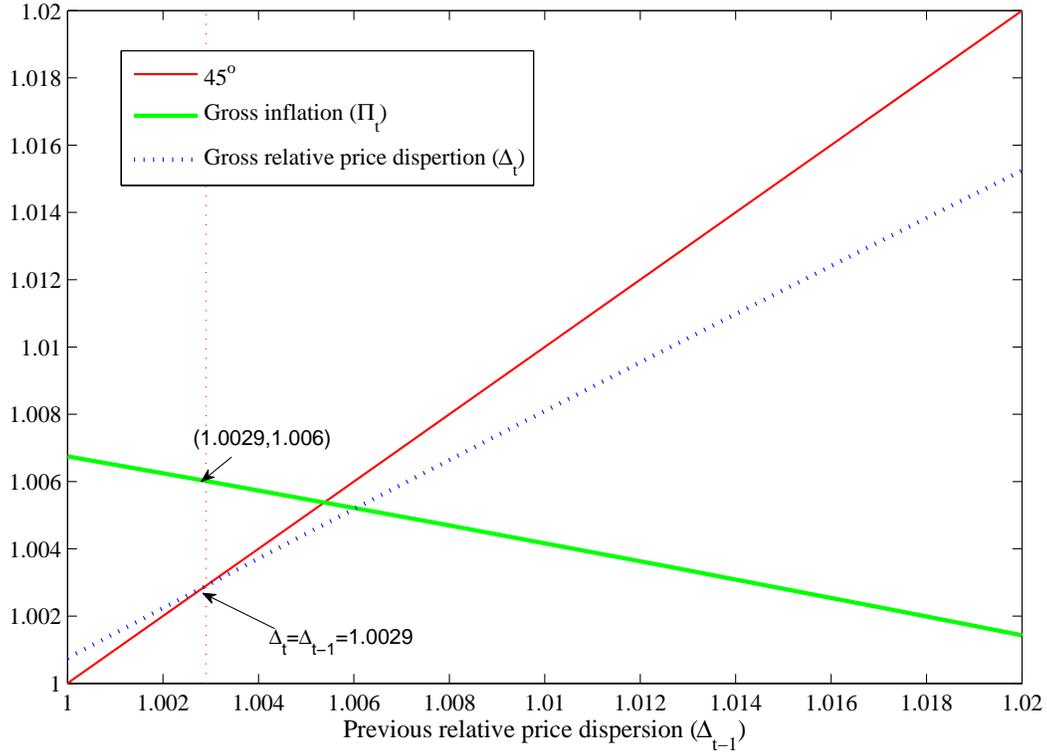


Figure 1: Inflation and current relative price dispersion. Note that preference is at the steady state ($\beta^{SS} = \bar{\beta} = 0.993$) and overall economic distortion (Φ) is 0.20.

To illustrate the impact of nonlinearity and overall economic distortion, I compute the deterministic steady state inflation in both LQ and FNL models with respect to different values of the overall economic distortion.⁹ The results are presented in Figure 2 and are similar to those of Anderson et al. (2010).

Two interesting features in Figure 2 are worth being addressed. First, there is a positive relationship between the steady state inflation and the size of overall

⁹Note that the monopolistic distortion is always 0.1. For each value of overall economic distortion (Φ), we can compute a corresponding value of income tax using equation (16). For example, if $\Phi = 0$, $\tau_w = -11\%$; if $\Phi = 0.2$, $\tau_w = 11\%$

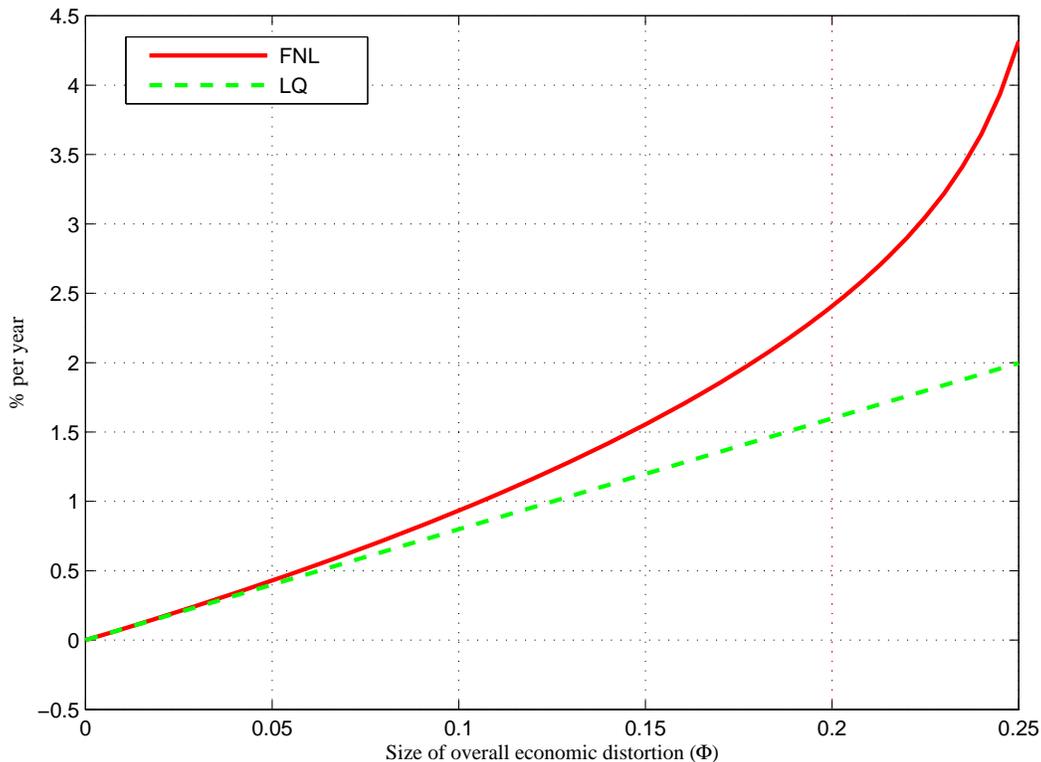


Figure 2: Steady state inflation.

economic distortion in both FNL and LQ models. Intuitively, the larger the size of the overall distortion, the higher the marginal benefit of inflation: a higher inflation rate can help to lower the real markup, stimulating output and employment. However, this comes at a cost. In fact, a higher inflation rate induces a firm to set a higher price when it has a chance to do so. This is because the firm knows that it may not be able to adjust its price in the future and that a higher inflation rate will erode its relative price and profit. This front-loading behavior in price setting causes the dispersion in relative prices to increase and lower the aggregate output, as in equation (17).

Second and more importantly, Figure 2 shows that the steady state interest rate in the true (FNL) model is a convex function with respect to the size of overall economic distortion. However, the steady state interest rates in the LQ model are only the first order approximation of the true value around $\Phi = 0$. Due to the convexity of the true function, the LQ model always underestimates the true steady state value.¹⁰ When the size of the overall economic distortion increases, the underestimation increases at an increasing rate.

Surprisingly, it is not difficult to prove that, under commitment, the steady state inflation rate in both LQ and FNL models is zero regardless of the size of overall economic distortion.¹¹ Therefore, the steady state interest rate is always equal to the steady state real interest rate, which is 2.8% annually.

With inflation and interest rates being smaller than the true ones, the ZLB is more likely to be reached in the LQ model. Therefore, given preference shocks that cause the ZLB to bind in both models, the LQ method generates more sizeable output losses than the FNL method. We will see this more clearly in the next section.

4.3. Optimal output, inflation, and interest rate policy

When there is a positive preference shock, households value their future consumption more. In other words, they are more patient so they tend to save more and consume less today, putting downward pressure on output and the price level. To restore consumption and output, we need a lower real rate. If the central bank was not restrained by the ZLB, it could adjust the nominal interest rate so that the actual real interest rate is the same as the natural real rate. However, because the

¹⁰While the degree of convexity depends negatively on the curvature of the labor supply (η) and the risk aversion parameter (γ), it is positively related to the price stickiness.

¹¹See Schmitt-Grohe and Uribe (2010) for the proof.

ZLB is allowed, a big positive preference shock causes the ZLB to bind. As a result, the actual real rate will be larger than the natural real rate because the nominal interest rate cannot be negative, resulting in a sizable output loss. For comparison, I experiment with different levels of overall economic distortion and relative price dispersion.

4.3.1. *The case of a non-distorted steady state*

In this efficient economy, there exists a labor income subsidy designed to fully offset the monopoly power so the overall economic distortion (Φ) is zero. Figure 3 shows the policy function at each state of the preference that is presented as a percentage deviation from the steady state $\bar{\beta}$, given the initial relative price dispersion at the steady state. The results are annualized. The solid blue lines represent the results from the FNL model with the ZLB, while the dashed green lines represent those from the FNL model without the ZLB. The dot-dashed red lines show the results from the LQ model with the ZLB.

The results from the FNL model have the same characteristics as those in Adam and Billi (2007) and Nakov (2008). First, in the absence of the ZLB, the central bank can achieve the target efficient output and price stabilization by adjusting the nominal interest rate as much as possible, even to its being negative. Second, when the ZLB presents, the central bank cannot stabilize output and inflation under shocks that cause the ZLB to bind. Third, the central bank cuts the nominal interest rate more aggressively, especially when the economy is near the ZLB, in the model with the stochastic ZLB than in the model without the ZLB or with perfect foresight binding ZLB. The aggressiveness occurs due to the *risk* of falling into the liquidity trap associated with deflation that forces the central bank to cut the interest rate

