Household Leverage, Housing Markets, and Macroeconomic Fluctuations

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Abstract

This paper examines implications of housing and household leverage on macroeconomic fluctuations under credit shocks in the presence of the zero lower bound (ZLB) on nominal interest rates. To this end, I build a housing model by incorporating houses into a standard deleveraging model and allow debt limits to be endogenous. I find that, under an adverse credit shock, only with high leverage can the housing model generate more macroeconomic fluctuations with the nominal interest rate being more likely to hit the ZLB, compared to the standard deleveraging model without housing. In addition, the relative amplification is more pronounced under a shock that causes the ZLB to bind in both the models. Importantly, the ZLB plays a key role in generating a significant decline in the housing price under a particularly adverse credit shock.

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Keywords: Household leverage, Housing markets, House prices, ZLB, Deleveraging, Credit shocks.

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1. Introduction

Mian and Sufi (2011) document two striking stylized facts from the last recession. First, there was a surge in household leverage, defined as a debt-to-income ratio, during the 2002-2006 period. Second, the recession was worse and housing prices fell more in regions where household leverage had increased more. In addition, the nominal interest rate reached the zero lower bound (ZLB) in December 2008, worsening the recession because conventional monetary policy became ineffective in reducing short-term nominal interest rates to stimulate the economy.

Apparently, the household leverage, the housing market, and the ZLB played an important role in causing the worst recession that the U.S. has ever observed since the Great Depression. However, the standard deleveraging and ZLB literature that models debt limits exogenously, including Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), has no implications of housing, household leverage, and the ZLB on asset prices and macroeconomic fluctuations under a credit shock.

This paper aims to fill the gap. Specifically, I am going to answer the following questions: How does the economy respond to a credit shock in a deleveraging model with housing and endogenous debt limits? How different is the response compared to that from the the standard deleveraging model without housing? How sensitive is the response with respect to household leverage? Will the nominal interest rate hit the ZLB more or less frequently in the housing model, compared to the model without housing? Is the housing model able to amplify house prices under a credit shock? If yes, under what condition?

To this end, I extend the standard deleveraging and ZLB model by incorporating houses and allowing debt limits to be endogenous. Specifically, the debt limit of borrowers is tied to both exogenous credit market conditions and the endogenous market value of collateral assets, which are houses. To see the impact of household leverage, I then consider two cases, high leverage and normal leverage, with more focus on the former. In the case of high leverage, the housing model is calibrated to match the high debt-to-income ratio in the U.S. at the onset of the last recession. In the case of normal leverage, the model is calibrated to match the normal debt-to-income ratio in 2002, right before the housing boom.

Solving the models with and without housing, I am able to show that, under an adverse credit shock, only with high leverage can the housing model amplify macroeconomic fluctuations with the nominal interest rate being more likely to hit the ZLB, compared to the standard deleveraging model without housing. This finding contradicts the widely held belief that incorporating houses and allowing endogenous debt limits will *always* amplify output and inflation fluctuations under an adverse credit shock because people believe that cutting housing goods under the initial shock would cause output and the debt limit to fall more, creating another round of deleveraging and so on.

The intuition for the results is as follows: An initial adverse shock to the credit market lowers the debt limit and makes the borrowing constraint tighter, given the other factors, so borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back more necessary non-durable goods. So the interest rate declines less in the housing model than in the model without housing. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially.

However, in a world with high leverage, the initial debt-to-value ratio is very high. The home equity of borrowers is substantially low, even negative. Therefore, selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility and adjusting houses is costly, the borrowers do not want to cut back their durable housing goods. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut back.

In both cases, with normal leverage and with high leverage, we see the debt level declines due to two reasons. First, the adverse shock to the credit market lowers the debt limit and tightens the borrowing constraint. Second, the initial decline in the debt limit will lead to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable goods consumption and the debt limit. However, only in the case of high leverage can the housing model generate a tighter borrowing constraint and a larger cut in the nominal interest rate, compared to the model without housing.

When the ZLB binds in the two models, macroeconomic fluctuations are more pronounced in the housing model with high leverage than in the model without housing. Apparently, in this case, housing is very important factor in transmitting the credit shock and amplifying macroeconomic fluctuations. As explained previously, in the case of high leverage, selling houses does more harm than good because it induces the budget and collateral constraints of borrowers to be even tighter. Therefore, it makes the ZLB problem more serious.

Another important contribution of this paper is about the role of a binding ZLB condition in amplifying housing price fluctuations under a particularly adverse credit shock. Without the presence of the ZLB, the credit shock is not able to generate significant declines in the housing price. This result is in line with the common finding in the existing literature of housing and macroeconomic fluctuations. The reason is that the housing price is the expected present discounted value of housing utility flows. The credit shock is not able to alter the flows significantly without the presence of the ZLB because the central bank has some power to stabilize the economy, including the borrowers' housing consumption and the marginal utility of houses. As a result, the housing price does not move much under a credit shock, see Liu et al. (2013) for more detailed discussion.

However, this is not the case when the ZLB is present. In a deep recession with binding ZLB, the housing price falls sharply. Specifically, under an adverse credit shock that causes the debt-to-value ratio to fall about 10 percent permanently, the decline in the housing price is three times greater in the model with the ZLB than in the model without the ZLB. Intuitively, due to the ZLB effect, the borrowers have to scale back their durable housing goods more, affecting the flow of housing services more. As a result, the housing price falls more when the ZLB condition is imposed.

The related literature on the ZLB has been inspired by the seminal work by Krugman (1998). After his work, extensive research related to the ZLB has been conducted, including Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) among others.[1].

Guerrieri and Lorenzoni (2011) model a debt limit and household heterogeneity in labor productivity. They show that a decline in the exogenous debt limit causes future consumption to be more volatile because with a lower debt limit, it will be more difficulty for households to insure their consumption risks. Therefore, savers will save more and borrowers will borrow less due to precautionary savings, resulting in a sharp decrease in the nominal interest rate and a binding ZLB.

Eggertsson and Krugman (2012) also model the debt limit and deleveraging as

[1]A more extensive list includes Eggertsson and Woodford (2003), Adam and Billi (2006, 2007), Nakov (2008), Levin et al. (2010), Bodenstein et al. (2010), Werning (2011), Ngo (2014), Fernandez-Villaverde et al. (2012), Hall (2011), and Judd et al. (2011).

a key factor driving the nominal interest rate to the ZLB. In contrast to Guerrieri and Lorenzoni, where savings come from precautionary behavior, Eggertsson and Krugman model savings based on the difference in the two types of representative households. One type is patient; the other is not. The patient representative household saves and lends his money to the impatient one. Like in Guerrieri and Lorenzoni (2011), the debt limit is exogenous in Eggertsson and Krugman (2012).

There are several recent independent working papers that incorporate and study housing in a general equilibrium model. Among these papers, Justiano et al. (2014) and Guerrieri and Iacoviello (2014) are closer to this paper. They also extend the standard deleveraging model by tying the debt limit to both the credit market condition (or the loan-to-value ratio) and the market value of collateral housing assets.

Like this paper, Justiano et al. (2014) also cast doubt on whether credit shocks could account for the behavior of house prices and mortgage debt during the housing boom and burst periods. However, they do not investigate the implication of different levels of household leverage. More importantly, they do not examine the impact of the ZLB on house prices because the ZLB is not binding in their paper.

Guerrieri and Iacoviello (2014) also study the role of housing and collateral constraints on generating asymmetric macroeconomic fluctuations in a DSGE model where debt limits are endogenously linked to the market value of houses. They show that a negative housing demand shock can push the economy into the ZLB and can generate large declines in inflation and output. Apparently, both their paper and this paper take into account the ZLB. However, the main difference between the two papers is that their paper focuses on housing demand shock whereas this paper focuses on the shock to the credit market, or the debt-to-value ratio shock.

Recent work by Midrigan and Philippon (2011) and Favilukas et al. (2013) also incorporates housing into a DSGE model with credit shocks. However, this paper is more focused on whether the housing margin amplifies or dampens credit shocks in the presence of the ZLB. In their papers, the ZLB is not present.

2. Model

The model in this paper is a standard two-representative agent model, as found in Eggertsson and Krugman (2012) and Iacoviello (2005). There are two types of households: credit-constrained households (or borrowers) of mass χ_b, and unconstrained households (or savers) of mass $\chi_s = 1 - \chi_b$. The borrowers are impatient while the savers are patient and act as the lenders. The households consume nondurable goods and enjoy housing goods that have a fixed supply.

Houses play two roles in the model. First, they provide housing services to the households. Second, they can be used as collateral assets for borrowing. One of the two key features in our model is an endogenous debt limit that is determined by both the endogenous market value of houses and exogenous financial market conditions (determined by a debt-to-value ratio), as in Kiyotaki and Moore (1997) and Iacoviello (2005).

