OVERBorrowING, DELEVERAGING AND A GREAT RECESSION *

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Abstract

This paper examines the role of overborrowing, deleveraging, and an incomplete financial market in driving an economy to a great recession with a binding zero lower bound on the nominal interest rate (ZLB). There are two key features that differentiate my work from the current literature of deleveraging and the ZLB. First, I endogenize the debt limit of borrowers by tying it to the market value of collateral assets. Second, and more importantly, I allow for overborrowing by calibrating the model to match with the high debt-to-income ratio at the onset of the Great Recession for the U.S. homeowners that owned a house in 1997. I am able to show that the second feature makes the ZLB more likely to bind under an adverse shock to the credit market. When the ZLB binds, a great recession emerges with a free fall in output and the price level, mostly due to the Fisherian debt deflation that puts more debt burden on the borrowers.


Keywords: overborrowing, deleveraging, the ZLB, liquidity trap, Taylor rule, Great Recession, Fisherian debt deflation.

*I would like to thank Jianjun Miao, Robert King, Francois Gourio, J.Christina Wang and Alisdair McKay for their discussion and comments. Also, I would like to thank the participants of the BU dissertation workshop, BU macro reading group, and Cleveland State University seminar, Fall 2013 Midwest Macro Meeting for their comments and suggestions. All errors in the paper are mine.

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1 Introduction

There are two striking stylized facts from the last recession. First, there was a surge in household leverage, defined as a debt-to-income ratio, during the 2002-2006 period. As documented in Mian and Sufi [2011], during this period, the debt-to-income ratio for the existing homeowners that owned a house in 1997 increased sharply by 0.7, from about 2.6. This increase occurred due to the flood of funds in the U.S., the boom of the housing market, and the willingness of lenders in making loans based on their expectations about the price of collateral assets, especially housing prices. Second, the recession was worse in the regions where household leverage had increased more.

Obviously, the overborrowing and housing market play an important role in causing the worst recession that the U.S. has ever observed since the Great Depression. However, the standard deleveraging and ZLB literature that models debt limits as an exogenous stochastic process, including Eggertsson and Krugman [2012] and Guerrieri and Lorenzoni [2011], hereafter called the EK and GL models, predicts that including durable goods, such as a house, would mitigate the impact of an adverse shock because households would use durable goods as a cushion to deal with the shock.

In this paper, I extend the standard deleveraging and ZLB literature by modeling the debt limit endogenously, instead of exogenously as in the EK and GL models. Specifically, the debt limit is tied to both exogenous credit market conditions and the endogenous market value of collateral assets, which are houses. More importantly, I allow for overborrowing by calibrating the model to match the observed debt-to-income ratio at the onset of the last recession.

Without the overborrowing characteristic, a model with endogenous debt limit generates a result supporting for the EK’s prediction that the borrowers would use durable goods as a cushion to fight against adverse shocks to the credit market. As a result, the ZLB is less likely to bind and output is less volatile compared to a model with exogenous debt limits, such as the EK model. The result contradicts the common belief that endogenizing the debt limit will always amplify output and inflation fluctuations
under an adverse shock to the credit market because cutting durable good might cause the debt limit to fall more.

More salient is that in the presence of overborrowing, I am able to show that the model with an endogenous debt limit generates a more powerful transmission mechanism. The economic variables are more responsive to a shock to the credit market and the ZLB is more likely to bind. When the ZLB binds, a great recession emerges with a discontinuous fall in output and the price level.

The intuition for the results is as follows: An adverse shock to the credit market lowers the debt limit and makes the borrowing constraint tighter given the other factors, so borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back more necessary non-durable goods. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially.

However, in a world with overborrowing, the initial debt-to-value ratio is very high. The home equity of borrowers is substantially low, even negative. Therefore, selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility, the borrowers do not want to cut back their durable housing goods. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when the nondurable goods consumption is cut back.

In both cases, with and without overborrowing, we see the debt level declines due to two reasons. First, the adverse shock to the credit market lowers the debt limit and tightens the borrowing constraint. Second, an initial decline in the debt limit will lead to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable goods consumption and the debt limit. However, only in the case of overborrowing can the model generate a tighter borrowing constraint compared to the standard EK model.

The monetary policy in this paper is a simple Taylor rule, so the central bank cannot
stabilize output and inflation under an adverse shock to the credit market. Therefore, output falls. In the framework of monopolistic competition, the price level falls, leading to a higher real debt burden of credit-constrained households, causing them to further reduce their consumption. This Fisherian debt deflation, associated with overborrowing, is more likely to drive the economy to the ZLB. I show quantitatively that the Fisherian debt deflation is extremely powerful when the ZLB binds. It generates a free fall in output and the price level.

The related literature on the ZLB has been inspired by seminal work by Krugman [1998], which extensively discusses the causes and consequences of the ZLB in a series of simple two-period perfect-foresight models. Since Krugman [1998], extensive research related to the ZLB has been conducted, including Eggertsson and Woodford [2003], Jung et al. [2005], Adam and Billi [2006, 2007], Nakov [2008], Levin et al. [2010], Bodenstein et al. [2010], Eggertsson and Krugman [2010], Werning [2011], Nakata [2011], Fernandez-Villaverde et al. [2012], and Judd et al. [2011]. These papers use preference shocks as a reduced form that drives an economy to the liquidity trap with a binding ZLB.