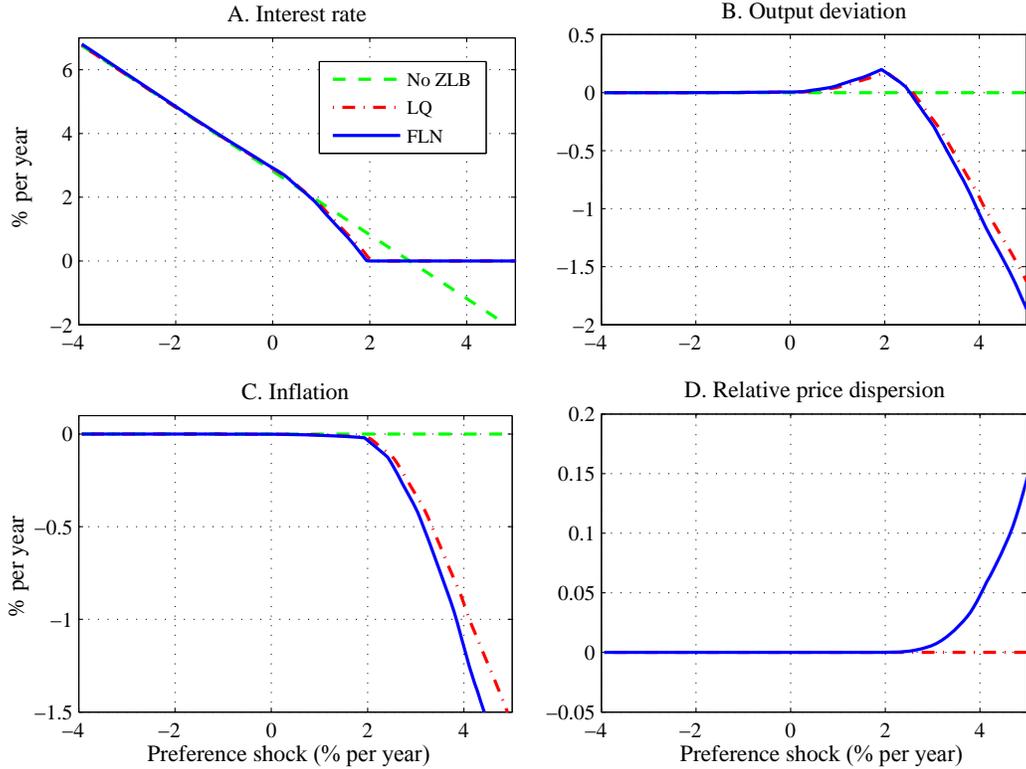


Figure 3: Optimal policy in the economy with a non-distorted steady state ($\Phi = 0$). Gross initial relative price dispersion is at the steady state ($\Delta_{t-1} = \Delta^{SS} = 1$). The preference shock is annualized percentage deviation from the steady state ($\beta^{SS} = \bar{\beta} = 0.993$).

more than it would be without the risk.¹²

Adam and Billi (2007) ask whether a fully-nonlinear model might generate a policy function different from theirs. By solving both the FNL and LQ models, I am able to answer that question.¹³ The dot-dashed red lines in Figure 3 present the

¹²See Adam and Billi (2007) and Nakov (2008) for more detailed explanation.

¹³Fernandez-Villaverde et al. (2012) claim that results from a fully nonlinear model are very different from those in the LQ model. However, they model monetary policy using a Taylor rule with an inflation target of 2% instead of zero inflation target as in this part. Judd et al. (2011) compare the results from their nonlinear method with those from the perturbation method, not with the LQ method.

policy function using the LQ framework. Without a particularly positive shock, the optimal policy is very similar in the two models. The finding is robust to the parameters and the nature of shocks. The reason is that, when $\Phi = 0$, the steady state inflation and interest rates are the same in both models regardless of the parameters and the nature of shocks. Also, the relative price dispersion is zero in the two models.

However, when a particularly positive shock occurs and the ZLB binds, the central bank cannot stabilize the price level, so the current relative price dispersion increases from the steady state and starts playing its role, as a negative technology shock, in the FNL model. In this case, the FNL model generates more output loss and more decline in the price level than does the LQ model, which keeps the relative price dispersion constant regardless of the state of the economy, as in Panel D of Figure 3. However, the differences between the FNL and LQ models are not significant.

To investigate the role of initial relative price dispersion under the ZLB, I plot the optimal policy using different values of initial relative price dispersion, as in Figure 4. The solid blue lines show the policy function when the initial relative price dispersion is 0% annually, while the dash-dotted red lines and dashed green lines show the policy function when the initial relative price dispersion is at 3.5% and 10% respectively.

Note that the initial relative price dispersion (Δ_{t-1}) can be very high due to a change in the tax regime, although the responses of current relative price dispersion (Δ_t) are relatively small under preference shocks. For example, when the labor income tax changes from $\tau_w = 16.67\%$ initially to $\tau_w = -11.0\%$ as in the case of a non-distorted steady state, the initial relative price dispersion is 5.2% per year. The larger the tax change, the greater the initial relative price dispersion.

As shown in Figure 4, when the initial relative price dispersion is 3.5%, the nominal interest rate is about 0.7% higher than it would otherwise be if the initial

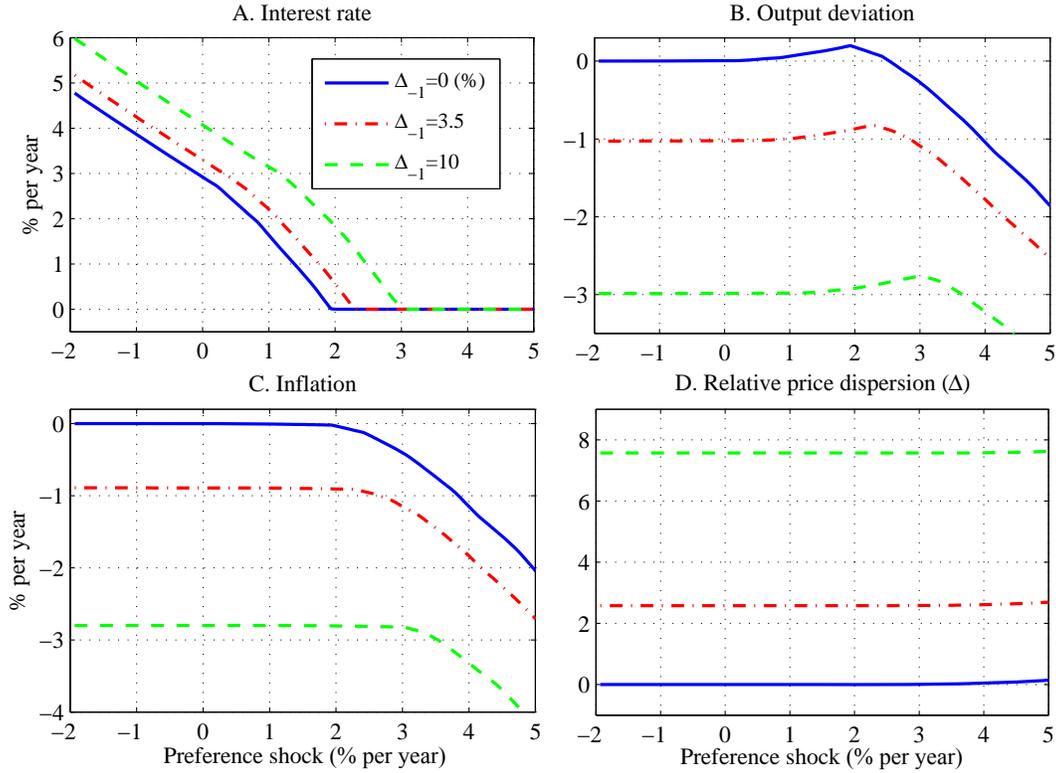


Figure 4: Optimal policy in the economy with a non-distorted steady state ($\Phi = 0$), with different values of initial relative price dispersion, in the FNL model. The preference shock is annualized percentage deviation from the steady state ($\beta^{SS} = \bar{\beta} = 0.993$).

relative price dispersion is zero. The current relative price dispersion is about 2.5%, which is 1.0% lower than the initial value. The output loss is about 1.0%, due to high dispersion of relative prices. The economy experiences deflation.

Intuitively, when the initial relative price dispersion is large, the inefficiency wedge is high, and the central bank would implement highly contractionary monetary policy by pursuing higher nominal interest rates on average than it would otherwise. By doing so, the central bank can lower the front-loading price setting behavior of firms and, as a result, lower the current relative price dispersion. In this case, the monetary

policy is so contractionary that it creates output losses and disinflation (or deflation in this case). Interestingly, due to high nominal interest rates on average, the ZLB is less likely to bind. For example, in the case of 3.5% initial relative price dispersion, the ZLB binds only when a shock with a magnitude of at least 2.5% occurs, compared with 2% in the zero initial relative price dispersion.

4.3.2. The case of a distorted steady state

As explained in the calibration section, in this case, the overall economic distortion (Φ) in this economy is 0.20. This means that at the steady state, the economy produces much less than the efficient output level. With this overall economic distortion, the central bank no longer targets zero inflation. The deterministic steady state inflation is about 2.4% that is associated with the steady state interest rate and price dispersion of 5.2% and 1.2%, respectively. Figure 5 shows the optimal policy where the relative price dispersion is set at the steady state of 1.0029 (or 1.2% annually).

The solid blue lines show the policy function in the FNL model. We can easily see that on average the central bank pursues higher inflation and nominal interest rates in the FNL model than in the LQ model. Without a particularly positive preference shock, the central bank implements the inflation rate and interest rate of around 2.4% and 5.2% respectively. The higher the average inflation and interest rates, the less likely the ZLB will bind.

In the first case of zero overall economic distortion, the steady state inflation and interest rates are 0 and 2.8% respectively. A positive preference shock with a magnitude of 1.2 standard deviations (or 2% per year), which reduces the natural real rate by 2% annually, can cause the ZLB to bind in the first case. However, in this case of a distorted steady state, with 2.4% inflation target and 5.2% steady state interest rate, a much more severe shock is required to drive the economy to the ZLB

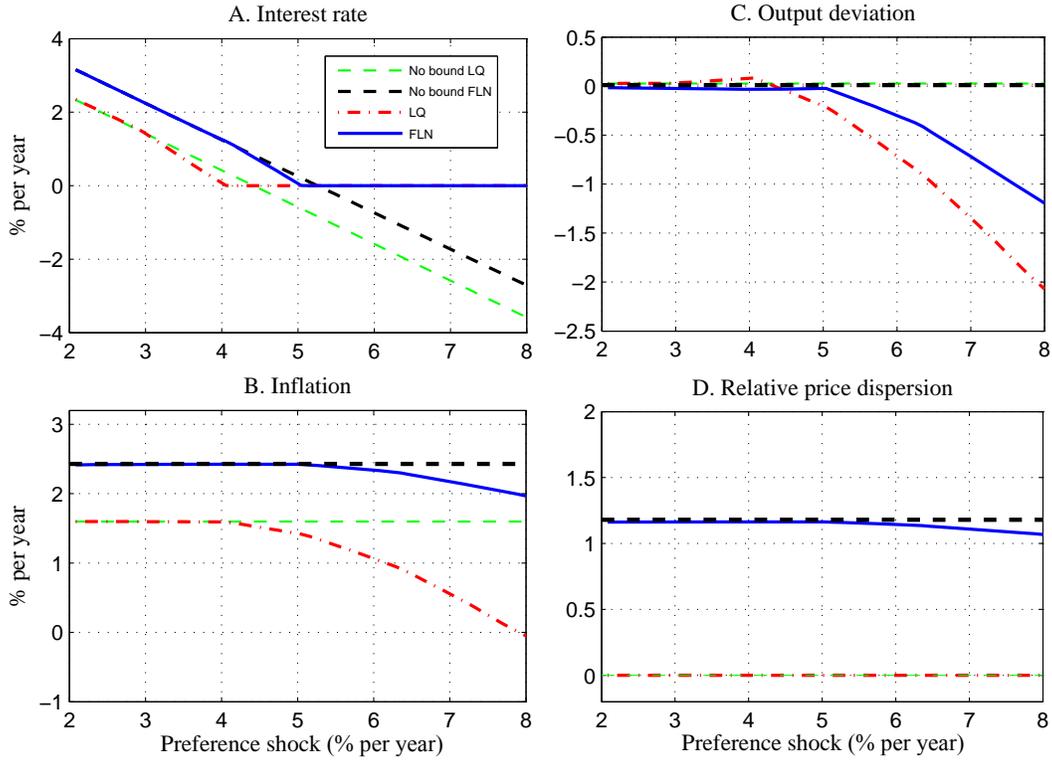


Figure 5: Optimal policy in the economy with a distorted steady state ($\Phi = 0.20$). Note that the initial relative price dispersion at the deterministic steady state ($\Delta_{t-1} = \Delta^{SS} = 1.0029$, or 1.2% per year). The preference shock is annualized percentage deviation from the steady state ($\beta^{SS} = \bar{\beta} = 0.993$).