2.1. The borrowing-constrained household's problem

The representative borrowing-constrained household chooses the path of nondurable goods, durable housing goods, real debts, and labor to maximize his expected present discounted lifetime utility subject to his budget constraint and borrowing constraint. His problem can be described mathematically as follows:

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta_b^t U_{bt}(C_{bt}, H_{bt}, N_{bt}) \tag{1}$$

subject to the budget constraint

$$C_{bt}+D_{bt-1}\left(1+r_{t-1}\right)+q_t\left(H_{bt}-H_{bt-1}\right)+\frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}}-1\right)^2 q_t H_{bt}=w_t N_{bt}+D_{bt} \quad (2)$$

and the borrowing constraint

$$D_{bt}\left(1+r_t\right)\leq \xi_t E_t\left[q_{t+1}H_{bt}\right], \quad (3)$$

where

$$1+r_t=\frac{1+i_t}{1+\pi_{t+1}}; \quad (4)$$

U is per-period utility; C_b, D_b, H_b, N_b are composite non-housing goods, real debts, housing quantity, and labor supply by the borrower respectively; i, π, r, q denote the nominal interest rate, the inflation rate, the real interest rate, and the real price of a house, respectively; ϕ_H is the housing adjustment cost parameter, showing how costly it is to adjust houses; and ξ reflects the credit market conditions. The credit market shock follows an AR(1) process:

$$\ln\left(\xi_{t+1}\right)=\left(1-\rho_\xi\right)\ln\left(\overline{\xi}\right)+\rho_\xi\ln\left(\xi_t\right)+\varepsilon_{\xi,t+1} \quad (5)$$

where $\varepsilon_{\xi,t}$ is is independently and identically distributed with mean 0 and variance σ_ε^2; ρ_ξ presents the persistence of the credit shock. In this paper, I will investigate the case of permanent shocks, where $\rho_\xi = 1$.

Because the credit market condition ξ is a very important parameter, I would like to clarify two issues that could potentially arise. First, I interpret the parameter as a debt-to-market value of collateral assets ratio, or debt-to-value (DTV) ratio. The debt-to-value ratio can be interpreted as a cumulative loan-to-value ratio instead of a normal loan-to-value ratio. Using a cumulative loan-to-value ratio allows us to capture high leverage, measured as the high debt-to-income ratio that we observed at the onset of the recent housing bust.

Second, I am not going to model why the parameter exists and why it is too high at some point. The rationale for the existence of the parameter could be an asymmetric information problem, and the rationale for why it is too high at some point comes from lenders' over-optimism about the likelihood of getting their money back. This over-optimism is grounded on an extended period of steady economic growth and/or rising asset prices, such as housing prices.

From Equations (3) and (4), the borrowing constraint can be rewritten as $D_{bt} \leq \xi_t E_t \left[q_{t+1} H_{bt}(1+\pi_{t+1})/(1+i_t)\right]$, which is similar to Equation (17) of Iacoviello (2005) that describes the endogenous borrowing constraint for impatient households. According to this equation, the debt limit is $D_{bt}^{limit} = \xi_t E_t \left[q_{t+1} H_{bt}(1+\pi_{t+1})/(1+i_t)\right]$. Apparently, the debt limit is a proportion ξ_t, which is exogenous, of the expected market value of collateral assets $E_t[q_{t+1} H_{bt}]$ discounted by $(1-r_t)$, which is endogenously determined in the model.[2]

This endogenous debt limit distinguishes this paper from the current literature of deleveraging and the ZLB, including Eggertsson and Krugman (2012), where the debt limit is exogenous. In their model, the deleveraging shock occurs when there is an exogenous downward revision of the debt limit due to a change in lenders' point of view toward the risk of borrowers or toward the collateral asset values. The sudden downward revision is called the Minsky moment.

In reality, it is hard to imagine how the debt limit is exogenously determined

[2]Although the debt-to-value ratio, ξ_t, is not endogenous as in Bernanke et al. (1999) or Carlstrom and Fuerst (1997), the debt limit is still endogenous because it depends on the market value of houses and the real interest rate that are endogenous in the model.

by lenders because most loans in the U.S. are actually collateralized debts. What is reasonably exogenous is the debt-to-value ratio, ξ_t. An exogenous change in the ratio can trigger deleveraging that could potentially affect the market value of collateral assets, leading to another round of deleveraging, and so on. In this paper, the time when a sudden change in the ratio ξ_t happens is considered as a Minsky moment.

Let λ_{bt}, ϕ_{bt} be the Lagrange multipliers with respect to the borrower's budget constraint and debt constraint. The optimal choices must satisfy the following conditions:

$$U_{bt,C} - \lambda_{bt} = 0, \tag{6}$$

$$\frac{-U_{bt,N}}{U_{bt,C}} = w_t, \tag{7}$$

$$\lambda_{bt} - \phi_{bt} E_t \left[1 + r_t\right] = \beta_b E_t \left[\lambda_{bt+1} \left(1 + r_t\right)\right], \tag{8}$$

$$\begin{aligned} & U_{bt,H} + \xi_t \phi_{bt} E_t \left[q_{t+1}\right] + \beta_b E_t \left[\lambda_{bt+1} q_{t+1} + \lambda_{bt+1} q_{t+1} \phi_H \left(\frac{H_{bt+1}}{H_{bt}} - 1\right) \left(\frac{H_{bt+1}}{H_{bt}}\right)^2\right] \\ = \; & \lambda_{bt} q_t + \lambda_{bt} q_t \frac{\phi_H}{2} \left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 + \lambda_{bt} q_t \phi_H \left(\frac{H_{bt}}{H_{bt-1}} - 1\right) \frac{H_{bt}}{H_{bt-1}}, \end{aligned} \tag{9}$$

$$\min \left\{\xi_t E_t \left[q_{t+1} H_{bt}\right] - D_{bt} \left(1 + r_t\right), \phi_{b,t}\right\} = 0, \tag{10}$$

$$\phi_{bt} \geq 0. \tag{11}$$

Equation (6) shows the marginal utility derived from consuming the composite non-durable goods. Equation (7) presents the intra-temporal trade-off between consumption and labor at the margin. Equation (8) is the Euler equation for the borrower, which is the inter-temporal trade-off between today's consumption and tomorrow's consumption. If the credit-constrained household consumes one unit of non-durable goods, he would receive utility from his consumption. In addition, he would put $(1 + r_t)$ more pressure on the collateral constraint that costs him $\phi_{bt} (1 + r_t)$ in terms of utility. Therefore, the left-hand side of the equation is the marginal benefit of consuming today, while the right-hand side is the marginal utility he has to forgo due to not saving.

The marginal trade-off between non-durable goods and durable housing goods is illustrated in equation (9). The left-hand side of the equation shows the marginal benefit of buying one more unit of houses. The marginal benefit includes: (i) housing services; (ii) the value of the debt limit that he would get by relaxing the collateral constraint due to owning more houses; and (iii) the next period's value of the houses in terms of utility and the housing adjustment cost saved due to having more houses today. The right-hand side of the equation is the marginal cost of buying houses. The borrowing constraint is rewritten as in equation (10). This equation is the combination of the collateral constraint and the non-negativity of the shadow value of debt, ϕ_{bt}.

2.2. The unconstrained household's problem

The representative unconstrained household never faces a borrowing constraint. He saves and lends to the credit-constrained households. He also owns intermediate-goods firms. His problem is as follows:

$$\max \quad E \sum_{t=0}^{\infty} \beta_s^t U_{st} \left(C_{st}, H_{st}, N_{st} \right) \tag{12}$$

subject to the budget constraint

$$\begin{aligned}
& C_{st} + D_{st-1}\left(1+r_{t-1}\right) + q_t\left(H_{st} - H_{st-1}\right) \\
& + \frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}} - 1\right)^2 q_t H_{st} = w_t N_{st} + \int_{i=0}^{1} Z_{it} di + D_{st},
\end{aligned} \tag{13}$$

where U is per-period utility; C_s, D_s, H_s, N_s are composite non-housing goods, real debts, housing quantity, and labor supply by the saver respectively; i, π, r, q denote the nominal interest rate, the inflation rate, the real interest rate, and the real price of a house, respectively; ϕ_H is the housing adjustment cost parameter, showing how costly it is to adjust houses; and Z denotes nominal profits from the i^{th} intermediate-goods firms that are owned by the savers only.