In contrast to the above-mentioned papers, some recent papers deal with different types of shocks that cause the ZLB to bind. Hall [2011] models excessive capital stock and a sharp decline in capital utilization as the reason for the nominal interest rate to be pinned at the ZLB. Curdia and Woodford [2009] model a shock to the wedge between deposit and lending rates as a driving force.

Guerrieri and Lorenzoni [2011] model a debt limit and household heterogeneity in labor productivity. They show that an exogenous decline in the debt limit acts as an increase in the subjective discount factor. The decline in the debt limit causes future consumption to be more volatile because with a lower debt limit, households will be less able to insure their consumption risks. Therefore, savers will save more and borrowers will borrow less due to precautionary savings. As a result, savings flood the financial market, resulting in a sharp decrease in the nominal interest rate, causing the ZLB to bind.

Eggertsson and Krugman [2012] also model the debt limit and deleveraging as a key
factor driving the nominal interest rate to the ZLB. In contrast to Guerrieri and Lorenzoni [2011] where savings come from precautionary behavior, Eggertsson and Krugman [2012] model savings based on the difference in the two types of representative households. One type is patient; the other is not. The patient representative household saves and lends his money to the impatient one. Similar to Guerrieri and Lorenzoni [2011], Eggertson and Krugman model the debt limit as an exogenous process.

The remainder of this paper is organized as follows. Section 2 presents the structure of the economy. Section 3 shows how key parameters are calibrated to match the U.S. stylized facts, and reports main results and intuition based on the two-period assumption. Section 4 reports the results from dynamic transition. Section 5 concludes. Appendices are presented in Section 6.

2 Model

The model in this paper is a standard two-representative agent model, such as those models found in Eggertsson and Krugman [2012] and Iacoviello [2005]. There are two types of households: credit-constrained households (or borrowers) of mass $\chi_b$, and unconstrained households (or savers) of mass $\chi_s = 1 - \chi_b$. The borrowers are impatient while the savers are patient and act as the lenders. The households consume non-housing goods and enjoy housing service from owning houses that have a fixed supply.

Houses play two roles in the model. First, they provide housing services to the households. Second, they can be used as collateral assets for borrowing. One of the two key features in our model is an endogenous debt limit that is determined by both the endogenous market value of houses and exogenous financial market conditions (determined by a debt-to-value ratio), as in Kiyotaki and Moore [1997] and Iacoviello [2005]. This feature distinguishes this paper from the current literature of deleveraging and the ZLB, as in Eggertsson and Krugman [2012] and Guerrieri and Lorenzoni [2011], who model debt limits as an exogenous process.
2.1 The borrowing-constrained household’s problem

The representative borrowing-constrained household chooses the path of non-durable goods, durable housing goods, and labor to maximize his expected present discounted lifetime utility subject to his budget constraint and borrowing constraint. His problem can be described mathematically as follows:

\[
\max \quad E_0 \sum_{t=0}^{\infty} \beta^t U_b(C_b, H_b, N_b) \tag{1}
\]

subject to:

\[
C_b + D_{b-1} (1 + r_{t-1}) + q_t (H_b - H_{b-1}) = (1 - \tau) w_t N_b + T_b + D_b \tag{2}
\]

\[
D_b \cdot (1 + r_t) \leq \xi_t E_t [q_{t+1} H_b] \tag{3}
\]

\[
1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \tag{4}
\]

where \(C, D_b, H_b, N_b, T\) are composite non-housing goods, real debt, housing quantity, labor supply by the borrower, and lump sum tax/transfer respectively; \(i, \pi, r, q, \tau\) denote nominal interest rate, inflation rate, real interest rate, the real price of a house, and labor income tax/subsidy; and \(\xi\) reflects the credit market conditions. The credit market shock follows an AR(1) process:

\[
\ln (\xi_{t+1}) = (1 - \rho_{\xi}) \ln (\xi_t) + \rho_{\xi} \ln (\xi_t) + \varepsilon_{\xi, t+1} \tag{5}
\]

Because the credit market condition \(\xi\) is a very important parameter, I would like to clarify two issues that could potentially arise. First, I would interpret the parameter as a debt to market value of collateral assets ratio. By studying a debt-to-value ratio, we are able to allow for overborrowing that results in a substantially high debt-to-income ratio, as documented by Mian and Sufi [2011]. Intuitively, this high debt-to-income ration occurs due to the fact that lenders are willing to lend money based on their expectations on existing collateral asset appreciation. Second, I am not going to model why the
parameter exists. The rationale for the existence could be an asymmetric information problem or an irrational exuberance problem, in which lenders lend money based on their own expectations that are formed based on overly optimism about the likelihood of getting their money back.