- about 3 standard deviations (or reducing the natural real rate by 5% annually).

More importantly, when the economy is near the ZLB, the central bank in the economy with zero overall economic distortion cuts interest rate more aggressively than in the economy with positive distortion. The intuition is that, a large overall economic distortion in this paper incentivizes the central bank to inflate the economy and, as a result, generate higher inflation and interest rates on average.

When the economy is near the ZLB, a particularly adverse preference shock might have occurred. Because the preference process is mean-reverting, it is rather unlikely

that another adverse shock will happen and push the economy into the liquidity trap with large output losses and low inflation. As a result, the downward pressure on the conditional expected inflation is very small, generating small pressure on further lowering the nominal interest rate. Therefore, an interest rate cut is not as big as it would be in an economy with a smaller overall economic distortion.

The logic also helps explain why inflationary biases are reduced substantially in the case of a sufficiently-distorted steady state. Nakov (2008) studies the case of a non-distorted steady state, and argues that, under discretionary optimization, the presence of the ZLB biases private sector expectations of inflation and the output gap downwards, resulting in average inflation below the target. Depending on the specific parameterization this negative bias can be quite large. However, as explained above, the downward pressure on both conditional and unconditional expected inflation is significantly smaller in this case of a distorted steady state, reducing the inflationary bias substantially.

The literature concerning the case of a non-distorted steady state, including Adam and Billi (2007), indicates that even when the economy escapes from a liquidity trap (or when the output gap is positive), the central bank still keeps the nominal interest rate at zero for some time until the risk of falling back to the trap is not considerable. Specifically, from Panels A and B of Figure 3, when the preference is between 2% and 2.8% higher than the steady state, the nominal interest rate is kept at zero even though the economy is not at the liquidity trap. However, with the positive overall economic distortion, the central bank is less likely to keep the nominal interest rate at zero when the economy escapes from the liquidity trap, as seen in Panels A and C of Figure 5.

Unlike the case of a non-distorted steady state, in this case the ZLB and liquidity trap are not necessarily associated with deflation because of higher expected inflation.

Even if a particularly adverse demand shock occurs and pushes the economy into the liquidity trap with output loss and binding ZLB, downward pressure on the price level may not be big enough to offset the high expected inflation. Hence, the actual inflation is positive. The results also shed light on the recent discussion about the "missing disinflation", e.g. Coibion et al. (2012).

For example, at the preference of 6% higher than the steady state value (or the natural real rate is 6% lower than its steady state rate), output loss is 0.3% that is associated with 2.3% inflation rate. The results with positive overall economic distortion are consistent with what we have observed since the last recession where the federal funds rate (FFR) is technically zero and inflation is moderately positive.

The policy function from the LQ framework is presented by the dash-dotted red lines in Figure 5, although they are less accurate compared with the true policy in the FNL model. Specifically, in the LQ model, a shock with a magnitude of 2.3 standard deviations (or 4% per year) is required to drive the economy to the ZLB, while it requires a shock with a magnitude of at least 3 standard deviations (or 5.2% per year) to make the ZLB binding in the FNL model. In addition, given that the ZLB is binding in both models, the output loss and inflation decline are much larger in the LQ model. Specifically, when the time discount factor is 8% higher than its steady state value (or the natural real rate is 8% smaller than its steady state), the bounds are binding in both models.¹⁴ The output falls by 2.2%, associated with an inflation rate decline of 1.7% in the LQ model, compared to 1.25% and 0.4% in the FNL model.

The central bank in the LQ approach pursues lower inflation and nominal interest rates on average because of the inaccuracy of the LQ approach. Intuitively, the

¹⁴The shock is slightly above three standard deviations.

inaccuracy of the LQ model comes from the fact that the LQ method eliminates an endogenous state variable called relative price dispersion. As shown in Panel D of Figure 5, in the LQ framework, the relative price dispersion is always 0% annually, while it is 1.2% in the FNL model. We know that higher relative price dispersion is associated with higher inflation, as in equation (21). Thus, an adverse shock that causes the ZLB to bind in the FNL model must generate more slackness in the LQ model. As a result, the output loss and decline in inflation are greater in the LQ model than in the FNL model.

Figure 5 also shows that when the economy is near the ZLB, the interest rate is cut more aggressively in the LQ model than in the FNL model. The intuition is the same as above. With a larger overall economic distortion, inflation and nominal interest rates in the LQ model are smaller than those in the true model, which is the FNL model. Therefore, the ZLB in the LQ model is more likely to bind and the economy is more likely to be pushed in the liquidity trap with lower inflation. When the economy is near the ZLB, downward pressure on the conditional expected inflation in the LQ model is larger than in the FNL model, resulting in a more aggressive cut in interest rates.

To provide a more detailed comparison of the FNL and the LQ model, it is useful to answer the question: to which extent are the equilibrium responses in the case of a distorted steady state really driven by the presence of the ZLB? From Figure 5, the central bank can obtain the inflation and output targets using both FNL and LQ models in the absence of the ZLB. There are only demand shocks (or preference shocks) in the model, so there is no trade-off between output and inflation stabilization. However, the targets based on the LQ method are inaccurate because this method does not capture the full nonlinearity of the model. Specifically from Panel B of Figure 5, the equilibrium inflation rate is always 2.4% per year in the FNL

model and 1.6% in the LQ model. The difference of 0.8% between the two models is solely due to the nonlinearity.

In the presence of the ZLB, the equilibrium responses in the FNL and LQ model are different due to two factors. The first factor is the nonlinearity effect as explained above. The second factor is the presence of the ZLB. To see the second effect, let us examine Panel B of Figure 5 again. Under a shock that increases the preference by 6% per year and the ZLB binds, the equilibrium inflation is about 2.3% in the FNL model and 1% in the LQ model. The total difference is about 1.3%. Because the nonlinearity accounts for approximately 0.8%, the ZLB presence accounts for approximately 0.5%.

To extend the results from Yun (2005), I investigate the role of initial relative price dispersion and report the results in Figure 6. When the initial dispersion is greater than the steady state value of 1.2%, both output gap and inflation fall further than they would if the initial dispersion were kept at the steady state value. The opposite results occur if the initial dispersion is smaller than the steady state value. Again, this happens because the relative price dispersion is positively correlated with the initial dispersion as we see in Figure 1.

In addition, the relative price dispersion plays the role of endogenous technology in the aggregate production function. The higher the relative price dispersion, the lower the technology and the lower the output. Therefore, the additional output loss (gain) depends on whether the initial relative price dispersion is greater (smaller) than its steady state value. From Figure 5 we also see that the greater the initial relative price dispersion, the higher the nominal interest rate. As a result, the ZLB is less likely to bind.

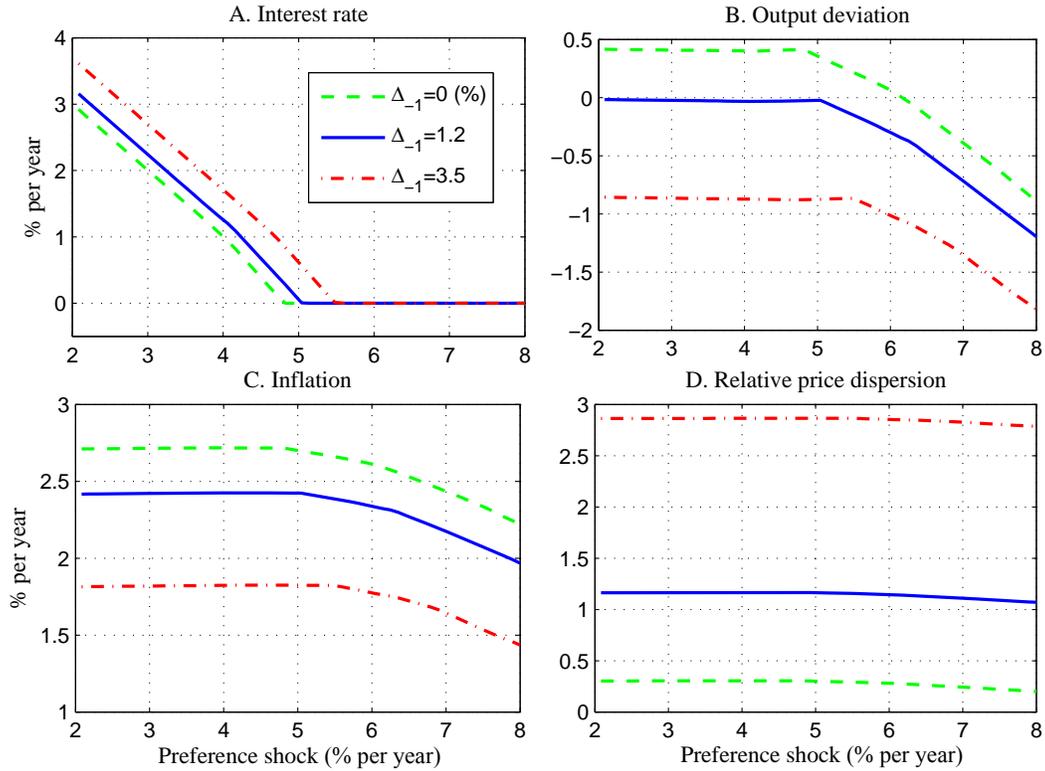


Figure 6: Optimal policy in the economy with a distorted steady state ($\Phi = 0.2$), with different values of initial relative price dispersion, in the FNL model. The preference shock is annualized percentage deviation from the steady state ($\beta^{SS} = \bar{\beta} = 0.993$).