Let λ_{st} be the Lagrange multiplier with respect to the budget constraint of the saver. The optimal choices of the saver must satisfy the following condition:

$$U_{st,C} - \lambda_{st} = 0, \tag{14}$$

$$\frac{-U_{st,N}}{U_{st,C}} = w_t, \tag{15}$$

$$\lambda_{st} - \beta_s E_t\left[\lambda_{st+1}\left(1+r_t\right)\right] = 0, \tag{16}$$

$$\begin{aligned}
& U_{st,H} + \beta_s E_t\left[\lambda_{st+1} q_{t+1} + \lambda_{st+1} q_{t+1} \phi_H \left(\frac{H_{st+1}}{H_{st}} - 1\right)\left(\frac{H_{st+1}}{H_{st}}\right)^2\right] \\
= \; & \lambda_{st} q_t + \lambda_{st} q_t \frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}} - 1\right)^2 + \lambda_{st} q_t \phi_H \left(\frac{H_{st}}{H_{st-1}} - 1\right)\frac{H_{st}}{H_{st-1}}.
\end{aligned} \tag{17}$$

Equation (14) shows the saver's marginal utility derived from non-durable goods consumption. Equation (15) presents his marginal trade-off between consumption and labor. Equation (16) is the Euler equation for the saver, which is the intertemporal trade-off between today's consumption and tomorrow's consumption. The marginal trade-off between non-durable goods consumption and housing goods is illustrated in equation (17).

2.3. Final goods producers

There is a mass 1 of final goods producers who operate in a perfectly competitive market. Each final goods producer produces the consumption goods by aggregating a variety of differentiated goods using a CES technology. His problem is to maximize his contemporaneous profit:

$$\max P_t Y_t - \int P_t(i) Y_t(i) \, di \tag{18}$$

subject to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{19}$$

where y_{it} is the input of intermediate goods $i \in [0,1]$ and ε is the elasticity of substitution between differentiated goods.

The optimal decision of the final goods producer gives rise to the demand for the i^{th} intermediate goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{20}$$

and the price level:

$$P_t = \left(\int P_t(i)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}. \tag{21}$$

2.4. Intermediate goods producers

There is a mass 1 of intermediate goods firms, which are owned by the savers and are operated in a monopolistically competitive market. The firms faces costs of adjusting prices, as in Rotemberg (1982). The problem of firm i is given below:

$$\max_{P_{it},N_{it}} E_t^i \sum_{j=0}^{\infty} Q_{st,t+j} Z_{it+j} \tag{22}$$

subject to its demand function (Eq.20), profit

$$Z_{it} = \frac{P_{it}}{P_t} Y_{it} - w_t N_{it} - \frac{\phi_P}{2} \left(\frac{P_{it}}{P_{it-1}} - (1 + \theta \pi^*) \right)^2 Y_t, \tag{23}$$

and a linear production technology

$$Y_{it} = N_{it}, \tag{24}$$

where π^* is the target inflation; θ is the degree of price indexation; and ϕ_P is the price adjustment cost parameter which determines the degree of nominal price rigidity.

In a symmetric equilibrium, the optimality conditions give rise to a nonlinear version of the New Keynesian Phillips Curve

$$\begin{aligned} &\left(1 - \varepsilon + \varepsilon \frac{w_t}{A_t} - \phi_P \left(\pi_t - \theta \pi^*\right) \left(1 + \pi_t\right)\right) Y_t \\ &+ \phi_P Q_{st,t+1} E_t \left[\left(\pi_{t+1} - \theta \pi^*\right) \left(1 + \pi_{t+1}\right) Y_{t+1} \right] = 0, \end{aligned} \tag{25}$$

where the stochastic discount factor

$$Q_{st,t+1} = \beta_s \frac{U_{st+1,C}}{U_{st,C}}. \tag{26}$$

2.5. The central bank

The central bank conducts monetary policy using a simple Taylor rule

$$\left(\frac{1+i_t}{1+i}\right)=\left(\frac{Y_t}{Y^*}\right)^{\phi_y}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi} \tag{27}$$

subject to the ZLB condition

$$i_t \geq 0, \tag{28}$$

where π^* and Y^* are the target inflation and output respectively. The ZLB is a key constraint in this model. It distinguishes this paper from the existing housing literature with endogenous borrowing constraints, such as Iacoviello (2005).

2.6. Equilibrium

In equilibrium, all the markets are cleared:

$$\chi_b H_{bt} + \chi_s H_{st} = \overline{H}, \tag{29}$$

$$\chi_b N_{bt} + \chi_s N_{st} = N_t, \tag{30}$$

$$\chi_b D_{bt} + \chi_s D_{st} = 0, \tag{31}$$

$$\begin{aligned}&\chi_b C_{bt} + \chi_s C_{st} + \chi_b \frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}}-1\right)^2 q_t H_{bt} + \chi_s \frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}}-1\right)^2 q_t H_{st}\\ &= (A_t N_t)\left(1-\frac{\phi_P}{2}\left(\pi_t-\theta\pi^*\right)^2\right).\end{aligned} \tag{32}$$

Equation (29) shows that the total demand for houses equals the total fixed housing supply. Equations (30), (31), and (32) present the market clearing conditions for labor, debts, and the non-housing composite goods markets respectively.

Equilibrium definition: An equilibrium consists of the path of allocation $\{C_{bt}, N_{bt}, D_{bt}, H_{bt}, C_{st}, N_{st}, D_{st}, H_{st}, N_t, Y_t, \phi_b\}_{t=0}^{\infty}$ and prices $\{i_t, w_t, \pi_t, q_t\}_{t=0}^{\infty}$ that satisfies: (i) the borrowers' and savers' optimality conditions; (ii) the firms' optimality conditions; (iii) the market clearing conditions; (iv) the Taylor rule and the ZLB; and (v) the motion equation for the credit shock.[3]

3. Calibration

The per-period utility functions for the borrower and saver are specified as below:

$$U_{bt} = \frac{C_{bt}^{1-\gamma_b}}{1-\gamma_b} + j_b \frac{H_{bt}^{1-\psi_b}}{1-\psi_b} - \eta_b \frac{N_{bt}^{1+\phi}}{1+\phi}$$
$$U_{st} = \frac{C_{st}^{1-\gamma_s}}{1-\gamma_s} + j_s \frac{H_{st}^{1-\psi_s}}{1-\psi_s} - \eta_s \frac{N_{st}^{1+\phi}}{1+\phi}$$

I calibrate the parameters based on the observed data in the U.S. and on the other studies. The key parameters are presented in Table 1. The subjective discount factor for the savers, β_s, is 0.99, corresponding to the real interest rate of 4% per year. The subjective discount factor for the borrower, β_b, is 0.96. The fraction of borrowers, χ_b, is 0.60, corresponding to the 60% of households that are borrowing constrained. These numbers are close to the values reported by empirical studies, and they are in the range used in the existing literature of housing and macroeconomic fluctuations, see Iacoviello (2005) for more detailed discussion.

The constant relative risk aversion parameters for the borrower and the saver are

[3] See Appendix A for the system of nonlinear difference equations governing the equilibrium of the model.

Table 1: Benchmark Parameterization

Symbol	Description	Value
χ_b	Fraction of borrowers	0.60
χ_s	Fraction of savers	0.40
β_s	Subjective discount of savers	0.99
β_b	Subjective discount of borrowers	0.96
$\overline{H}$	Fixed stock of housing supply	1
γ_b, γ_s	CRRA parameters	1
ψ_b, ψ_s	Housing utility parameters	1
$\bar{\xi}_{high}$	Debt-to-value (DTV) ratio, the case of high leverage	0.91
$\bar{\xi}_{normal}$	Debt-to-value (DTV) ratio, the case of normal leverage	0.68
j_s	Borrower's housing utility parameter	0.02
j_b	Saver's housing utility parameter	0.03
η_b	Borrowers' labor disutility parameter	1.02
η_s	Savers' labor disutility parameter	0.86
ϕ	Inverse labor supply elasticity	1/2
ϕ_P	Price adjustment cost parameter	168
ε	Elasticity of substitution among differentiated goods	20
ϕ_H	Housing adjustment cost parameter	0.1
θ	Inflation indexation parameter	0
π^*	Target inflation rate of 2% per year	0.005
ϕ_π	Weight of inflation in the Taylor rule	1.5
ϕ_y	Weight of output in the Taylor rule	0.5

calibrated to be 1, or γ_b=$\gamma_s = 1$, corresponding to log utility function with respect to composite nondurable consumption goods. The housing demand elasticities for the borrower and the saver are 1, or $\psi_b = \psi_s = 1$. These parameters are commonly used in the existing literature, see Iacoviello (2005). The labor supply elasticity with respect to wages is chosen to be 2, or $\phi = 1/2$, which is in the range commonly used in the literature of ZLB and deleveraging, such as Eggertsson and Krugman (2012).

The borrower labor disutility parameter, η_b, and the saver labor disutility parameter, η_s, are calibrated to match the steady state labor supply that is normalized to

1 for both the borrower and the saver. The fixed stock of housing supply, $\overline{H}$, is also normalized to 1. These parameters can change the scale of some steady state values, but they are almost irrelevant to the results of the paper.