Let $\lambda_{bt}, \phi_{bt}$ be the Lagrange multipliers with respect to budget constraint and debt constraint. The optimal choices of the constrained households must satisfy the following conditions:

$$U_{bt,C} - \lambda_{bt} = 0 \quad (6)$$

$$-\frac{U_{bt,N}}{U_{bt,C}} = (1 - \tau) w_t \quad (7)$$

$$\lambda_{bt} - \phi_{bt} E_t [1 + r_t] = \beta_b E_t [\lambda_{bt+1} (1 + r_t)] \quad (8)$$

$$U_{bt,H} + \xi_t \phi_{bt} E_t [q_{t+1}] + \beta_b E_t [\lambda_{bt+1} \cdot q_{t+1}] = \lambda_{bt} q_t \quad (9)$$

$$\min \{\xi_t E_t [q_{t+1} H_{bt}] - D_{bt} (1 + r_t), \phi_{bt, t}\} = 0 \quad (10)$$

$$\phi_{bt} \geq 0 \quad (11)$$

Equation (6) shows the marginal utility derived from consuming the composite non-durable goods. Equation (7) presents the intra-temporal trade-off between consumption and labor at the margin. Equation (8) is the Euler equation for the borrower, which is the inter-temporal trade-off between today’s consumption and tomorrow’s consumption. If the credit-constrained household consumes one unit of nondurable goods, he would receive utility from his consumption. In addition, he would put $(1 + r_t)$ more pressure on the collateral constraint that costs him $\phi_{bt} (1 + r_t)$ in terms of utility. Therefore, the left-hand side of the equation is the marginal benefit of consuming today, while the right-hand side is the marginal utility he has to forgo due to not saving.

The marginal trade-off between non-durable goods and durable housing goods is illustrated in equation (9). The left-hand side of the equation shows the marginal benefit of buying one more unit of houses. The marginal benefit includes housing services, the value of the debt limit that he would get by relaxing the collateral constraint due to
owning more houses, and the next period value of the houses in terms of utility. The right-hand side of the equation is the marginal cost of buying houses. The borrowing constraint is rewritten as in equation (10). This equation is the combination of the collateral constraint and the non-negativity of the shadow value of debt, \( \phi_{bt} \).

### 2.2 The unconstrained household’s problem

The representative unconstrained household never faces a borrowing constraint. He saves and lends to the credit-constrained households. He also owns intermediate-goods firms. His problem is as follows:

\[
\max_t E \sum_{t=0}^{\infty} \beta^t U_{st} (C_{st}, H_{st}, N_{st}) \quad (12)
\]

subject to:

\[
C_{st} + D_s - 1 (1 + r_{t-1}) + q_t (H_{st} - H_{st-1}) = (1 - \tau) w_t N_{st} + \int_{i=0}^{1} \frac{Z_{st}}{P_t} di + T_{st} + D_{st} \quad (13)
\]

where \( C, D, H, N, T \) are composite non-housing goods, real debt, housing quantity, labor supply and lump sum tax/transfer respectively; \( i, \pi, r, q, \tau \) denote nominal interest rate, inflation rate, real interest rate, the real price of a house, and labor income tax/subsidy; and \( Z \) is nominal profit from the \( i^{th} \) intermediate-goods firms that are owned by the savers only.

Let \( \lambda_{st} \) be the Lagrange multiplier with respect to the budget constraint of the the saver. The optimal choices of the saver must satisfy the following condition:

\[
U_{st,C} - \lambda_{st} = 0 \quad (14)
\]

\[
-\frac{U_{st,N}}{U_{st,C}} = (1 - \tau) w_t \quad (15)
\]

\[
\lambda_{st} - \beta_s E_t [\lambda_{st+1} (1 + r_t)] = 0 \quad (16)
\]

\[
U_{st,H} + \beta_s E_t [\lambda_{st+1} \cdot q_{t+1}] = \lambda_{st} q_t \quad (17)
\]
Equation (14) shows the saver’s marginal utility derived from non-durable goods consumption. Equation (15) presents his marginal trade-off between consumption and labor. Equation (16) is the Euler equation for the saver, which is the intertemporal trade-off between today’s consumption and tomorrow’s consumption. The marginal trade-off between non-durable goods consumption and housing goods is illustrated in equation (17).

2.3 Final goods producers

There is a mass 1 of final goods producers who operate in a perfectly competitive market. Each final goods producer produces the consumption goods by aggregating a variety of differentiated goods using a CES technology. His problem is to maximize his contemporaneous profit:

$$\max P_t Y_t - \int P_t (i) Y_t (i) \, di$$

subject to

$$Y_t = \left( \int_{0}^{1} Y_t (i)^{\varepsilon - 1} \, di \right)^{\frac{1}{\varepsilon - 1}}$$

where \(y_{it}\) is the input of intermediate goods \(i \in [0, 1]\) and \(\varepsilon\) is the elasticity of substitution between differentiated goods.