4.4. What is the optimal inflation rate?

Since the late 1990s when Japan fell into the liquidity trap with binding ZLB, economists, such as Krugman (1998), have debated whether central banks should target a significantly positive inflation target and what the optimal inflation rate is. These topics are even more important today as the U.S. federal funds rate has been at zero since December 2008 and the U.S. economy is experiencing its greatest slump since the Great Depression. Blanchard et al. (2010) suggest that policymakers might consider an optimal inflation target of around 4%. The suggestion lies under

the argument that, in the presence of the ZLB, significantly positive inflation creates leeway for the central bank to deal with a particularly adverse demand shock that would drive the economy into the liquidity trap with a binding ZLB.

However, positive inflation comes at a price. Higher inflation is always associated with more front-loading behavior of firms when they have a chance to reset their prices. As a result, it is associated with higher relative price dispersion and lower output. This occurs because if the firms know that inflation is high and they cannot adjust their prices flexibly in the future, they will set higher prices today, causing higher relative price dispersion.

In this section, I am going to use the FNL model to answer a very important policy question - what is the optimal inflation target the social planner should pursue by setting the size of overall economic distortion or tax rate accordingly, as in equation (16)? For example, the social planner can choose a 2% inflation target by setting overall economic distortion of 0.18, or 8.89% income tax. It is reasonable to think of the social planner having two separate decision-making bodies. One is the Treasury Department that conducts tax policy, and the other is the central bank that conducts monetary policy. The social planner is assumed to be able to choose and commit to a labor income tax policy knowing that he will implement optional discretionary monetary policy later.

The social planner's problem boils down to comparing the social welfare for each tax/subsidy policy under discretionary monetary policy, choosing the best tax policy once, then committing to the policy. The setup is very similar to the case of discretionary policy without model misspecification as in Billi (2011), where the social planner chooses the inflation goal once and for all at time $t=0$. Absent the ZLB, the social planner should choose a zero inflation goal and, as a result, zero optimal long-run inflation. However, in the presence of the ZLB, the social planner should

choose a positive inflation target as a guard against the incidence of a binding ZLB.

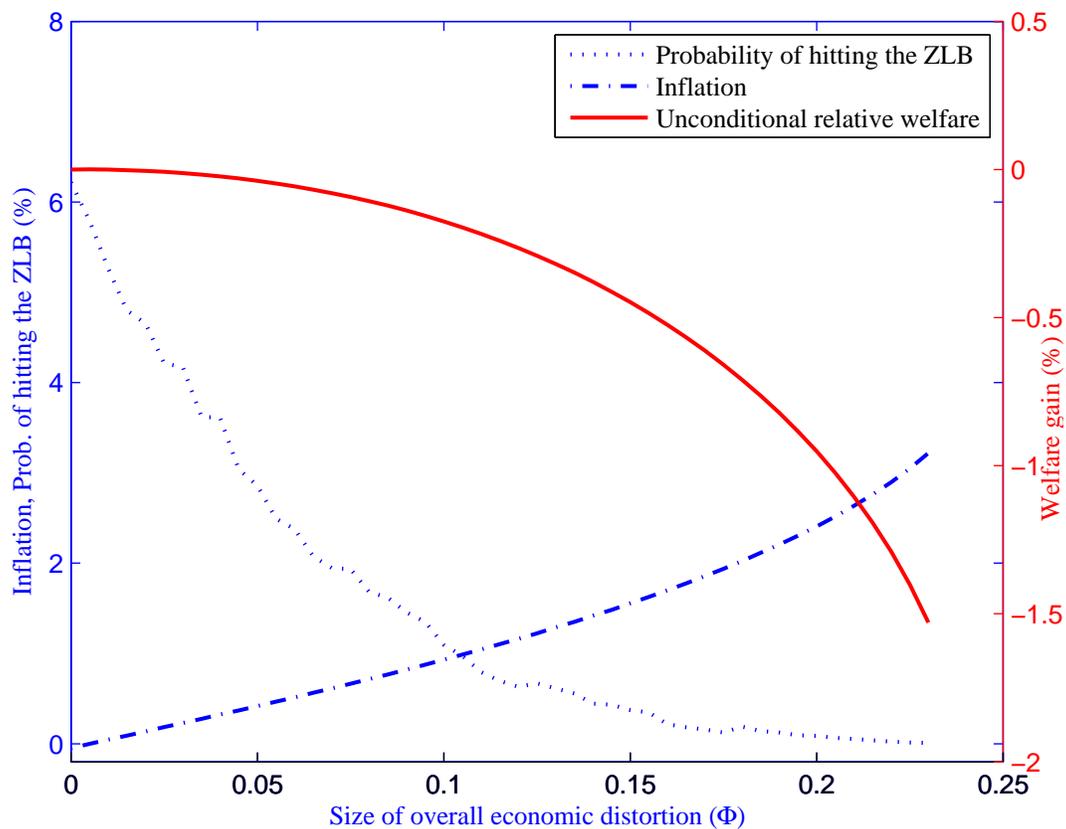


Figure 7: Unconditional welfare gain relative to the case of non-distorted steady state. The probability of hitting the ZLB is around 6% in the non-distorted case.

To find out the optimal inflation rate, I first solve the FNL model with respect to different values of the overall economic distortion. Then, based on the optimal policy, I compute the corresponding long-run inflation, which is the average inflation from a simulation of 300,000 periods. I also compute the simulated probability of hitting the ZLB using these 300,000 periods. As shown in Figure 7 with the y-axis on the left, the higher the overall economic distortion, the higher the long-run inflation, and

the lower the probability of hitting the ZLB. For example, if the overall economic distortion is zero, or the income tax rate is -11% , the average long-run inflation is around -0.02% and the probability of hitting the ZLB is around 6% .

The solid red line in Figure 7, with the y-axis on the right, presents the unconditional welfare relative to the one associated with the non-distorted steady state, as the function of overall economic distortion. To compute this unconditional relative welfare, for each value of overall economic distortion, I first solve for the value function as the function of the initial relative price distortion and the preference shock. Then, I take a random sample of 300,000 preference shocks, and I compute the average welfare, given the initial relative price distortion at the steady state. Eventually, I compute the unconditional welfare gain as a percentage change from the one associated with the non-distorted steady state.

It is surprising that the unconditional welfare is decreasing in the size of the overall distortion. In other words, the unconditional expected welfare is maximal when the overall economic distortion is around zero. This happens even when the average long-run inflation is approximately zero and the probability of hitting the ZLB is the greatest. Therefore, the optimal inflation rate is approximately zero.

The intuition is that : although the benefit of a significantly positive inflation target relative to the non-distorted case is large when a particularly adverse demand shock occurs, as shown by the red lines in Figure 8, such a shock is rather unlikely to occur. Without such a shock, the economy has to incur welfare loss associated with positive inflation every period. In sum, the unconditional welfare declines in inflation and overall economic distortion. For example, with the overall economic distortion of 0.02 or the average long-run inflation of 0.25% per year, the welfare is smaller than the non-distorted welfare by about 0.013 in almost all states of the preference shock, as presented by the thick solid red line in Figure 8. Only with a shock of at

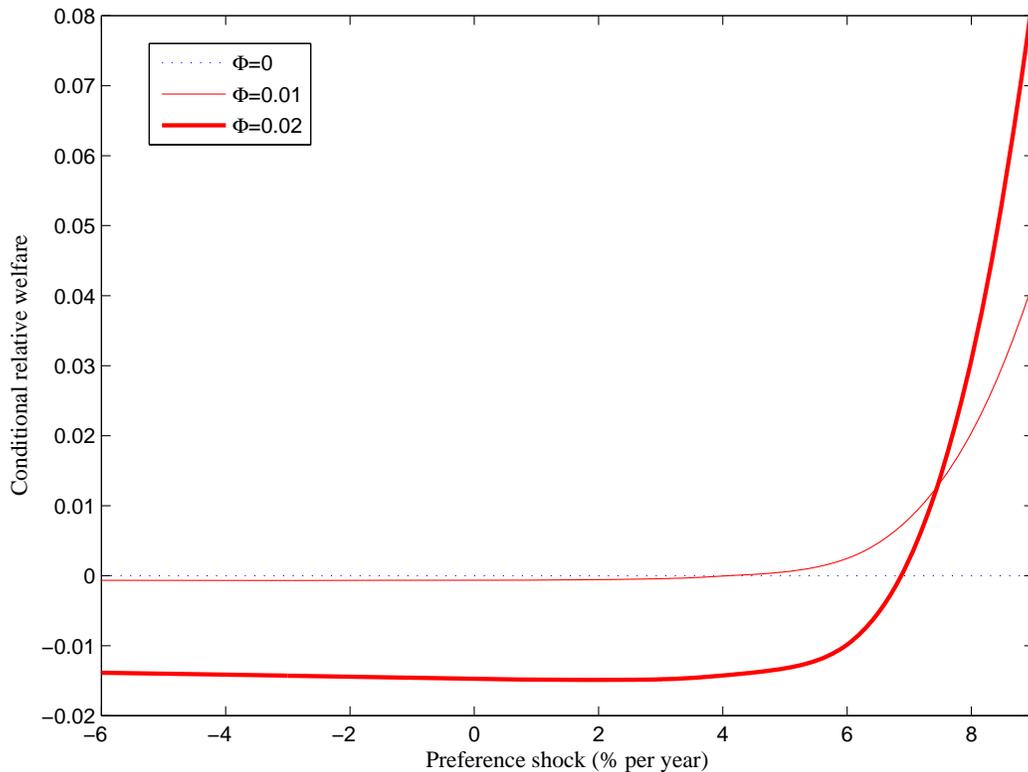


Figure 8: Conditional welfare relative to the case of non-distorted steady state. The probability of hitting the ZLB is around 6% in the non-distorted case.

least 6.7% (or 4 standard deviations), the welfare gain relative to the non-distorted case is positive. Therefore, on average, the unconditional welfare corresponding to the long-run inflation of 0.25% per year is smaller than the one with the inflation rate of -0.02% .