I calibrate the initial debt-to-value ratio, $\bar{\xi}$, to match three facts: (i) the total debt to income ratio is 1.32 in the fourth quarter of 2006 - the onset of the housing burst.[4] The total debt is measured by the total liabilities of households and nonprofit organizations, while the total income is measured by the personal disposable income of households and nonprofit organization; (ii) the total housing wealth to income ratio is around 2.45. The total housing wealth is measured by the households and nonprofit organization real estate at market value. The values of total debt, income, and housing wealth are from the Flows of Funds Tables reported by the Federal Reserve System; and (iii) on average, each household owns a house, or H_b/H_s is 1.

These facts result in the debt-to-value ratio, $\bar{\xi}$, of around 0.91, which I consider to be high leverage. This value is smaller than 0.95, the maximal value of cumulative loan-to-value ratio for the first-time home buyers in the U.S. reported by Duca et al. (2011). However, it is larger than some values estimated in the existing literature, i.e. 0.89 in Iacoviello (2005). These facts are also used to uniquely determine the values of the housing utility parameters for the saver and the borrower, j_s and j_b.

In addition to using the high debt-to-income ratio of 1.32, I investigate the case when the debt-to-income ratio is 1, the value that we observed in the U.S. in the first quarter of 2002, right before the housing boom period. This lower value of the debt-to-income ratio results in the debt-to-value ratio, $\bar{\xi}$, or 0.68, which I consider to be normal leverage.

[4] I use the debt-to-income ratio instead of the debt-to-GDP ratio to measure the leverage because income captures the flow of wealth more precisely.

For the Taylor rule, I choose the inflation target to be 2 percent per year, or $\overline{\pi} = 0.005$. The weight of inflation, ϕ_π, and the weight of output, ϕ_y are set at the conventional values, 1.5 and 0.5, respectively. Note that the conventional value, $\phi_\pi = 1.5$, is used when the ZLB is not imposed. In the presence of the ZLB and in this framework, using this conventional value is likely to produce very unstable results if prices are very sticky, i.e. when the average duration of keeping prices unchanged is 4 quarters or longer.

The demand elasticity for differentiated goods is calibrated to be 20, corresponding to a net markup of 5%, which is similar to the value used in Iacoviello (2005). The benchmark inflation indexation parameter, θ, is set at 0.

The price adjustment cost parameter, ϕ_P, is calibrated to be 168, corresponding to keeping a price unchanged for 3 quarters on average.[5] The duration is in line with empirical studies, see Nakamura and Steinsson (2008) for more discussion. The housing adjustment cost parameter, ϕ_H, is set at 0.1, which corresponds to a realtor fee of 5% of the total transaction value for selling and buying a house of the size at the steady state. This realtor fee is also within the range used in the literature, see Duca et al. (2011).

4. Results

In this section, I first follow Eggertsson and Krugman (2012) and assume the timing of the model as follows: At time 0, the economy stands at an initial steady state associated with a certain debt-to-value ratio $\xi = \overline{\xi}$. Then, a permanent shock to the credit market occurs at time 1, so ξ changes to ξ'. The representative households choose new debts, housing quantities, nondurable consumption goods, and labor at

[5]See Miao and Ngo (2014) for how to calculate this duration.

time 1. The economy returns to the new steady state associated with the new value of the debt-to-value ratio, ξ', at time 2 and afterward.[6],[7]

In the last part of this section, I relax the timing assumption by allowing the economy to gradually go to the new steady state. In this case, I will plot the impulse response functions under a permanent shock that causes the debt-to-value ratio to decrease by 12% permanently.

In the case with the timing assumption, Eggertsson and Krugman (2012) call period 2 the long-run and they call period 1 the short-run. I will plot the short-run (or period 1) responses of selected economic variables for both the models with and without housing under different scenarios regarding the presence of the ZLB and the level of household leverage.

4.1. The case without the ZLB

To understand the role of household leverage I first provide the results from the case with high leverage and without the ZLB, as presented in Figure 1. The x-axis shows a permanent annualized percentage change in the credit market parameter (ξ) from the initial steady state value. The short-run responses of selected macro economic variables in period 1 are presented in the y-axis.

The solid blue lines present the results from the housing model with endogenous debt limit, while the dashed red lines show the results from the model without housing. The intersections between the solid blue lines and the dashed red lines present the initial steady state values. In the model without housing, the housing

[6] The reason for using the timing assumption is that I am able to compute and plot the impact response against different values of the credit shock without spending too much time on solving the models.

[7] Note that there are also several endogenous state variables in Eggertsson and Krugman (2012), including the borrower's debt and the real interest rate.

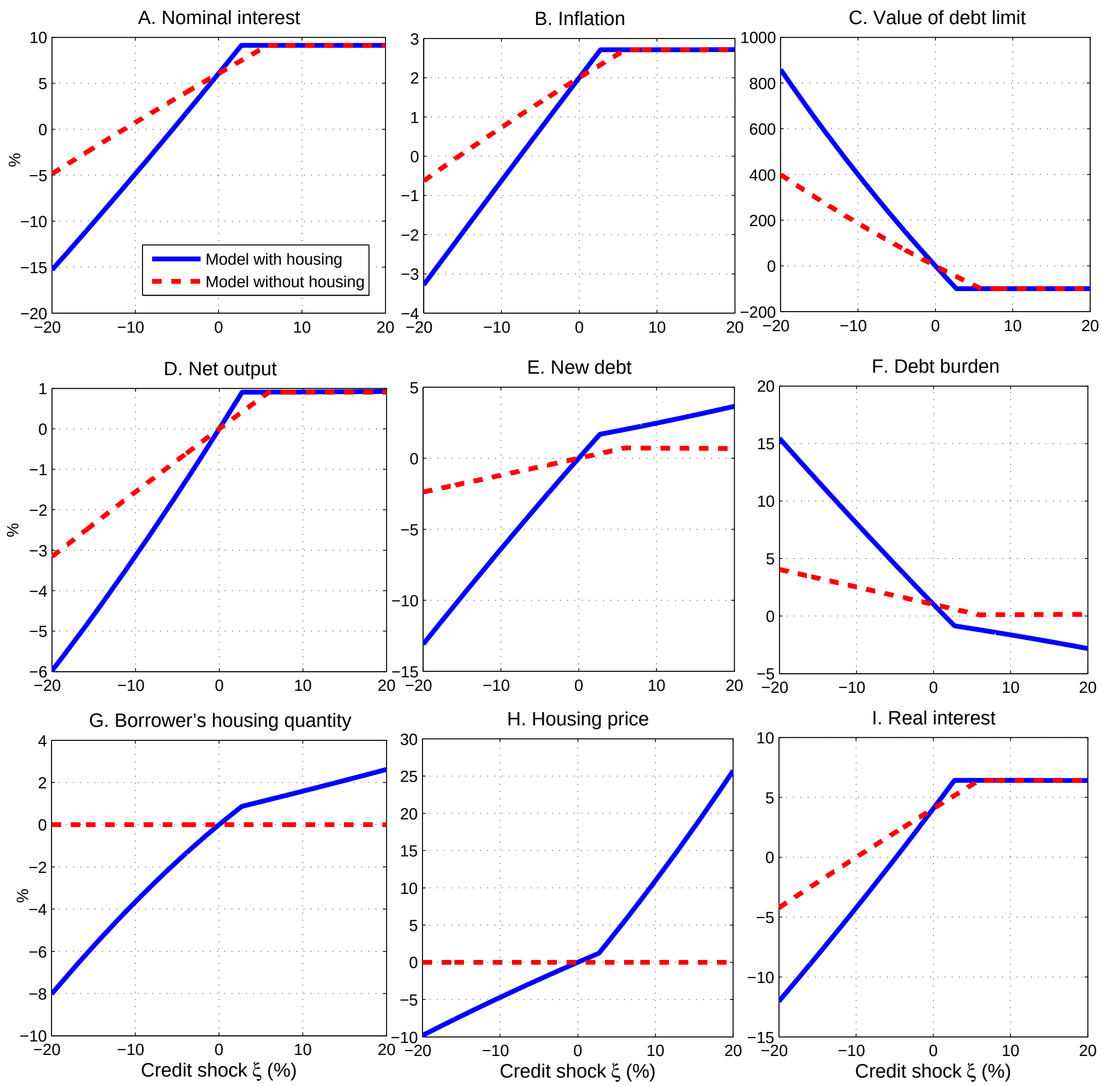

Figure 1: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with high leverage and without the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

price and housing quantities for both borrowers and savers are kept unchanged at the steady state.

Panels D, E, G, and H of Figure 1 show the percentage change of net output, new debt, housing quantity of the borrowers, and housing price from the initial steady state values, respectively. The net output is the total output minus the price adjustment cost. Panel F of Figure 1 shows debt burden as a percentage of the initial debt, where debt burden is computed as debt payment minus new debt. A decline in debt burden means that the borrowers are able to borrow more.