The optimal decision of the final goods producer gives rise to the demand for the \(i^{th}\) intermediate goods:

$$Y_t (i) = \left( \frac{P_t (i)}{P_t} \right)^{-\varepsilon} Y_t$$

where \(P_t\) is the price level:

$$P_t = \left( \int P_t (i)^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}$$

2.4 Intermediate goods producers

There is a mass 1 of intermediate goods firms. These firms are owned by the savers and are operated in a monopolistically competitive market. A firm’s objective is to maximize its expected present discounted flows of profit. The firms adjust their prices according
to a quadratic adjustment cost of Rotemberg’s type. Firm $i$’s problem is given below:

$$\max_{P_{it}, N_{it}} \sum_{j=0}^{\infty} Q_{st,t+j} Z_{it+j}$$

subject to

$$Z_{it} = P_{it} Y_{it} - P_{it} w_t N_{it} - P_{t} \frac{\varphi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2 Y_t$$

$$Y_{it} = A_t N_{it}$$

$$Y_{it} = \left( \frac{P_{it}}{P_{it}} \right)^{-\varepsilon} Y_t$$

$$P_{i0} = P_0$$

where $\varphi$ is the adjustment cost parameter and $A_t$ presents technology shocks that follow an AR(1) process:

$$\ln (A_{t+1}) = \rho_A \ln (A_t) + \varepsilon_{A,t+1}$$

The optimality conditions give rise to the following condition:

$$\left( 1 - \varepsilon + \varepsilon \frac{w_t}{A_t} - \varphi \pi_t (1 + \pi_t) \right) Y_t + \varphi Q_{st,t+1} E_t \left[ \pi_{t+1} (1 + \pi_{t+1})^2 Y_{t+1} \right] = 0$$

where

$$Q_{st,t+1} = \beta_s \frac{U_{st+1,C}}{U_{st,C}} \frac{1}{(1 + \pi_{t+1})}$$

is the stochastic discount factor.

### 2.5 Aggregate conditions

In equilibrium, all the markets are cleared:

$$\chi_b H_{bt} + \chi_s H_{st} = \bar{H}$$

$$\chi_b N_{bt} + \chi_s N_{st} = N_t$$
\[ \chi_b D_{bt} + \chi_s D_{st} = B_{gt} \tag{32} \]

\[ C_t = \left( \frac{A_t N_t}{\Delta_t} \right) \left( 1 - \frac{\phi}{2 \pi^2} \right) \tag{33} \]

\[ \chi_b C_{bt} + \chi_s C_{st} = C_t \tag{34} \]

\[ \Delta_t = \int \left( \frac{P_{tt}}{P_t} \right)^{-\varepsilon} di \tag{35} \]

Equation (30) shows that the total demand for houses equals the total fixed housing supply. Equation (31), (32), (33) present market clearing conditions for the non-housing composite goods, labor, and debt markets respectively.

### 2.6 Government policy

**Monetary policy:** The central bank conducts monetary policy using a simple Taylor rule as following:

\[ \left( \frac{1 + i_t}{1 + i} \right) = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{\phi_i} \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\phi_\pi} \tag{36} \]

\[ i_t \geq 0 \tag{37} \]

Equation (37) implies that the nominal interest rate is not allowed to be negative. This is the key condition in the literature of deleveraging and ZLB.

**Fiscal policy:** The government collects labor income taxes and issues a fixed quantity of short-term government bonds to cover lump sum transfers to households:

\[ \chi_b T_{bt} + \chi_s T_{st} = \tau (\chi_b N_{bt} + \chi_b N_{st}) w_t + B_{gt} \tag{39} \]

### 2.7 Equilibrium

**Definition 1** An equilibrium consists of the path of prices \( \{i_t, w_t, \pi_t\}_{t=0}^{\infty} \) and allocation \( \{H_{bt}, D_{bt}, C_{bt}, N_{bt}, T_{bt}, H_{st}, D_{st}, C_{st}, N_{st}, T_{st}, C_t, N_t, Y_t\}_{t=0}^{\infty} \) that satisfies the following con-
(1) Borrowers’ and savers’ optimization conditions.

(2) Firms’s optimization conditions.

(3) Aggregate conditions.

(4) Taylor rule and the ZLB.

(5) Balanced government budget.

3 Results: A two-period deleveraging model

To provide intuition about the transmission mechanism of the model, in this section, I work with a simple two-period deleveraging model. The timing of the model is similar to the one in Eggertsson and Krugman [2012]. Initially, at time 0, the economy stands at a steady state with a certain credit market condition \( \xi = \bar{\xi} \). Then a permanent shock to the credit market occurs (\( \xi \) changes to \( \xi' \)) at time 1. The representative households choose new debts, housing quantities, nondurable consumption goods, and labor. The economy converges to a new steady state at time 2.

3.1 Calibration

The preferences for the borrowers and savers are specified as below:

\[
U_{bt} = \ln C_{bt} + j_b \ln H_{bt} - \eta_b \frac{N_{bt}^{1+\phi}}{1 + \phi}
\]

\[
U_{st} = \ln C_{st} + j_s \ln H_{st} - \eta_s \frac{N_{st}^{1+\phi}}{1 + \phi}
\]