4.5. Sensitivity analysis

For a robustness check, in this section, I raise the variance of the preference shock such that the unconditional probability of hitting the ZLB increases from 6% to around 9.5% in the economy with a non-distorted steady state. In this case,

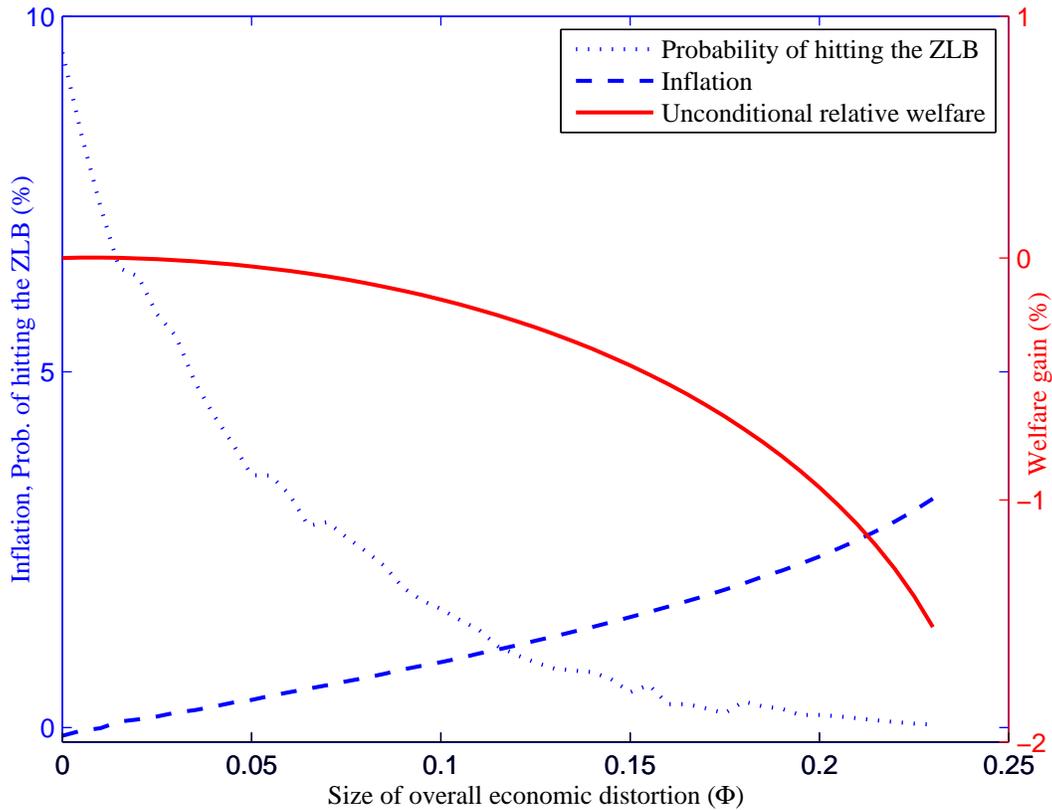


Figure 9: Unconditional welfare gain relative to the case of non-distorted steady state. The probability of hitting the ZLB is 9.5% in the non-distorted case.

although the optimal inflation target is no longer zero, it is not significantly greater than zero. Figure 9 shows that the unconditional welfare increases in inflation when inflation is small, and, after a point, it decreases in inflation. The welfare is maximal when the overall economic distortion is set around 0.01, corresponding to the average long-run inflation of 0.02% per year.

Figure 10 shows that, when the size of the overall economic distortion is zero or the long-run inflation is around -0.07% per year, a small increase in the size of

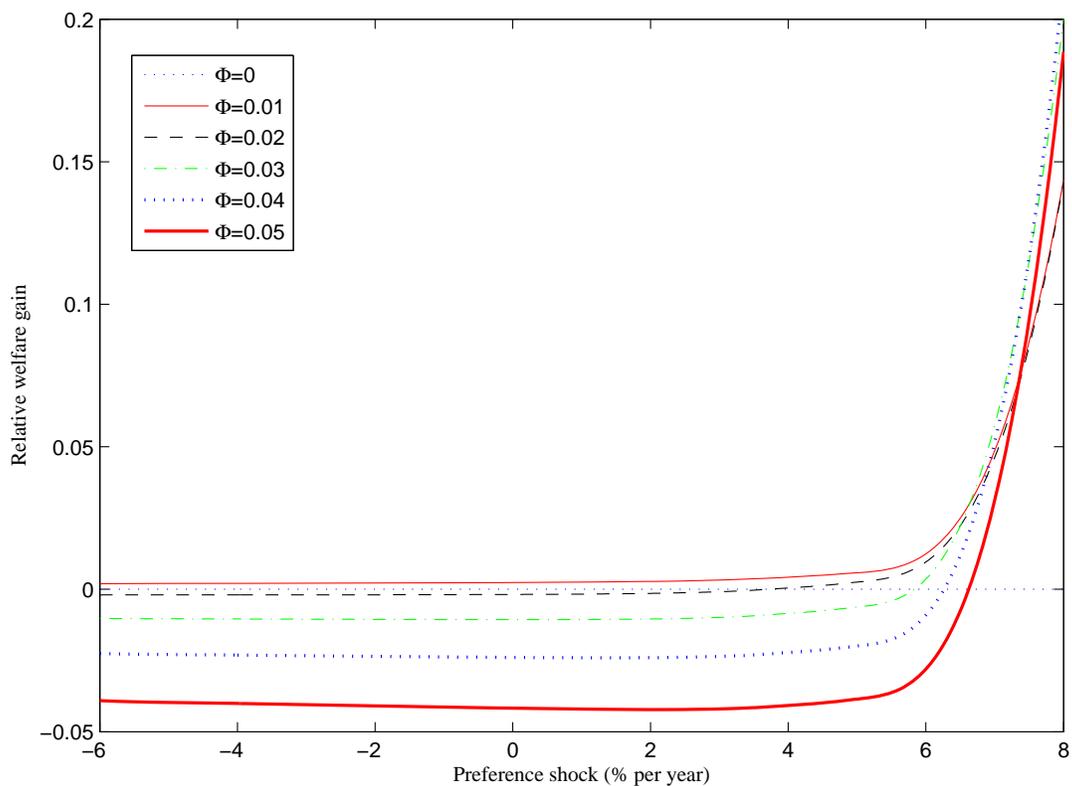


Figure 10: Conditional welfare relative to the case of non-distorted steady state. The probability of hitting the ZLB is 9.5% in the non-distorted case.

distortion will result in welfare gain relative to the case of no distortion given a particularly adverse shock, while welfare loss is not significant without such a shock. On average, the unconditional welfare is larger in the case of small positive distortion compared to the case of no distortion. However, when the size of the overall economic distortion is large, the cost of inflation increases substantially. Although welfare gain is relatively high under a particularly adverse shock, it is much lower when the economy is in normal times. On average, the unconditional welfare is smaller relative to the non-distorted case.

Billi (2011) and Coibion et al. (2012) both find strictly positive optimal long-run inflation rates because they include some factors in their models that cause larger welfare losses when the ZLB binds. My model is just the canonical New Keynesian, and we can view the exercise here as a complement to their work.¹⁵

Specifically, Billi (2011) allows inflation indexation that results in very persistent inflation in the Phillips curve. Due to this characteristic, under a particularly adverse shock that causes the ZLB to bind, his model might generate very persistent deflationary episodes associated with binding ZLB and output losses. Consequently, a binding ZLB would be more damaging in his model than in my model. Hence, the central bank is very “afraid” of the ZLB, and, as a result, pursues much higher optimal inflation: about 13.4%. In his framework, the probability of hitting the ZLB is approximately zero.

The optimal inflation in the benchmark model of Coibion et al. (2012) is around 1.5% per year and greater than the one in this paper. The reason is that, in this paper, the central bank conducts optimal discretionary monetary policy instead of a simple Taylor rule as in the benchmark model of Coibion et al. (2012). We know from the existing literature, i.e. Nakov (2008), that when the ZLB binds, welfare losses are smaller under an optimal discretionary monetary policy than under a simple Taylor rule. So the desire for higher inflation is greater in Coibion et al. (2012).

More importantly, Coibion et al. (2012) allow habit formation. The habit formation makes the central bank very “afraid” of unstable output and inflation, especially when the ZLB binds. In addition, the shock used to cause a binding ZLB in Coibion et al. (2012) is more persistent than in my paper. Therefore, the social planner in

¹⁵Adding more realistic factors in the model and studying the optimal long-run inflation based on the FNL model is the author’s future research agenda.

their model is willing to pursue a higher inflation target and, as a result, a greater optimal long-run inflation rate.

5. Conclusion

In this paper, I study optimal monetary policy under discretion in the presence of an occasionally binding ZLB. My contribution to the existing literature of the ZLB is twofold. First, in contrast to the existing literature assuming an efficient steady state, I allow for the case of a distorted steady state due to monopoly and taxation, and study the impact of this overall distortion on optimal discretionary monetary policy in the presence of the ZLB. Second, unlike the existing literature that uses the LQ approach, I solve FNL models. I also provide a comparison between the FNL and LQ models.

Using the FNL method, I find some implications for optimal discretionary monetary policy. First, under a particularly adverse shock driving the economy near the ZLB, the central bank in an economy with a larger economic distortion would cut the interest rate less aggressively. Second, the social planner should choose approximately zero inflation target even when the unconditional probability of hitting the ZLB is as high as 9.5%.

I show that the FNL and LQ models produce very different optimal policy in the case of a highly distorted steady state. In this case, simplifying the FNL model using the LQ approach will result in two main inaccuracies. First, the probability of hitting the ZLB is higher in the LQ approach than in the FNL method. Second, with an inaccurately higher probability of hitting the ZLB, the central bank cuts the nominal interest rate more aggressively in the LQ model than it would otherwise do in the FNL model, given a particularly adverse demand shock that drives the economy close to the ZLB.

There are different directions to extending my paper and to investigating the ZLB as a friction that transmits adverse shocks and amplifies macroeconomic fluctuations. First, we can use the FNL method in this paper to find out if commitment policy is still better than discretion policy when we allow for a large economic distortion. Assuming zero overall economic distortion and comparing optimal policy under discretion and commitment, as done in the literature, is misleading because the role of inflation is underestimated. Second, we can apply the FNL method to examine the interaction between the ZLB and other frictions as a transmission mechanism that magnifies macroeconomic fluctuations. Ngo (2013) studies the interaction among overborrowing, incomplete financial markets and the ZLB in generating a great recession.

6. Acknowledgements

I am grateful to two anonymous referees for their very helpful comments. This paper is a substantial revision of the first chapter in my doctoral dissertation at Boston University, which was completed when I was a graduate research fellow at the Federal Reserve Bank of Boston. I am grateful to my advisors Jianjun Miao, Bob King and François Gourio for their advice and encouragement. I also thank Simon Gilchrist, Alisdair McKay, Leena Rudanko, J.Christina Wang, Luigi Paciello, David Seymour, Mike Gechter, and participants at the BU Macro Dissertation Workshop, Green Line Macro Meeting, Federal Reserve Bank of Boston Seminar, Midwest Macro Meeting, and Econometric Society North American Summer Meeting for their comments and suggestions. The views in this paper are those of the author. They do not necessarily represent the views of the Federal Reserve Bank of Boston or the views of the Federal Reserve System.