Under a positive shock to the debt-to-value ratio, from Figure 1 we see that, in the housing model, the value of debt limit and debt burden decline, while borrowers' new debt and new housing quantity, total output, inflation, and interest rates all increase in equilibrium. Intuitively, when the debt-to-value ratio increases initially, the debt limit faced by the borrowers increases, and they are allowed to borrow more given the other factors. As a result, the shadow value of the debt limit decreases and the borrowers borrow more. Hence, both borrowers' nondurable and durable housing consumptions increase.

The increase in the housing quantity leads to another increase in the debt limit, encouraging borrowers to borrow and spend more, and creating another round of expansion. The debt burden declines because the borrowers are not only able to roll over their debt, but are also able to borrow more. In the monopolistic framework, inflation rises when output increases. Under a simple Taylor rule, the nominal interest rate increases.

However, up to a certain value, a further increase in the debt-to-value ratio would not alter the responses of some macroeconomic variables. After that threshold, borrowers are allowed to borrow up to the amount they desire, and the credit constraint is no longer binding. Therefore, the shadow value of the debt limit is zero. The inflation and nominal interest rate hit the upper bounds of around 2.8% and 10.0% per year respectively.

The opposite mechanism occurs under a negative shock to the credit market. In this circumstance, borrowers are not able to borrow as much as before, given the other factors. The debt limit decreases, while the shadow value of the debt limit rises. Therefore, new debt falls and debt burden increases. As a result, the borrowers' nondurable and housing consumption fall due to deleveraging. Because the monetary policy is a simple Taylor rule, it is not powerful enough to stabilize output and inflation. Hence, output declines and, as a result, disinflation occurs. The responses are also amplified by the collateral effect and debt deflation effect.

It is not surprising that the model with housing generates more amplified responses of macroeconomic variables to a credit market shock compared to the model without housing. Let us look at the scenario under a negative shock. Panel A in Figure 1 shows that the nominal interest rate in the housing model falls more than in the model without housing. Although we do not study the impact of the ZLB in this section, it is worth noting that, compared to the model without housing, the nominal interest rate reaches zero and below more frequently in the housing model with high leverage. In other words, to drive the nominal interest rate to zero and below, it requires a smaller negative credit shock in the housing model than in the model without housing.

In addition, the housing model produces a bigger decline in inflation and output, as in Panels B and D of Figure 1. The borrowers suffer a tighter collateral constraint in the housing model than in the model without housing, as shown in Panel C where the shadow value of debt limit is higher in the housing model than in the model without housing.

The most interesting and important finding is that the more amplified transmission mechanism of the housing model can be found only when we allow for high leverage. This contradicts to the widely-held belief that allowing endogenous debt limits always amplifies macroeconomic fluctuations under a shock. As explained above, by high leverage, I calibrate the parameter $\overline{\xi}$ to match the high debt-to-income ratio at the onset of the housing bubble burst. The ratio is substantially high: around 1.32. Without the high leverage, we cannot generate such results.

To demonstrate the role of high leverage, I report the results from the case with normal leverage and without the ZLB in Figure 2. In the case of normal leverage, I calibrate the initial debt-to-value of collateral asset ratio to match with the debt-to-income ratio in the first quarter of 2002, right before the 2002-2006 period of housing and credit boom. As documented by the Federal Reserve System, in the Flows of Funds Tables, the debt-to-income ratio was around 1 in the first quarter of 2002, instead of 1.32 as in the benchmark calibration. This results in a lower debt-to-value ratio that is around 0.68, smaller than 0.91 as in the case of high leverage.

As in Panel A of Figure 2, under a negative demand shock, the decrease in the nominal interest rate in the housing model is not as big as in the model without housing. Compared to the model without housing, the nominal interest rate reaches zero and below less frequently. In other words, to drive the nominal interest rate to zero and below, it requires a larger credit shock in the housing model than in the model without housing.

Inflation and output fluctuate less in the housing model with normal leverage than in the model without housing. Although, we see a higher debt burden and a smaller new debt in the housing model, the shadow value of debt limit is actually smaller in this model compared to the model without housing. This means that the slackness of the collateral constraint is smaller in the housing model. The result contradicts to the common belief that the endogenous debt limit would always amplify macroeconomic

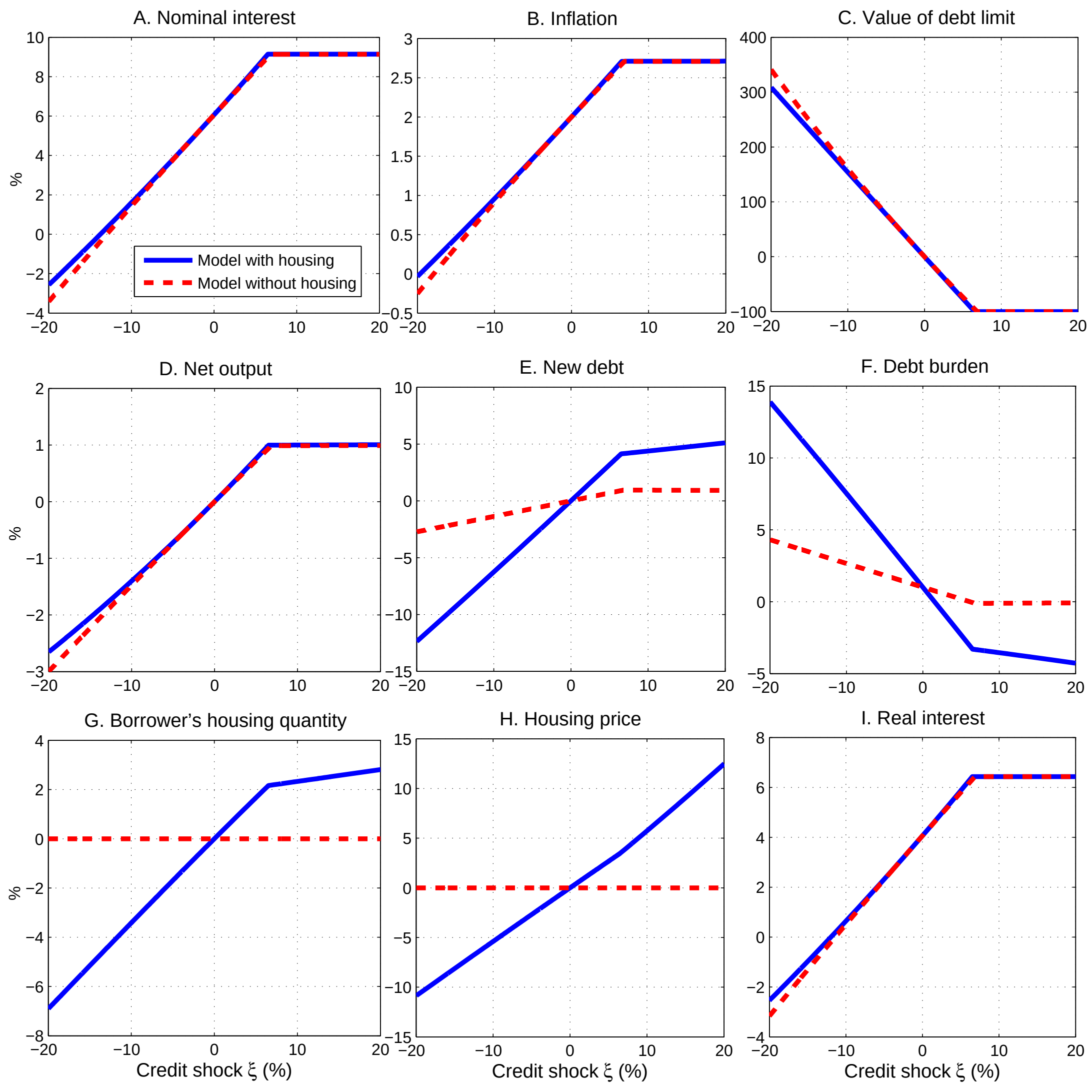

Figure 2: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with normal leverage and without the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

fluctuations under shocks.

The intuition for the results is as follows: An adverse shock to the credit market lowers the debt limit initially and makes the borrowing constraint tighter, given the other factors. Therefore, the borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back the more necessary non-durable goods. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially compared to the model without housing.

However, in a world with high leverage, the initial debt-to-value ratio is very high. The home equity of borrowers is substantially low, even negative. Therefore, selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility and adjusting houses is costly, the borrowers do not want to cut back their durable housing consumption. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut back.