The key parameters are calibrated in Table 1. The fraction of borrowers is 0.58. Subjective discount factors are 0.99 and 0.96 for the borrower and saver respectively. These numbers are taken directly from Iacoviello [2005]. In this economy, 58% of households are credit constrained. The fixed stock of housing supply are normalized to 1. I calibrate three parameters \( \xi, j_s, \) and \( j_b \) to match three stylized facts that we observed in the U.S. economy. First, the total debt to income ratio is 3.2 at the onset of the last
Table 1: Benchmark parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Note</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_b$</td>
<td>0.58</td>
<td>Fraction of borrowers</td>
<td>Iacoviello [2005]</td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>0.42</td>
<td>Fraction of savers</td>
<td>Iacoviello [2005]</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.99</td>
<td>Subjective discount of savers</td>
<td>Iacoviello [2005]</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>0.96</td>
<td>Subjective discount of borrowers</td>
<td>Iacoviello [2005]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.95</td>
<td>Calibrated to match $\chi_bD_b/Y = 3.2$</td>
<td>Mian and Sufi [2011]</td>
</tr>
<tr>
<td>$j_s$</td>
<td>0.038</td>
<td>Calibrated to match $H_b/H_s = 1$</td>
<td>Flow of funds table</td>
</tr>
<tr>
<td>$j_b$</td>
<td>0.049</td>
<td>Calibrated to match $qH/Y = 4$</td>
<td>Flow of funds table</td>
</tr>
<tr>
<td>$\eta_b$</td>
<td>1.04</td>
<td>Borrowers’ labor disutility parameter</td>
<td>To calibrate SS $Nb = 1$</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>0.76</td>
<td>Savers’ labor disutility parameter</td>
<td>To calibrate SS $Ns = 1$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>Inverse labor supply elasticity</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>150</td>
<td>Mean price duration of 3.3 quarters</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>20</td>
<td>Markup is 5%</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>0</td>
<td>Labor income tax</td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>0</td>
<td>Target inflation rate</td>
<td></td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>2.5</td>
<td>Weight of inflation in Taylor rule</td>
<td></td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>0</td>
<td>Weight of last period interest rate</td>
<td></td>
</tr>
</tbody>
</table>

Note: The flow of funds table is from the Federal Reserve Bank.

financial crisis, as reported in Mian and Sufi [2011]. Second, the total housing asset to income ratio is around 4, as in the flow of funds table reported by the Federal Reserve Bank. Third, on average, each household owns a house, or $H_b/H_s = 1$. The calibration results in a debt-to-value ratio as high as 0.95, which is higher than 0.89 estimated in Iacoviello [2005].

I calibrate $\eta_b$ and $\eta_s$ to be 1.04 and 0.76 respectively in order to have the initial steady state labor of 1 for both borrowers and savers. The inverse labor supply elasticity, $\phi$, is chosen to be 1. The demand elasticity for differentiated goods is calibrated to be 20, corresponding to a net markup of 5%. In other words, the only non-trivial economic distortion in the economy is due to the monopoly power. The adjustment cost parameter ($\psi$) is calibrated to be 150, which corresponds to the average duration of about 3.3 quarters without prices changing. For the Taylor rule, I choose the inflation target to be 2 percent.
3.2 Without the ZLB

To understand the role of the ZLB in the following section, in this section I first provide the results from the case without the ZLB, as presented in Figures 1 and 2. The x-axis shows a permanent percentage change in the credit market parameter (ξ) from the initial steady state value. The short-run responses of selected macro economic variables in period 1 are presented in the y-axis. The solid blue lines present the results from the housing model, while the dashed red lines show the results from the EK model. The intersections between the solid blue lines and the dashed red lines presents the initial steady state values. In the EK model, I fix the housing price and housing quantities for both borrowers and the savers.

Panel D of Figure 1 and Panels A, C and D of Figure 2 show the percentage change of total output, new debt, housing quantity of the borrowers, and housing price from per year. The weight of inflation and past interest rates are 2.5 and 0 respectively.

Figure 1: Responses of macro economic variables, the case of overborrowing and without the ZLB.
Figure 2: Responses of macro economic variables, the case of overborrowing and without the ZLB. (cont.)

the initial steady state values respectively. Panel B of Figure 2 shows debt service as a percentage of the initial debt, where debt service is computed as debt payment minus new debt. A decline in debt service means that the borrowers can borrow more than the amount of debt payment.

Under a positive shock to the debt-to-value ratio, from Figures 1 and 2, we see that, in the housing model, the value of debt limit and debt service decline, while borrowers’ new debt and new housing quantity, total output, inflation, and interest rates all increase in equilibrium. Intuitively, when the debt-to-value ratio increases initially, the debt limit faced by the borrowers increases, and they are allowed to borrow more given the other factors. As a result, the shadow value of the debt limit decreases and the borrowers attempt to borrow more. Hence, both borrowers’ nondurable and durable housing consumptions increase.

The increase in the housing quantity leads to another increase in the debt limit, encouraging borrowers to borrow and spend more and creating another round of expan-
sion. Debt service declines because the borrowers are not only able to roll over their debts, but are also able to borrow more. In the monopolistic framework, inflation rises when output increases. Under a simple Taylor rule, the nominal interest rate increases.

However, up to a certain value, a further increase in the debt-to-value ratio would not alter the responses of some macroeconomic variables. After that value, borrowers are allowed to borrow up to the amount they want. In other words, the borrowers no longer face a credit constraint. Therefore, the shadow value of the debt limit is zero and the borrowers no longer accumulate more houses to lift up their potential debt limit. The inflation and nominal interest rate hit the upper bounds of around 1% and 6.8% per year respectively.