7. References

- Adam, K., Billi, M., 2006. Optimal monetary policy under commitment with a zero bound on nominal interest rates. *Journal of Money, Credit, and Banking* 38(3), 1877–1905.
- Adam, K., Billi, M., 2007. Discretionary monetary policy and the zero lower bound on nominal interest rates. *Journal of Monetary Economics* 54, 728–752.
- Alvarez, F., Gonzalez-Rozada, M., Neumeyer, A., Beraja, M., 2011. From hyper inflation to stable prices: Argentina’s evidence on menu cost models. Working Paper.
- Amano, R., Shukayev, M., 2012. Risk premium shocks and the zero bound on nominal interest rates. *Journal of Money, Credit and Banking* 44 (8).
- Anderson, G., Kim, J., Yun, T., 2010. Using a projection method to analyze inflation bias in a micro-founded model. *Journal of Economic Dynamics and Control* 34, 1572 – 1581.
- Billi, R., 2011. Optimal inflation for the us economy. *American Economic Journal: Macroeconomics* 3, 29–52.
- Blanchard, O., Dell’Ariccia, G., Mauro, P., 2010. Rethinking macroeconomic policy. IMF Staff Position Note.
- Bodenstein, M., Hebden, J., Nunes, R., 2010. Imperfect credibility and the zero lower bound on the nominal interest rate. Federal Reserve Board Working Paper.
- Calvo, A., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 383–398.

- Coibion, O., Gorodnichenko, Y., Wieland, J., 2012. The optimal inflation rate in new keynesian models: Should central banks raise their inflation targets in light of the zero lower bound? *Review of Economics Studies* (forthcoming).
- Curdia, V., Woodford, M., 2009. Credit frictions and optimal monetary policy. Federal Reserve Bank of New York Working Paper.
- Diewert, E., Fox, K., 2008. On the estimation of return to scale, technical progress and monopolistic markups. *Journal of Econometrics* 145, 174–193.
- Eggertsson, G., Krugman, P., 2010. Debt, deleveraging and the liquidity trap: A fisher - minsky - koo approach. Working Paper (QJE 2012 forthcoming).
- Eggertsson, G., Woodford, M., 2003. The zero bound on interest rates and optimal monetary policy. NBER Working Paper.
- Fernandez-Villaverde, J., Gordon, G., Guerron-Quintana, P., Rubio-Ramirez, F. J., 2012. Nonlinear adventures at the zero lower bound. NBER Working Paper.
- Guerrieri, V., Lorenzoni, G., 2011. Credit crises, precautionary savings and the liquidity trap. Working Paper.
- Hall, R., 2011. The long slump. *American Economic Review* 101, 431–469.
- Judd, L. K., Maliar, L., Maliar, S., 2011. A cluster-grid algorithm: Solving problems with high dimensionality. Working Paper.
- Jung, T., Teranishi, Y., Watanabe, T., 2005. Optimal monetary policy at the zero-interest-rate bound. *Journal of Money, Credit and Banking* 37, 813–835.
- Krugman, P., 1998. It's baaack: Japan's slump and the return of the liquidity trap. *Brookings Papers on Economic Activity* 2, 137–205.

- Levin, A., Lopez-Salido, D., Nelson, E., Yun, T., 2010. Limitations on the effectiveness of forward guidance at the zero lower bound. *International Journal of Central Banking* 6 (1), 143–189.
- McGrattan, E., 1994. The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics* 33, 573–601.
- Miranda, M. J., Fackler, L. P., 2002. *Applied Computational Economics and Finance*. MIT Press.
- Nakata, T., 2011. Optimal fiscal and monetary policy with occasionally binding zero lower bound. *New York University Working Paper*.
- Nakov, A., 2008. Optimal and simple monetary policy rules with zero floor on the nominal interest rate. *International Journal of Central Banking* 4 (2), 73–127.
- Ngo, P., 2013. Overborrowing, deleveraging and a great recession. *Mimeo Boston University*.
- Rotemberg, J., Woodford, M., 1997. An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. *NBER Macroeconomics Annual*.
- Schmitt-Grohe, S., Uribe, M., 2010. The optimal rate of inflation. *NBER Working Paper No.16054*.
- Werning, I., 2011. Managing a liquidity trap: Fiscal and monetary policy. *MIT Working Paper*.
- Woodford, M., 2001. Inflation stabilization and welfare. *NBER Working Paper No.8071*.

Woodford, M., 2003. Interest and Prices. Princeton University Press.

Yun, T., 1996. Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics* 37, 345–70.

Yun, T., 2005. Optimal monetary policy with relative price distortions. *American Economics Review* 95, 89–108.

Zandweghe, V., Wolman, A., 2010. Discretionary monetary policy in the calvo model. FRB Kansas Working Paper.

Appendix A. Discretionary optimal policy under ZLB

$$V(\Delta_{t-1}, \beta_t) = \max_{\{i_t, C_t, N_t, S_t, F_t, \pi_t, \Delta_t\}} \left\{ \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + \beta_t E_t V(\Delta_t, \beta_{t+1}) \right\}$$

Subject to

$$\frac{C_t^{-\gamma}}{(1+i_t)} = \beta_t E_t \left[\frac{C_{t+1}^{-\gamma}}{\Pi_{t+1}} \right] = \beta_t E_t [Z_1(\Delta_t, \beta_{t+1})]$$

$$F_t - C_t^{-\gamma+1} = \theta \beta_t E_t [\Pi_{t+1}^{\varepsilon-1} F_{t+1}] = \theta \beta_t E_t [Z_2(\Delta_t, \beta_{t+1})]$$

$$S_t - \frac{\chi C_t N_t^\eta}{(1-\Phi)} = \theta \beta_t E_t [\Pi_{t+1}^\varepsilon S_{t+1}] = \theta \beta_t E_t [Z_3(\Delta_t, \beta_{t+1})]$$

$$\Delta_t = (1-\theta) \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi_t^\varepsilon \Delta_{t-1}$$

$$S_t = F_t \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}$$

$$C_t = N_t (\Delta_t)^{-1}$$

$$i_t \geq 0$$

I rewrite the problem in the Lagrangian form as follow:

$$\begin{aligned}
L = & \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + \beta_t E_t V(\Delta_t, \beta_{t+1}) \\
& - \lambda_{1t} \left[\frac{C_t^{-\gamma}}{(1+i_t)} - \beta_t E_t [Z_1(\Delta_t, \beta_{t+1})] \right] \\
& - \lambda_{2t} [F_t - C_t^{-\gamma+1} - \theta \beta_t E_t [Z_2(\Delta_t, \beta_{t+1})]] \\
& - \lambda_{3t} \left[S_t - \frac{\chi C_t N_t^\eta}{(1-\Phi)} - \theta \beta_t E_t [Z_3(\Delta_t, \beta_{t+1})] \right] \\
& - \lambda_{4t} \left[\Delta_t - (1-\theta) \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \theta \Pi_t^\varepsilon \Delta_{t-1} \right] \\
& - \lambda_{5t} \left[S_t - F_t \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}} \right] \\
& - \lambda_{6t} [C_t - N_t (\Delta_t)^{-1}]
\end{aligned}$$

The first order conditions:

$$i_t : \lambda_{1t} C_t^{-\gamma} (1+i_t)^{-2} \leq 0 \text{ with equality if } (1+i_t) > 1$$

$$C_t : 0 = C_t^{-\gamma} + \lambda_{1t} \gamma C_t^{-\gamma-1} / (1+i_t) + (1-\gamma) \lambda_{2t} C_t^{-\gamma} + \lambda_{3t} \frac{\chi N_t^\eta}{(1-\Phi)} - \lambda_{6t}$$

$$N_t : 0 = -\chi N_t^\eta + \lambda_{3t} \frac{\chi \eta C_t N_t^{\eta-1}}{(1-\Phi)} + \lambda_{6t} \Delta_t^{-1},$$

$$F_t : 0 = -\lambda_{2t} + \lambda_{5t} \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}$$

$$S_t : 0 = -\lambda_{3t} - \lambda_{5t}$$

$$\begin{aligned}
\pi_t : 0 &= -\lambda_{4t}\varepsilon \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{\varepsilon-1}} \theta\Pi_t^{\varepsilon-2} + \lambda_{4t}\varepsilon\theta\Pi_t^{\varepsilon-1}\Delta_{t-1} \\
&+ \lambda_{5t}F_t \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{-\varepsilon}{\varepsilon-1}} \frac{\theta\Pi_t^{\varepsilon-2}}{1 - \theta} \\
\Delta_t : 0 &= \beta_t E_t V_\Delta (\Delta_t, \beta_{t+1}) + \lambda_{1t}\beta_t E_t [Z_{1\Delta} (\Delta_t, \beta_{t+1})] \\
&+ \lambda_{2t}\theta\beta_t E_t [Z_{2\Delta} (\Delta_t, \beta_{t+1})] + \lambda_{3t}\theta\beta_t E_t [Z_{3\Delta} (\Delta_t, \beta_{t+1})] \\
&- \lambda_{4t} - \lambda_{6t}N_t\Delta_t^{-2}
\end{aligned}$$

and the envelope theorem:

$$V_\Delta(\Delta_{t-1}, \beta_t) = \lambda_{4t}\theta\Pi_t^\varepsilon$$

where $Z_{i\Delta}$ denotes partial derivative of Z_i with respect to Δ . Simplifying and combining with equilibrium conditions, we obtain 13 equations with 13 variables $(R, C, N, S, F, \Pi, \Delta, \lambda_1, \dots, \lambda_6)'$

$$(1) \quad i_t : 0 = \max(\lambda_{1t}C_t^{-\gamma} (1 + i_t)^{-2}, i_t)$$

$$(2) \quad C_t : 0 = C_t^{-\gamma} + \lambda_{1t}\gamma C_t^{-\gamma-1}/(1 + i_t) + (1 - \gamma)\lambda_{2t}C_t^{-\gamma} + \lambda_{3t}\frac{\chi N_t^\eta}{(1 - \Phi)} - \lambda_{6t}$$

$$(3) \quad N_t : 0 = -\chi N_t^\eta + \lambda_{3t}\frac{\chi\eta C_t N_t^{\eta-1}}{(1 - \Phi)} + \lambda_{6t}\Delta_t^{-1},$$

$$(4) \quad F_t : 0 = -\lambda_{2t} + \lambda_{5t} \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}}$$

$$(5) \quad S_t : 0 = -\lambda_{3t} - \lambda_{5t}$$

$$(6) \quad \pi_t : 0 = -\lambda_{4t}\varepsilon \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{\varepsilon-1}} + \lambda_{4t}\varepsilon\Pi_t\Delta_{t-1} \\ + \lambda_{5t}F_t \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{-\varepsilon}{\varepsilon-1}} \frac{1}{1 - \theta}$$