In both cases, with leverage and with high leverage, we see the debt level decline due to two reasons. First, the adverse shock to the credit market lowers the debt limit initially and tightens the borrowing constraint. Second, the initial decline in the debt limit leads to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable goods consumption and the debt limit. However, only in the case of high leverage can the model generate a tighter borrowing constraint, compared to the standard model without housing.

4.2. The case with the ZLB

In this section, I study the impact of an adverse shock to the credit market in the case of high leverage and in the presence of the ZLB. The results are presented in Figure 3.

Given the fact that that housing and household leverage are key ingredients of the analysis and that comparison with the standard deleveraging model without housing is an important theme of this paper, we would like to see how the housing model with high leverage affects macroeconomic variables at the ZLB, relative to the model without housing. As seen in Panel A of Figure 3, it is not surprising that the nominal interest is more likely to hit the ZLB in the housing model with high leverage than in the model without housing. When the ZLB binds in both models, the housing model with high leverage generates more severe recessions than the model without housing does.[8]

Apparently, the housing model with high leverage amplifies macroeconomic fluctuations substantially. As explained previously, with high leverage, selling houses under an adverse credit shocks does more harm than good. Under a binding ZLB, the deleveraging by the borrower is even more pronounced due to the ineffectiveness of monetary policy. As a result, the borrower is forced to sell more houses because housing goods and consumption goods are complementary. The interaction between the ZLB channel and the housing channel causes the transmission mechanism more powerful in the housing model than in the standard model without housing.

The total output and inflation in the economy drop under a shock that causes the ZLB to bind, as seen in Panels B and D of Figure 3. The fall in output and inflation

[8] The housing model with normal leverage generates less severe recessions than the model without housing does. The intuition is as explained above. To save space, I do not report and discuss the case of normal leverage.

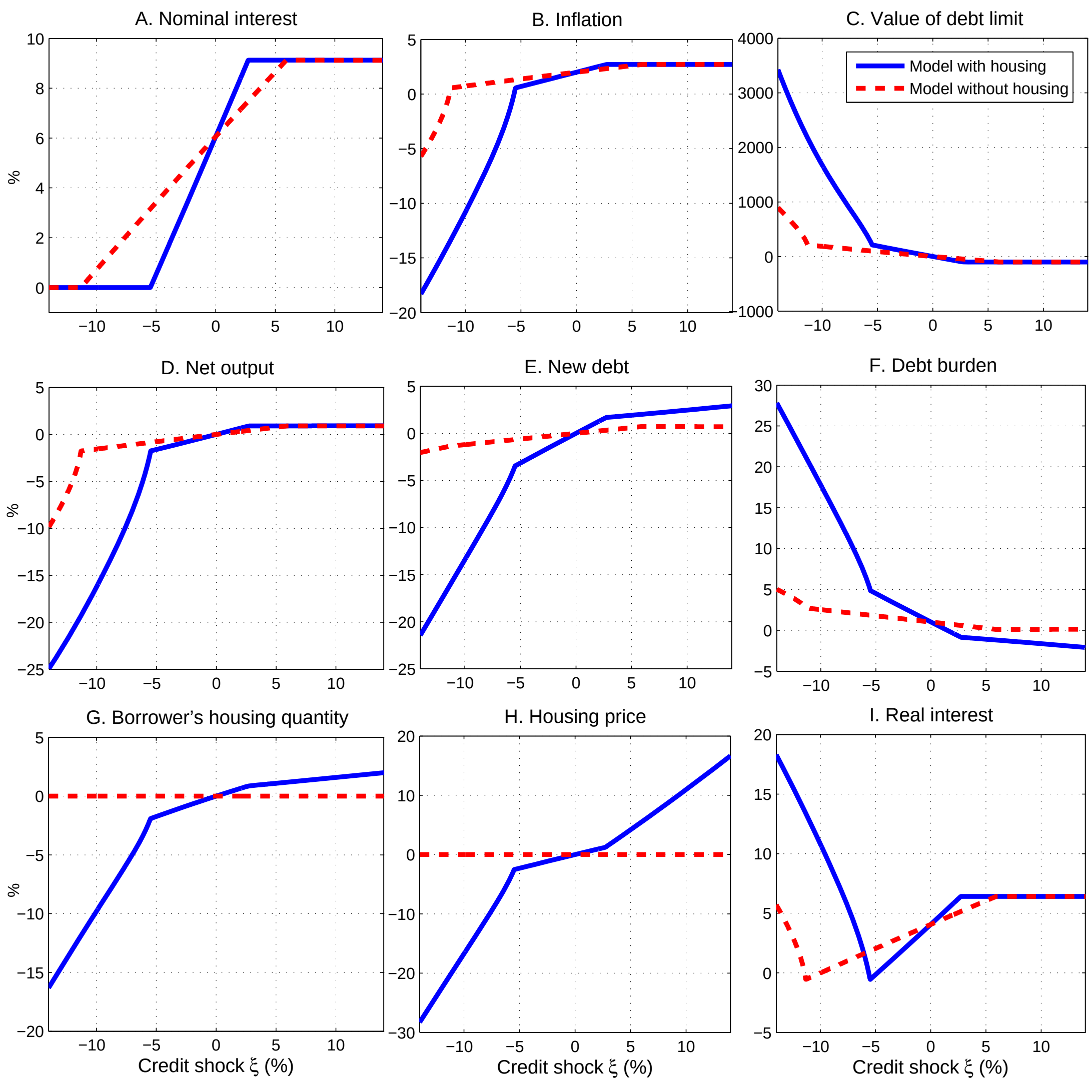

Figure 3: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with high leverage and with the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

result from two main channels. First, from the collateral constraint, as in equation (3), when there is an adverse shock to the credit market conditions ξ_t, both the nominal interest rate i_t and the new debt of borrowers D_{bt} fall.[9] When the nominal interest hits the zero bound, more downward pressure is put on the new debt, leading to a larger deleveraging compared to the case without the ZLB. In other words, a binding ZLB amplifies the collateral effect.

Second, compared to the model without the ZLB, the real interest payment is higher when the ZLB binds. From Panel I of Figure 3, the real interest rate soars up when the economy is in a deep recession with a binding ZLB. Specifically, when the economy is hit by an adverse credit shock that causes the debt-to-value to be 10% permanently lower than the initial value, the nominal interest reaches the ZLB and the real interest rate increases to around 10% per year. This real interest rate is much higher than the desired level of around -4%, as in Panel I of Figure 1, when the ZLB condition is not imposed. The increase in the real interest rate causes the debt burden to rise, making the borrowers' budget for consumption shrink substantially. This channel is called the Fisherian debt deflation.

To illustrate this powerful amplification mechanism of Fisherian debt deflation, I provide a comparison between the model with and without a Fisherian debt deflation channel in Figure 4. In the case without Fisherian debt deflation, the nominal interest rates are indexed by inflation. Hence, the real debt is constant no matter how large inflation or deflation is.

The solid blue lines show the results from the model with Fisherian debt deflation, while the dashed red lines present the results from the model without Fisherian debt

[9] The new debt falls because of the fact that the central bank uses a simple Taylor rule, and, as a result, is not able to completely stabilize the economy under a credit shock.

deflation. Including the Fisherian debt deflation not only makes the ZLB to bind more easily, as in Panel A of Figure 4, but also makes a recession worse when the ZLB binds, as in Panels B and D of Figure 4.

Now it is time for us to discuss the role of the credit shock in explaining the housing price fluctuations. It seems to be a consensus in the literature of housing and macroeconomic fluctuations that the credit shock is not able to generate a sharp decline in the housing price. It would be interesting to see if the point of view is supported in the analysis.

In the case without the ZLB. Panel H of Figure 1 shows that a 10% permanent decline in the debt-to-value ratio causes the housing price to decrease only about 5% per year. This result is in line with the existing literature, see more discussion in Liu et al. (2013). The intuition is that the housing price is the present discounted flow of housing services. Without the ZLB, the central bank is able to mitigate the fluctuation of the economy, including the fluctuation of the shadow value of debt limit and the borrowers' housing consumption, by adjusting the nominal interest rate freely, even to a negative level.

However, in the presence of the ZLB, the central bank is not able to lower the nominal interest rate below zero under an adverse credit shock. As a result, the borrowers have to deleverage by scaling back durable housing goods substantially. In the presence of the high leverage, the deleveraging causes the shadow value of debt limit to increase and the marginal benefit of owning houses to decrease. Consequently, the housing price drops. Particularly, under the negative 10% permanent credit shock, the housing price falls more than 15% per year, three times higher than the case without the ZLB.