The opposite mechanism occurs under a negative shock to the credit market. In this circumstance, borrowers are not able to borrow as much as before given the other factors. The debt limit decreases, while the shadow value of the debt limit rises. Therefore, new amount of debts falls and debt service increases. As a result, the borrowers’ nondurable and housing consumption fall due to the deleverage. Because the monetary policy is a simple Taylor rule, it is not powerful enough to stabilize output and inflation. Hence, output declines and, as a result, disinflation occurs. The responses are also amplified by the collateral effect and debt deflation effect.

The most interesting and important feature is that the model with housing generates more amplified responses of macroeconomic variables to a credit market shock compared to the conventional EK model. The results come from both overborrowing and an endogenous debt limit. The finding contradicts the common belief that a model with endogenous debt limit always generate such results. We will discuss this issue more in the following section.

Let us look at the scenario under a negative shock. Panel A in Figure 1 shows that the nominal interest rate in the housing model falls more than in the EK model. Also, the housing model produces a bigger decline in inflation and output, as in Panels B and D in Figure 1. The borrowers suffer a tighter collateral constraint in the housing model than in the EK model because the shadow value of debt limit is higher in the housing
model than in the EK model.

The more amplified transmission mechanism of the housing model can be found only when we allow for overborrowing. As explained above, by overborrowing, I calibrate the parameter $\xi$ to match the high debt-to-income ratio at the onset of the housing bubble burst. The ratio is substantially high: around 3.2. Without the overborrowing, we cannot generate such results.

To demonstrate the role of overborrowing, I also report the results from the case of normal borrowing and without the ZLB in Figure 3 and 4. In the case of normal borrowing, I calibrate the initial debt-to-value of collateral asset to match with a lower debt-to-income ratio before the credit boom in 2002-2006. As documented by Mian and Sufi [2011], before 2002, the debt-to-income ratio of the households who owned a house was 2.6 instead of 3.2 as in the benchmark. The corresponding debt-to-value of collateral asset ratio was around 0.77, which is smaller than 0.95 as in the case of overborrowing.

As in Panel A, Figure 3, under a negative demand shock, the decrease in the nominal
Figure 4: Responses of macro economic variables, the case of normal borrowing and without the ZLB. (cont.)

The interest rate in the housing model is not as big as in the EK model. Inflation and output also fall less in the housing model than in the EK model. Although, we see a higher debt service and a smaller new debt in the housing model, the value of debt limit is actually smaller in this model compared to the EK model. This means that the slackness of the collateral constraint is smaller in the housing model.

The intuition for the results is as follows: An adverse shock to the credit market lowers the debt limit and makes the borrowing constraint tighter given the other factors. So the borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back the more necessary non-durable goods. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially compared to the EK model.

However, in a world with overborrowing, the intial debt-to-value ratio is very high.
The home equity of borrowers is substantially low, even negative. Therefore, selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility, the borrowers do not want to cut back their durable housing consumption. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut back.

In both cases, with and without overborrowing, we see the the debt level decline due to two reasons. First, the adverse shock to the credit market lowers the debt limit and tightens the borrowing constraint. Second, an initial decline in the debt limit leads to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable goods consumption and the debt limit. However, only in the case of overborrowing can the model generate a tighter borrowing constraint, compared to the standard EK model.

There is another way to explain why the endogenous debt model with overborrowing can generate more amplified responses of economic variables to credit market shocks. In general, through the budget constraint (2), a one-dollar decrease in durable goods consumption would help to lower the current debt by one dollar. As a result, from the collateral constraint (3), the reduction relaxes the collateral constraint by $R_t$ dollar, where $R_t$ is the gross real interest rate. However, by reducing one dollar of durable housing goods, the borrowers have to give up the utility from the housing service.

More importantly, reducing one dollar of durable goods will lead to a fall of the debt limit, which depends on the expected housing price and financial market conditions, as in equation (2). In the model with overborrowing, the initial debt-to-value ratio is high. Therefore, it is costly to cut durable housing goods because it will put more pressure on the collateral constraint due to the reduction of collateral assets. This additional pressure is more than the relaxation thanks to a lower debt, which results from cutting back durable goods. Hence, the borrowers do not want to cut durable housing goods. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut.
However, in the model without overborrowing, the initial debt-to-value is low. It is not costly to cut durable goods because the additional pressure on the collateral constraint due to the reduction of the collateral asset can be offset by a lower level of debt resulting from scaling back durable good consumption. Therefore, cutting back durable good consumption is desirable. In both cases, we see a spiral decline in both durable goods consumption and the debt limit. However, only in the case of overborrowing can the model generate a tighter borrowing constraint compared to the standard EK model.

### 3.3 With ZLB

In this section, I study the impact of an adverse shock to the credit market in the case of overborrowing and in the presence of the ZLB. The results are presented in Figures 5 and 6. The transmission mechanism becomes drastically more powerful when the ZLB binds. The total output and inflation in the economy drop discontinuously under a shock that cause the ZLB to bind.
We can see the discontinuity of the short-run equilibrium output in Panel D of Figure 6. At the point where the ZLB binds, the output falls further from around $-4.5\%$ per year (or $-1.4\%$ per quarter) to around $-12\%$ per year (or $-3.1\%$ per quarter). Inflation falls more, from around $-1.6\%$ per year (or $-0.4\%$) to around $-4\%$ per year (or $-1\%$ per quarter).