$$(7) \quad \Delta_t : 0 = \theta\beta_t E_t [\lambda_{4t+1}\Pi_{t+1}^\varepsilon] + \lambda_{1t}\beta_t E_t [Z_{1\Delta}(\Delta_t, \beta_{t+1})] \\ + \lambda_{2t}\theta\beta_t E_t [Z_{2\Delta}(\Delta_t, \beta_{t+1})] + \lambda_{3t}\theta\beta_t E_t [Z_{3\Delta}(\Delta_t, \beta_{t+1})] \\ - \lambda_{4t} - \lambda_{6t}N_t\Delta_t^{-2}$$

$$(8) \quad \lambda_{1t} : 0 = \frac{C_t^{-\gamma}}{(1 + i_t)} - \beta_t E_t [Z_1(\Delta_t, \beta_{t+1})]$$

$$(9) \quad \lambda_{2t} : 0 = F_t - C_t^{-\gamma+1} - \theta\beta_t E_t [Z_2(\Delta_t, \beta_{t+1})]$$

$$(10) \quad \lambda_{3t} : 0 = S_t - \frac{\chi C_t N_t^\eta}{(1 - \Phi)} - \theta\beta_t E_t [Z_3(\Delta_t, \beta_{t+1})]$$

$$(11) \quad \lambda_{4t} : 0 = \Delta_t - (1 - \theta) \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \theta\Pi_t^\varepsilon\Delta_{t-1}$$

$$(12) \quad \lambda_{5t} : 0 = S_t - F_t \left(\frac{1 - \theta\Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}}$$

$$(13) \quad \lambda_{6t} : 0 = C_t - N_t(\Delta_t)^{-1}$$

where

$$Z_1(\Delta_t, \beta_{t+1}) = \frac{C_{t+1}^{-\gamma}}{\Pi_{t+1}}$$

$$Z_2(\Delta_t, \beta_{t+1}) = \Pi_{t+1}^{\varepsilon-1} F_{t+1}$$

$$Z_3(\Delta_t, \beta_{t+1}) = \Pi_{t+1}^\varepsilon S_{t+1}$$

$$Z_4(\Delta_t, \beta_{t+1}) = \Pi_{t+1}^\varepsilon \lambda_{4t+1}$$

The solution of the above nonlinear system is the function of the state, $s_t = (\Delta_{t-1}, \beta_t)$, where Δ_{t-1} is the endogenous and β_t are the exogenous states with the following law of motion:

$$\Delta_t = (1 - \theta) \left(\frac{1 - \theta \Pi_t^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi_t^\varepsilon \Delta_{t-1}$$

$$\ln(\beta_{t+1}) = (1 - \rho_\beta) \ln(\bar{\beta}) + \rho_\beta \ln(\beta_t) + \varepsilon_{t+1}^\beta$$

Appendix B. Solution method

I rewrite the above 13 functional equations with 13 unknown policy functions in a more compact form:

$$f(s, X(s), E[Z(X(s'))], E[Z_\Delta(X(s'))]) = 0$$

Here $f : R^{2+13+4+4} \rightarrow R^{13}$ is the equilibrium relationship.

where

$s = (\Delta, \beta)$ is the current state of the economy

$X(s) = (R(s), C(s), N(s), F(s), S(s), \Pi(s), \Delta(s), \lambda_1(s), \dots, \lambda_6(s))'$ is the policy function we need to solve, $X : R^3 \rightarrow R^{13}$.

s' is next period state that evolves according to the following motion equation:

$$s' = g(s, X(s), \varepsilon) = \begin{bmatrix} \Delta' = (1 - \theta) \left(\frac{1 - \theta \Pi(s)^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi(s)^\varepsilon \Delta \\ \beta' = \bar{\beta}^{(1-\rho_\beta)} \beta^{\rho_\beta} \exp(\varepsilon_\beta) \end{bmatrix}$$

$\varepsilon = [\varepsilon_\beta]$ is the innovations of the preference shock.

$$Z(X(s')) = \begin{pmatrix} Z_1(X(s')) = \frac{C(s')^{-\gamma}}{\Pi(s')} \\ Z_2(X(s')) = \Pi(s')^{\varepsilon-1} F(s') \\ Z_3(X(s')) = \Pi(s')^\varepsilon S(s') \\ Z_4(X(s')) = \Pi(s')^\varepsilon \lambda_4(s') \end{pmatrix}$$

$$Z_{\Delta}(X(s')) = \begin{pmatrix} Z_{1\Delta}(X(s')) = -\frac{\gamma C(s')^{-\gamma-1} C_{\Delta}(s')}{\Pi(s')} - \frac{C(s')^{-\gamma} \Pi_{\Delta}(s')}{\Pi(s')^2} \\ Z_{2\Delta}(X(s')) = (\varepsilon - 1) \Pi(s')^{\varepsilon-2} \Pi_{\Delta}(s') F(s') + \Pi(s')^{\varepsilon-1} F_{\Delta}(s') \\ Z_{3\Delta}(X(s')) = \varepsilon \Pi(s')^{\varepsilon-1} \Pi_{\Delta}(s') S(s') + \Pi(s')^{\varepsilon} S_{\Delta}(s') \\ Z_{4\Delta}(X(s')) = \varepsilon \Pi(s')^{\varepsilon-1} \Pi_{\Delta}(s') \lambda_4(s') + \Pi(s')^{\varepsilon} \lambda_{4\Delta}(s') \end{pmatrix}$$

I solve the above equilibrium relationship using a collocation method, where collocation nodes are non-equidistant in the sense that these nodes concentrate around potential kinks. First, I use equidistant nodes to solve the model using the below algorithm and find out policy function based on which I investigate potential kinds. I redistribute these nodes by clustering them around potential kinks, then resolve the model.

Below is the simplified algorithm of the collocation method:

Step 1: Define the space of the approximating functions and collocation nodes $S = (S_1, \dots, S_N)$, where $N = N_{\Delta} \times N_{\beta}$ and $(N_{\Delta} \times N_{\beta})$ is the polynomial degrees in each dimension of the space. In this paper, I use cubic spline method where $N_{\Delta} \times N_{\beta}$ are number of collocations nodes along each state dimension.

$$X(s) = (\phi(s)\theta_R, \phi(s)\theta_C, \phi(s)\theta_N, \phi(s)\theta_F, \phi(s)\theta_S, \phi(s)\theta_{\Pi}, \phi(s)\theta_{\Delta}, \phi(s)\theta_{\lambda_1}, \dots, \phi(s)\theta_{\lambda_6})';$$

or $X(s) = \phi(s)\Theta$

where

- $\phi(s)$ is a $1 \times N$ matrix of cubic spline basis functions evaluated at state $s \in S = (S_1, \dots, S_N)$.
- $\Theta = (\theta_R; \theta_C; \theta_N; \theta_F; \theta_S; \theta_{\Pi}; \theta_{\Delta}; \theta_{\lambda_1}; \dots; \theta_{\lambda_6})$ is $N \times 13$ coefficient matrix that we want to approximate.

Step 2: Initialize the coefficient matrix Θ^0 , and set up stopping rules.

Step 3: At each iteration j we have a corresponding Θ^j , implement the following substep:

1. At each collocation node s_i , $s_i \in \{S_1..S_N\}$: compute $E [Z(X (s'))]$, $E [Z_{\Delta}(X (s'))]$:

- $E [Z(X (s'))] = \sum_j w_j [Z(X (g (s, X(s'), e_j)))]$
- $E [Z_{\Delta}(X (s'))] = \sum_j w_j [Z_{\Delta}(X (g (s, X(s'), e_j)))]$

2. Solve for $X(s_i)$ s.t. $f (s_i, X(s_i), E [Z(X (s'))]$, $E [Z_{\Delta}(X (s'))]) = 0$, I solve this complementarity problem using the Newton method and the semi-smooth root-finding method laid in Miranda and Fackler (2003). I also provide an "analytical" Jacobian matrix f_X (see the formula below).¹⁶ The semi-smooth root-finding method and "analytical" Jacobian matrices help to significantly boost the speed of solving the problem and make the solver very reliable.

Step 4: Update $\Theta^{j+1} = \Phi^{-1}\Theta^j$, where $\Phi = (\phi(s_1), \dots, \phi(s_N))'$.

Step 5: Check the stopping rules. If not satisfied go to Step 3; otherwise go to Step 6.

Step 6: Report results.

In the paper, I obtain the maximum absolute error of 1e-4 across equilibrium conditions for some states, smaller than 1e-6 for almost all other states. By using more points, I can obtain more accurate results. There is another way to solve for the policy functions. I can define the residual function, $r(s, \Theta) = f(s, X (s), E [Z(X (s'))]$, $E [Z_{\Delta}(X (s'))])$ and use Newton's method to solve $r(s, \Theta) = 0$ by updating $\Theta^{j+1} = \Theta^j - \alpha [r_{\Theta} (s, \Theta^j)]^{-1} r(s, \Theta^j)$, where $r_{\Theta} (s, \Theta^j)$ is an "analytical" Jacobian matrix. I tried this method but it is extremely slow due to the inverse of a large coefficient

¹⁶"Analytical" because the first and second derivatives are computed using the approximating functions. Before solving the model, it is not obvious that the value function and the policy functions (for inflation, auxiliary variables F and S, consumption, and lambda 4 - the Lagrangian multiplier associated with the law of motion of the relative price dispersion) are differentiable with respect to the initial relative price dispersion. However, the numerical results show that these functions are smooth and differentiable with respect to the initial relative price dispersion.

matrix.