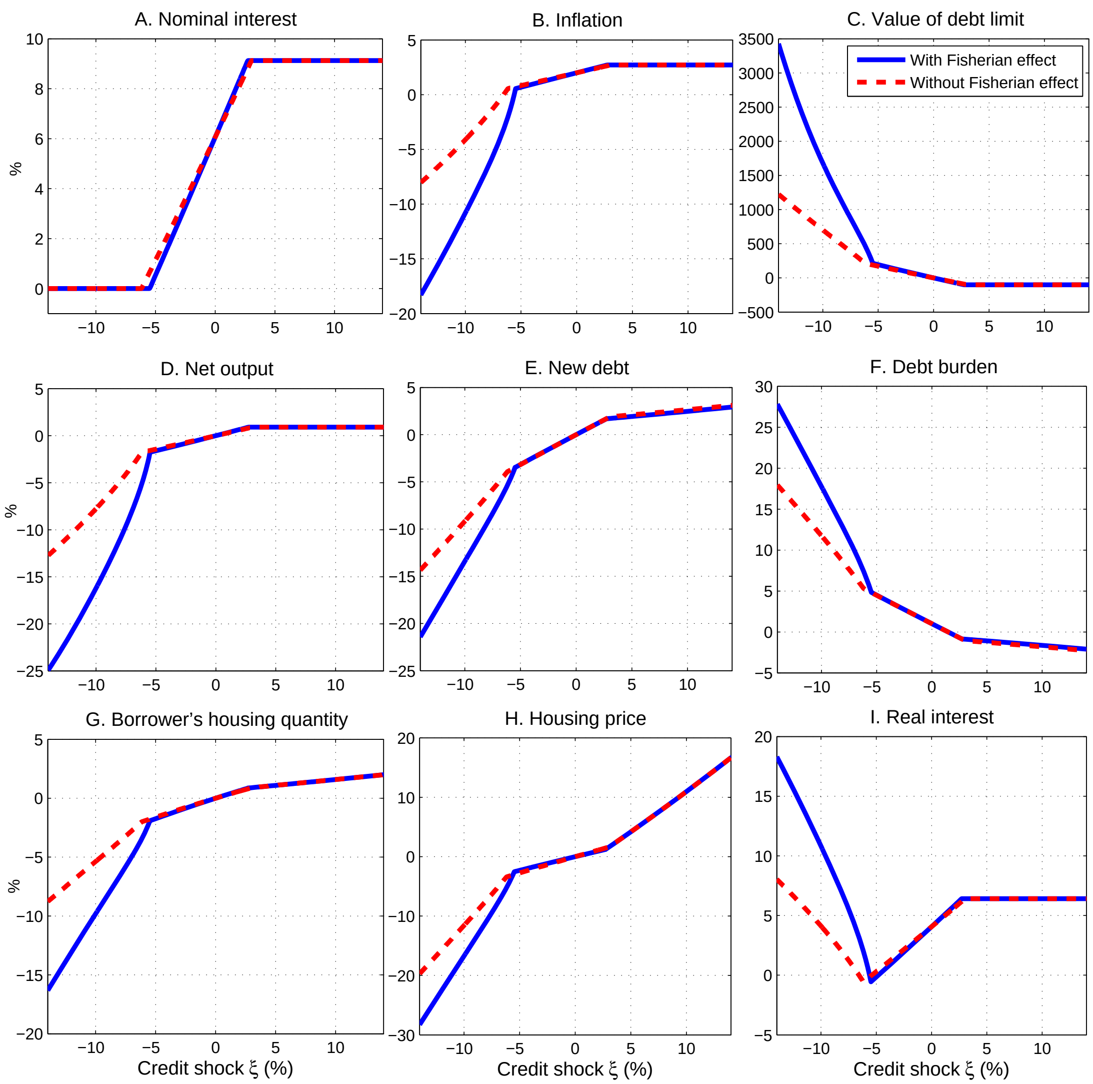

Figure 4: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with housing, high leverage, and the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

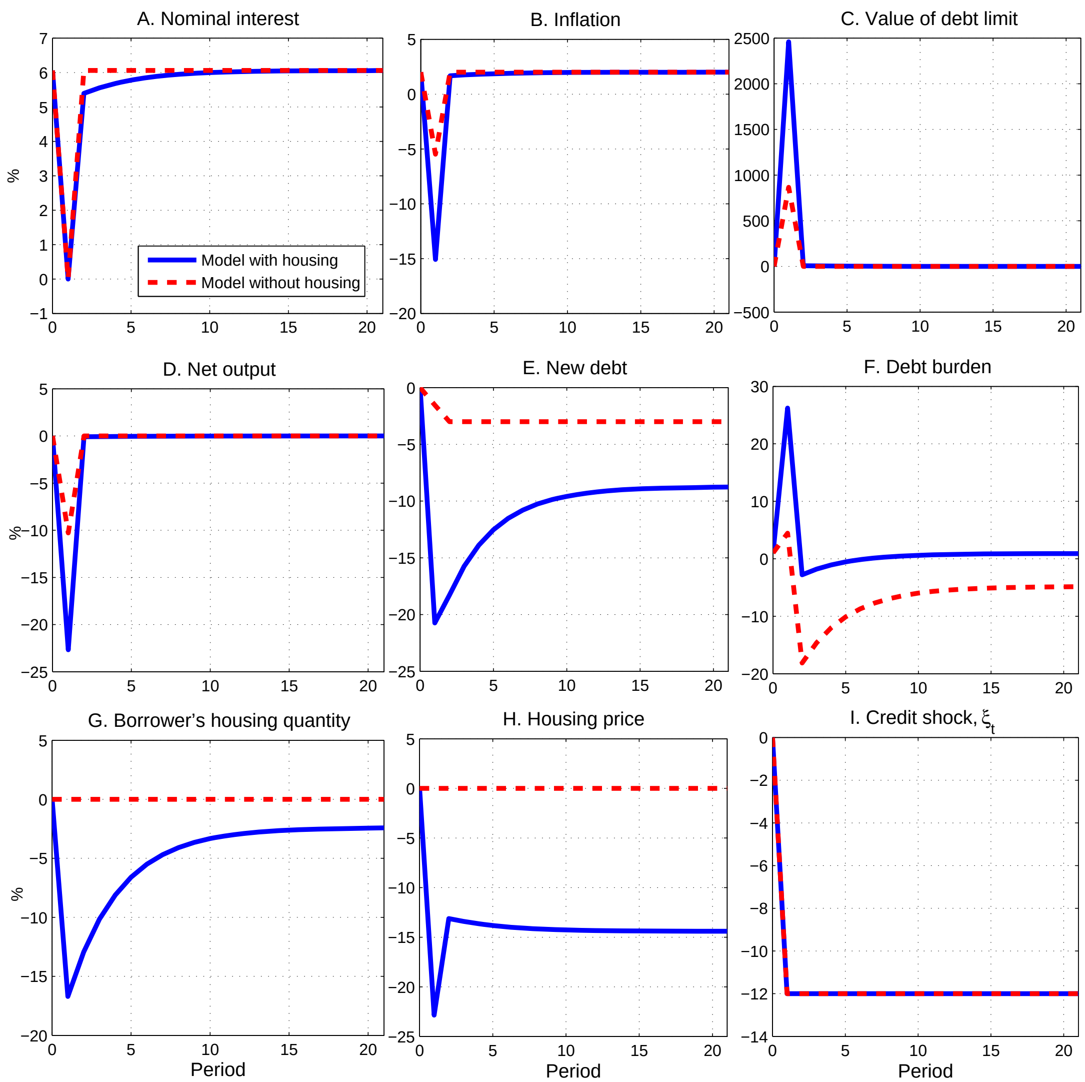

Figure 5: Impulse responses of selected economic variables under a permanent credit shock (ξ). The case with high leverage and with the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

4.3. Relaxing the timing assumption

In this section, instead of using the timing assumption as discussed above, I allow the variables to adjust gradually toward the steady state after the occurrence of a permanent credit shock that causes the debt-to-value ratio to decrease by 12% permanently. The transition from the old steady state to the new steady state can take long time.

As we know from the previous discussion, in the housing model with high leverage, a negative shock of around 5% is able to make the nominal interest rate reach the ZLB. However, I decide to choose a negative shock of 12% to ensure that the ZLB also binds in both the models with and without housing. Thus, we are able to see the difference between the two models at the time of a binding ZLB.

The impulse response functions are presented in Figure 5. The solid blue lines present the results from the housing model with high leverage and with the ZLB, while the dashed red lines show the results from the model without housing. Panel I of Figure 5 shows that the debt-to-value ratio declines 12% in period 1, then stays there permanently.

It is interesting to note that although we relax the timing assumption, the forced deleveraging phenomenon that can be seen previously still exists. From Figure 5, in the housing model, the borrowers' new debt drops and overshoots the new steady state value at time the shock occurs. The nominal interest rate stays at the ZLB only one period. The economy experiences a deep recession in the period when the shock occurs. Afterward, the debt increases and returns to the new steady state value, which is lower than the initial value. The new steady state values for the borrowers' housing quantity and the housing price are also lower than the initial steady state values.