The free fall in output and inflation result from two main channels. First, from the collateral constraint equation (3), when there is an adverse shock to the credit market conditions, $\xi_t$, both $i_t$ and $D_{bt}$ fall. When the nominal interest hits the zero bound, more downward pressure will be put on the current debt, leading to a larger deleveraging compared to the case without the ZLB. In other words, a binding ZLB amplifies the collateral effect. Additionally, compared to the model without the ZLB, the real interest rate is higher in the ZLB model due to the inability of the central bank to set a negative nominal interest rate. From the budget constraint to the borrowers, the real debt burden increases substantially. In total, the ZLB creates a very powerful
To see this powerful amplification mechanism of Fisherian debt deflation, I provide a comparison between the model with and without a Fisherian debt deflation channel in Figures 7 and 8. In the case without Fisherian debt deflation, the nominal interest rates are indexed by inflation. The real debt is constant no matter how large inflation or deflation is.

The solid blue lines show the results from the model with Fisherian debt deflation, while the dashed red lines present the results from the model without Fisherian debt deflation. Including the Fisherian debt deflation not only makes the ZLB to bind more easily, as in Panel A of Figure 7, but also makes a recession much worse when the ZLB binds, as in Panel B of Figure 7.

Figure 7: Responses of macro economic variables, the case with overborrowing and the ZLB.

amplification mechanism.
4 Results: Dynamic transition

In this section, I present the dynamic transition of interest rates and output under a 1% permanent decline in the credit market condition, $\xi$. I assume that the economy converges to the steady state after a certain number of periods, $T$. The shock is also assumed to gradually return to the new steady state, which is 1% lower than the initial one after $T$ periods. Figure 9 shows the transition of the shock for three cases corresponding with $T = \{1, 2, 3\}$, where $T = 1$ presents the baseline two-period model and $T = 2, 3$ corresponds to the three-period and four-period models.

The dynamic responses of interest rate and output are presented in Figure 10 and 11. The solid blue lines present the results of the baseline two-period model, while the dashed red lines and dash-dotted green lines show the results of the three- and four-period models respectively. It is important to note that in the third model, the ZLB is not binding after the first period, even though it looks like binding.

It is not surprising that the more sudden the change in the credit market, the more
Figure 9: Transition path of the shock to the credit market condition, $\xi_t$

Figure 10: Dynamics of the nominal interest rate
severe the recession. With the same 1% permanent decrease in the credit market condition, the baseline two-period model generates more output decline compared to the other two cases.

Interestingly, the recession becomes worse over time as long as the ZLB binds, as seen in the case of the three-period model. In this case, output falls by around \(-2.5\%\) in the first period, then falls more to around \(-4.2\%\) in the second period. Note that in this case, the ZLB binds in both periods.

5 Conclusion

I have already shown that including overborrowing and endogenizing the debt limit can generate a powerful mechanism that transmits a credit shock to a deep recession. With these features, the ZLB is more likely to bind in the housing model than in the EK model. When it binds, a great recession emerges, mostly due to the reinforcement between the
increase in the real debt burden faced by credit-constrained households and the fall of
the endogenous debt limit due to deleveraging.

There are several directions in which to extend the paper. It would be interesting
to extend the model by allowing for occasionally binding ZLB instead of assuming per-
fect foresight as done in this paper. We can also investigate different monetary policy
regimes instead of utilizing a simple Taylor rule. By doing so, we might answer how
monetary policy would be conducted optimally in this framework. Eventually, it would
be interesting to see what kind of fiscal policy would be best in cases where the ZLB
binds.

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6 Appendices

6.1 Equilibrium equations

We obtain the system of equations governing the equilibrium of the model as follows:

\[ C_{bt} + D_{bt-1} \left( \frac{1 + i_{t-1}}{1 + \pi_t} \right) + q_t (H_{bt} - H_{bt-1}) - w_t N_{bt} - D_{bt} = 0 \]  \hspace{1cm} (40)

\[ \frac{\eta_b N_{bt}^s}{C_{bt}^{-\gamma}} - (1 - \tau) w_t = 0 \]  \hspace{1cm} (41)

\[ C_{bt}^{-\gamma} - \phi_{bt} (1 + i_t) E_t \left[ \frac{1}{1 + \pi_{t+1}} \right] - \beta_b (1 + i_t) E_t \left[ \frac{C_{bt+1}^{-\gamma}}{1 + \pi_{t+1}} \right] = 0 \]  \hspace{1cm} (42)

\[ j_b H_{bt}^{-\psi} + \xi_t \phi_{bt} E_t [q_{t+1}] + \beta_b E_t [C_{bt+1}^{-\gamma} q_{t+1}] - C_{bt}^{-\gamma} q_t = 0 \]  \hspace{1cm} (43)

\[ \max \left\{ -\xi_t H_{bt} E_t [q_{t+1}] + D_{bt} (1 + i_t) E_t \left( \frac{1}{1 + \pi_{t+1}} \right), 0 - \phi_{bt} \right\} = 0 \]  \hspace{1cm} (44)