Appendix C. "Analytical" Jacobian matrix, f_X

$$\begin{aligned}
f_{1X} &= \begin{pmatrix} -2R_t^{-3}C_t^{-\gamma}\lambda_{1t}, R_t^{-2}(-\gamma)C_t^{-\gamma-1}\lambda_{1t}, 0, 0, 0, 0, 0, C_t^{-\gamma}R_t^{-2}, 0, 0, 0, 0, 0; \text{ if } \lambda_{1t}C_t^{-\gamma}R_t^{-2} > R_t - 1 \\ -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0; \text{ if } \lambda_{1t}C_t^{-\gamma}R_t^{-2} \leq R_t - 1 \end{pmatrix} \\
f_{2X} &= \begin{pmatrix} -\frac{\lambda_{1t}\gamma C_t^{-\gamma-1}}{(1+i_t)^2}, -\gamma C_t^{-\gamma-1} - \frac{\lambda_{1t}\gamma(\gamma+1)C_t^{-\gamma-2}}{(1+i_t)} - (1-\gamma)\gamma\lambda_{2t}C_t^{-\gamma-1}, \lambda_{3t}\frac{\chi\eta N_t^{\eta-1}}{(1-\Phi)}, \dots \\ 0, 0, 0, 0, \frac{\gamma C_t^{-\gamma-1}}{(1+i_t)}, (1-\gamma)C_t^{-\gamma}, \frac{\chi N_t^\eta}{(1-\Phi)}, 0, 0, -1; \end{pmatrix} \\
f_{3X} &= \left(0, \lambda_{3t}\frac{\chi\eta N_t^{\eta-1}}{(1-\Phi)}, -\chi\eta N_t^{\eta-1} + \lambda_{3t}\frac{\chi\eta(\eta-1)C_t N_t^{\eta-2}}{(1-\Phi)}, 0, 0, 0, -\frac{\lambda_{6t}}{\Delta_t^2}, 0, 0, \frac{\chi\eta C_t N_t^{\eta-1}}{(1-\Phi)}, 0, 0, \frac{1}{\Delta_t}; \right) \\
f_{4X} &= \left(0, 0, 0, 0, 0, \lambda_{5t} \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{-\varepsilon}{1-\varepsilon}} \frac{\theta\Pi_t^{\varepsilon-2}}{1-\theta}, 0, 0, -1, 0, 0, \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}, 0; \right) \\
f_{5X} &= (0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1, 0;) \\
f_{6X} &= \begin{pmatrix} 0, 0, 0, \lambda_{5t} \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{-\varepsilon}{\varepsilon-1}} \frac{1}{1-\theta}, 0, \lambda_{4t} \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{2-\varepsilon}{\varepsilon-1}} \frac{\varepsilon\theta\Pi_t^{\varepsilon-2}}{1-\theta} + \lambda_{4t}\varepsilon\Delta_{t-1} + \dots \\ \lambda_{5t}F_t \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{-2\varepsilon+1}{\varepsilon-1}} \frac{\varepsilon\theta\Pi_t^{\varepsilon-2}}{(1-\theta)^2}, 0, 0, 0, 0, -\varepsilon \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{\varepsilon-1}} + \varepsilon\Pi_t\Delta_{t-1}, \dots \\ F_t \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{-\varepsilon}{\varepsilon-1}} \frac{1}{1-\theta}, 0; \end{pmatrix} \\
f_{7X} &= \begin{pmatrix} 0, 0, -\frac{\lambda_{6t}}{\Delta_t^2}, 0, 0, 0, \dots \\ \theta\beta_t(E_t[Z_{4\Delta t+1}] + \lambda_{1t}E_t[Z_{1\Delta\Delta t+1}]/\theta + \lambda_{2t}E_t[Z_{2\Delta\Delta t+1}] + \lambda_{3t}E_t[Z_{3\Delta\Delta t+1}]) + \frac{2\lambda_{6t}N_t}{\Delta_t^3}, \\ \beta_t E_t[Z_{1\Delta t+1}], \theta\beta_t E_t[Z_{2\Delta t+1}], \theta\beta_t E_t[Z_{3\Delta t+1}], -1, 0, -\frac{N_t}{\Delta_t^2}; \end{pmatrix} \\
f_{8X} &= \left(-\frac{C_t^{-\gamma}}{(1+i_t)^2}, \frac{-\gamma C_t^{-\gamma-1}}{(1+i_t)}, 0, 0, 0, 0, \beta_t E_t[Z_{1\Delta t+1}], 0, 0, 0, 0, 0, 0; \right) \\
f_{9X} &= (0, -(-\gamma+1)C_t^{-\gamma}, 0, 1, 0, 0, \theta\beta_t E_t[Z_{2\Delta t+1}], 0, 0, 0, 0, 0, 0;)
\end{aligned}$$

$$\begin{aligned}
f_{10X} &= \left(0, -\frac{\chi N_t^\eta}{(1-\Phi)}, -\frac{\chi \eta C_t N_t^{\eta-1}}{(1-\Phi)}, 0, 1, 0, \theta \beta_t E_t [Z_{3\Delta t+1}], 0, 0, 0, 0, 0, 0; \right) \\
f_{11X} &= \left(0, 0, 0, 0, 0, \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{\varepsilon-1}} \varepsilon \theta \Pi_t^{\varepsilon-2} - \theta \varepsilon \Pi_t^{\varepsilon-1} \Delta_{t-1}, 1, 0, 0, 0, 0, 0, 0; \right) \\
f_{12X} &= \left(0, 0, 0, -\left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{1}{1-\varepsilon}}, 1, -F_t \left(\frac{1-\theta \Pi_t^{\varepsilon-1}}{1-\theta} \right)^{\frac{\varepsilon}{1-\varepsilon}} \frac{\theta \Pi_t^{\varepsilon-2}}{1-\theta}, 0, 0, 0, 0, 0, 0, 0; \right) \\
f_{13X} &= \left(0, 1, -\frac{1}{\Delta_t}, 0, 0, 0, \frac{N_t}{\Delta_t^2}, 0, 0, 0, 0, 0, 0; \right)
\end{aligned}$$

where

$$\begin{aligned}
Z_{1\Delta t+1} &= -\frac{\gamma C_{t+1}^{-\gamma-1} C_{\Delta t+1}}{\Pi_{t+1}} - \frac{C_{t+1}^{-\gamma} \Pi_{\Delta t+1}}{\Pi_{t+1}^2} \\
Z_{2\Delta t+1} &= (\varepsilon - 1) \Pi_{t+1}^{\varepsilon-2} \Pi_{\Delta t+1} F_{t+1} + \Pi_{t+1}^{\varepsilon-1} F_{\Delta t+1} \\
Z_{3\Delta t+1} &= \varepsilon \Pi_{t+1}^{\varepsilon-1} \Pi_{\Delta t+1} S_{t+1} + \Pi_{t+1}^\varepsilon S_{\Delta t+1} \\
Z_{4\Delta t+1} &= \varepsilon \Pi_{t+1}^{\varepsilon-1} \Pi_{\Delta t+1} \lambda_{4t+1} + \Pi_{t+1}^\varepsilon \lambda_{4\Delta t+1}
\end{aligned}$$

$$\begin{aligned}
Z_{1\Delta\Delta t+1} &= \frac{\gamma(\gamma+1) C_{t+1}^{-\gamma-2} C_{\Delta t+1}^2}{\Pi_{t+1}} - \gamma C_{t+1}^{-\gamma-1} \left(\frac{C_{\Delta\Delta t+1}}{\Pi_{t+1}} - \frac{C_{\Delta t+1} \Pi_{\Delta t+1}}{\Pi_{t+1}^2} \right) \\
&\quad \frac{\gamma C_{t+1}^{-\gamma-1} C_{\Delta t+1} \Pi_{\Delta t+1}}{\Pi_{t+1}^2} - C_{t+1}^{-\gamma} \left(\frac{\Pi_{\Delta\Delta t+1}}{\Pi_{t+1}^2} - \frac{2\Pi_{\Delta t+1}^2}{\Pi_{t+1}^3} \right)
\end{aligned}$$

$$\begin{aligned}
Z_{2\Delta\Delta t+1} &= (\varepsilon - 1)(\varepsilon - 2) \Pi_{t+1}^{\varepsilon-3} \Pi_{\Delta t+1}^2 F_{t+1} + (\varepsilon - 1) \Pi_{t+1}^{\varepsilon-2} (\Pi_{\Delta\Delta t+1} F_{t+1} + \Pi_{\Delta t+1} F_{\Delta t+1}) \\
&\quad + (\varepsilon - 1) \Pi_{t+1}^{\varepsilon-2} \Pi_{\Delta t+1} F_{\Delta t+1} + \Pi_{t+1}^{\varepsilon-1} F_{\Delta\Delta t+1}
\end{aligned}$$

$$\begin{aligned}
Z_{3\Delta\Delta t+1} &= \varepsilon(\varepsilon - 1) \Pi_{t+1}^{\varepsilon-2} \Pi_{\Delta t+1}^2 S_{t+1} + \varepsilon \Pi_{t+1}^{\varepsilon-1} (\Pi_{\Delta\Delta t+1} S_{t+1} + \Pi_{\Delta t+1} S_{\Delta t+1}) \\
&\quad + \varepsilon \Pi_{t+1}^{\varepsilon-1} \Pi_{\Delta t+1} S_{\Delta t+1} + \Pi_{t+1}^\varepsilon S_{\Delta\Delta t+1}
\end{aligned}$$

$$\begin{aligned}
Z_{3\Delta\Delta t+1} &= \varepsilon(\varepsilon - 1) \Pi_{t+1}^{\varepsilon-2} \Pi_{\Delta t+1}^2 \lambda_{4t+1} + \varepsilon \Pi_{t+1}^{\varepsilon-1} \Pi_{\Delta\Delta t+1} \lambda_{4t+1} + \Pi_{\Delta t+1} \lambda_{4\Delta t+1} \\
&\quad + \varepsilon \Pi_{t+1}^{\varepsilon-1} \Pi_{\Delta t+1} \lambda_{4\Delta t+1} + \Pi_{t+1}^\varepsilon \lambda_{4\Delta\Delta t+1}
\end{aligned}$$

Appendix D. Simplified LQ model of the FNL model

The LQ version of the FNL model is:

$$Max \left\{ -\Omega E_0 \sum_{t=0}^{\infty} \beta^t \{ \widehat{\pi}_t^2 + \lambda (x_t - x^*)^2 \} \right\}$$

subject to

$$x_t = E_t x_{t+1} - \frac{1}{\gamma} [\widehat{i}_t - E_t \widehat{\pi}_{t+1}] + \frac{1}{\gamma} [-\widehat{\beta}_t]$$

$$\widehat{\pi}_t = \beta E_t [\widehat{\pi}_{t+1}] + \kappa x_t$$

$$\widehat{\beta}_t = \ln(\beta_t) - \ln \beta = \rho_\beta \widehat{\beta}_{t-1} + \varepsilon_{\beta,t}$$

$$\widehat{i}_t \geq \log(\beta)$$

where

$$\widehat{\pi}_t = \log(1 + \pi_t)$$

$$\widehat{i}_t = \log(1 + i_t) - \log(1/\beta)$$

$$x_t = (\log(Y_t) - \log(\bar{Y})) - (\log(Y_t^f) - \log(\bar{Y}^f))$$

$$\Omega = -\frac{1}{2}(\gamma + \eta) \frac{\varepsilon}{(\gamma + \eta)} \frac{\theta}{(1 - \theta)(1 - \theta\beta)} \bar{C}^{-\gamma+1}$$

$$\lambda = \frac{(\gamma + \eta)(1 - \theta)(1 - \theta\beta)}{\varepsilon \theta} = \frac{\kappa}{\varepsilon}$$

$$x^* = \frac{\Phi}{\gamma + \eta}$$

$$\Phi = 1 - (1 - \tau_w)(1 - \varepsilon^{-1})$$

It is not difficult to show that the steady state values of this model are as follows:

$$\bar{\pi} = \frac{\lambda\kappa}{\kappa^2 + \lambda(1 - \beta)} x^*$$

$$\bar{i} = \bar{\pi}$$

$$\bar{x} = \frac{(1 - \beta)\bar{\pi}}{\kappa}$$