\[ C_{st} \frac{\chi_b D_{bt-1}}{\chi_s} \left( \frac{1 + i_{t-1}}{1 + \pi_t} \right) - q_t \frac{\chi_b}{\chi_s} (H_{bt} - H_{bt-1}) - \frac{1}{\chi_s} (Y_t - w_t \chi_b N_{bt} - \frac{\phi}{2} (\Pi_t - 1)^2 Y_t) + \frac{\chi_b D_{bt}}{\chi_s} = 0 \]  \hspace{1cm} (45)

\[ \frac{\eta_s N_{st}^s}{C_{st}^{-\gamma}} - (1 - \tau) w_t = 0 \]  \hspace{1cm} (46)

\[ C_{st}^{-\gamma} - \beta_s (1 + i_t) E_t \left[ \frac{C_{st+1}^{-\gamma}}{1 + \pi_{t+1}} \right] = 0 \]  \hspace{1cm} (47)

\[ j_s \left( \frac{H - \chi_b H_{bt}}{\chi_s} \right)^{-\psi} + \beta_s E_t [C_{st+1}^{-\gamma} q_{t+1}] - C_{st}^{-\gamma} q_t = 0 \]  \hspace{1cm} (48)

\[ \left( 1 - \varepsilon + \varepsilon w_t \frac{\Pi_t}{\Pi_s} - \varphi \pi_t (1 + \pi_t) \right) Y_t + \varphi \beta_s C_{st}^{-\gamma} E_t \left[ \frac{\pi_{t+1} (1 + \pi_{t+1}) Y_{t+1}}{C_{st+1}^{-\gamma}} \right] = 0 \]  \hspace{1cm} (49)

\[ \chi_b N_{bt} + \chi_s N_{st} - Y_t = 0 \]  \hspace{1cm} (50)

\[ \max \left\{ (1 + i) \left( \frac{1 + i_{t-1}}{1 + i} \right) \phi_t \left( \frac{1 + \pi_t}{1 + \pi} \right) \phi_s - (1 + i_t), 0 - i_t \right\} = 0 \]  \hspace{1cm} (51)
6.2 Steady state

\[ C_b = wN_b - rD_b \]

\[ \frac{\eta_b N_b^\phi}{C_b^{-\gamma}} = (1 - \tau) w \]

\[ C_b^{-\gamma} - \phi_b R - \beta_b C_b^{-\gamma} R = 0 \]

\[ j_b H_b^{-\psi} + \xi \phi_b q + \beta_b C_b^{-\gamma} q = C_b^{-\gamma} q \]

\[ \xi q H_b - D_b R = 0 \]

\[ C_s = \frac{1}{\chi_s} (Y - \chi_b w N_b) + \frac{\chi_b}{\chi_s} r D_b \]

\[ \frac{\eta_s N_s^\phi}{C_s^{-\gamma}} = (1 - \tau) w \]

\[ R = \frac{1}{\beta_s} \]

\[ j_s \left( \frac{H - \chi_b H_b}{\chi_s} \right)^{-\psi} + \beta_s C_s^{-\gamma} q = C_s^{-\gamma} q \]

\[ w = \frac{\varepsilon - 1}{\varepsilon} \]

\[ \chi_b N_b + \chi_s N_s = Y \]

or

\[ \Pi = 1 \]

\[ N_b = N_s = N^* = 1 = Y \]

\[ R = \frac{1}{\beta_s} \]

\[ w = \frac{\varepsilon - 1}{\varepsilon} \]

Auxiliary equations:

\[ \frac{\eta_b N_b^\phi}{(1 - \tau) w} = C_b^{-\gamma} \]
\[
\frac{\eta_s N_s^\phi}{(1 - \tau) w} = C_s^{-\gamma}
\]
\[
\frac{\eta_s}{\eta_b} = \left(\frac{C_s}{C_b}\right)^{-\gamma}, \text{ calibrate } \eta_s, \eta_b \text{ accordingly}
\]
\[
\phi_b = C_b^{-\gamma}(\beta_s - \beta_b)
\]
\[
D_b = \frac{\xi_q H_b}{R}
\]
\[
C_b = w N_b - (1 - \beta_s) \xi q H_b
\]
\[
C_s = \frac{1}{\chi_s} Y (1 - \chi_b w) + \frac{\chi_b}{\chi_s} (1 - \beta_s) \xi q H_b
\]

Core equations:
\[
j_b H_b^{-\psi} = C_b^{-\gamma} q (1 - \xi (\beta_s - \beta_b) - \beta_b)
\]
\[
\left(\frac{H - \chi_b H_b}{\chi_s}\right)^{-\psi} = \frac{C_s^{-\gamma} q (1 - \beta_s)}{J_s}
\]

Note that we calibrate \(\xi, j_b\) and \(j_s\) to match the total debt to income ratio of 3.2, such as Mian and Sufi [2011], to match the total housing asset to income ratio of about 4, as in the flow of funds table, and to match the saver’s housing to borrower’s housing ratio of 1.

\[
\xi = \frac{(\chi_b D_b) R}{\chi_b \left(\frac{q H}{Y}\right) \left(\frac{H_b}{H}\right)}
\]

where

\[
\frac{H_b}{H} = \frac{1}{\chi_b + \chi_s \left(\frac{H}{H_b}\right)}
\]