Compared to the case with the timing assumption, in this case it takes about

15 quarters for the economy to reach the new steady state. However, the dynamic responses of some variables, including the nominal interest rate, inflation, net output, and the value of debt limit, are strikingly similar to those in the previous case. Specifically, these variables return to the new steady state almost immediately after the occurrence of the permanent credit shock.

5. Conclusion

This paper provides a new insight into the impact of a credit shock on macroeconomic fluctuations in a deleveraging model with housing, endogenous debt limits, and the ZLB. In this paper, I show that incorporating housing in a standard deleveraging model and allowing the debt limit of borrowers to tie to the market value of houses will amplify macroeconomic fluctuations under a credit shock, compared with the standard deleveraging model without housing and with exogenous debt limits, only when household leverage is high. This result challenges the widely-held belief that endogenous debt limits (or endogenous borrowing constraints) would always produce more macroeconomic fluctuations.

With the high leverage feature, the ZLB is more likely to bind in the housing model than in the standard deleveraging model without housing. When it binds in both the models, the housing model generates a much deeper recession with larger declines in output and inflation.

In addition, the housing model is able to generate a drop in the housing price under an adverse credit shock when the ZLB is binding. Therefore, imposing the ZLB condition plays a key role in solving the puzzle in the literature of housing and macroeconomic fluctuations that a credit shock is not able to generate a significant decline in the housing price.

There are several directions in which to extend the paper. It would be interesting to extend the model by investigating different monetary policy regimes instead of using a simple Taylor rule. By doing so, we might answer how monetary policy would be conducted optimally in this framework. In addition, it is important to see what kind of fiscal policy would be best in cases where the ZLB binds.[10]

6. Acknowledgments

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[10]Investigating how large the government expenditure multiplier is in the framework of endogenous debt limit and the ZLB, as described in this paper, is part of my future research agenda.

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Appendix A. Equilibrium equations

The system of equations governing the equilibrium of the model is as follows:

$$\begin{aligned} & C_{bt} + D_{bt-1}\left(\frac{1+i_{t-1}}{1+\pi_t}\right) + q_t\left(H_{bt} - H_{bt-1}\right) \\ & + \frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 q_t H_{bt} - w_t N_{bt} - D_{bt} = 0 \end{aligned} \tag{A.1}$$

$$\frac{\eta_b N_{bt}^{\phi}}{C_{bt}^{-\gamma}} - w_t = 0 \tag{A.2}$$

$$C_{bt}^{-\gamma_b} - \phi_{bt}\left(1+i_t\right) E_t\left[\frac{1}{1+\pi_{t+1}}\right] - \beta_b\left(1+i_t\right) E_t\left[\frac{C_{bt+1}^{-\gamma_b}}{1+\pi_{t+1}}\right] = 0 \tag{A.3}$$

$$\begin{aligned} & j_b H_{bt}^{-\psi} + \xi_t \phi_{bt} E_t\left[q_{t+1}\right] + \beta_b E_t\left[C_{bt+1}^{-\gamma} q_{t+1}\left(1+\phi_H\left(\frac{H_{bt+1}}{H_{bt}}-1\right)\left(\frac{H_{bt+1}}{H_{bt}}\right)^2\right)\right] \\ & -C_{bt}^{-\gamma} q_t\left(1+\frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}}-1\right)^2+\phi_H\left(\frac{H_{bt}}{H_{bt-1}}-1\right)\frac{H_{bt}}{H_{bt-1}}\right) = 0 \end{aligned} \tag{A.4}$$

$$\max\left\{-\xi_t H_{bt} E_t\left[q_{t+1}\right] + D_{bt}\left(1+i_t\right) E_t\left(\frac{1}{1+\pi_{t+1}}\right), 0 - \phi_{bt}\right\} = 0 \tag{A.5}$$

$$\begin{aligned} & \chi_b C_{bt} + \chi_b \frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}}-1\right)^2 q_t H_{bt} + \chi_s C_{st} + \chi_s \frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}}-1\right)^2 q_t H_{st} \\ & -Y_t\left(1-\frac{\phi_P}{2}\left(\pi_t-\theta\pi^*\right)^2\right) = 0 \end{aligned} \tag{A.6}$$

$$\frac{\eta_s N_{st}^{\phi}}{C_{st}^{-\gamma}} - w_t = 0 \tag{A.7}$$

$$C_{st}^{-\gamma} - \beta_s\left(1+i_t\right) E_t\left[\frac{C_{st+1}^{-\gamma}}{1+\pi_{t+1}}\right] = 0 \tag{A.8}$$

$$j_s\left(\frac{\overline{H}-\chi_b H_{bt}}{\chi_s}\right)^{-\psi}+\beta_s E_t\left[C_{st+1}^{-\gamma}q_{t+1}\left(1+\phi_H\left(\frac{H_{st+1}}{H_{st}}-1\right)\left(\frac{H_{st+1}}{H_{st}}\right)^2\right)\right]$$
$$-C_{st}^{-\gamma}q_t\left(1+\frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}}-1\right)^2+\phi_H\left(\frac{H_{st}}{H_{st-1}}-1\right)\frac{H_{st}}{H_{st-1}}\right)=0 \tag{A.9}$$

$$\left(1-\varepsilon+\varepsilon\frac{w_t}{A_t}-\phi_P\left(\pi_t-\theta\pi^*\right)\left(1+\pi_t\right)\right)Y_t$$
$$+\phi_P\beta_s C_{st}^{\gamma}E_t\left[\frac{\left(\pi_{t+1}-\theta\pi^*\right)\left(1+\pi_{t+1}\right)Y_{t+1}}{C_{st+1}^{\gamma}}\right]=0 \tag{A.10}$$

$$\chi_b N_{bt}+\chi_s N_{st}-Y_t=0 \tag{A.11}$$

$$\max\left\{(1+i)\left(\frac{Y_t}{Y^*}\right)^{\phi y}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi}-(1+i_t),0-i_t\right\}=0 \tag{A.12}$$

Appendix B. Initial steady state and parameter calibration

At the initial steady state, the inflation is the same as the target inflation. The equilibrium labor supply is calibrated to be 1/3 for both the borrower and the saver. Thus, the target output is the same as the labor supply. The total housing quantity is normalized to be 1.

$$\pi=\pi^*=0.005$$

$$N_b=N_s=1/3$$

$$Y^*=Y=\chi_b N_b+\chi_s N_s=1/3$$

$$H=1$$

$$\text{Eq.(A.10)}: w = \frac{\varepsilon - 1}{\varepsilon} + \frac{\phi_P (1-\theta)\pi^* (1+\pi^*)(1-\beta_s)}{\varepsilon}$$

$$\text{Eq.(A.8)}: \frac{1+i}{1+\pi^*} = 1 + r = R = \frac{1}{\beta_s}$$

First, I calibrate ξ to match the total debt to income ratio of 1.32, the total housing asset to income ratio of about 2.45, as in the Flow of Funds Tables reported by the Federal Reserve System, and the saver's housing to borrower's housing ratio of 1:

$$\text{Eq.(A.5)}: \xi = \frac{\left(\frac{\chi_b D_b}{Y}\right) R}{\chi_b \left(\frac{qH}{Y}\right)\left(\frac{H_b}{H}\right)}$$

$$\frac{H_b}{H} = \frac{1}{\chi_b + \chi_s \left(\frac{H_s}{H_b}\right)}$$

I then compute the other steady state values:

$$H_b = H_s = H = 1$$

$$q = \left(\frac{qH}{Y}\right)\frac{Y}{H}$$

$$\text{Eq.(A.5)}: D_b = \frac{\xi H_b q}{R}$$

$$\text{Eq.(A.1)}: C_b = wN_b - rD_b$$

$$\text{Eq.(A.6)}, \text{Eq.(A.1)}: C_s = \frac{1}{\chi_s} Y \left(1 - \frac{\phi_P}{2}(\pi - \theta\pi^*)^2\right) - \frac{\chi_b}{\chi_s} C_s$$

Finally, I calibrate the free parameters j_b, j_s, η_b, η_s:

$$\text{Eq.(A.4)}: j_b = \frac{C_b^{-\gamma} q (1 - \xi(\beta_s - \beta_b) - \beta_b)}{H_b^{-\psi}}$$

$$\text{Eq.(A.9)}: j_s = \frac{C_s^{-\gamma} q (1-\beta_s)}{\left(\frac{\overline{H}-\chi_b H_b}{\chi_s}\right)^{-\psi}}$$

$$\text{Eq.(A.2)}: \eta_b = \frac{w C_b^{-\gamma}}{N_b^{\phi}}$$

$$\text{Eq.(A.7)}: \eta_s = \frac{w C_s^{-\gamma}}{N_s^{\phi}}$$

In sum, I calibrate the 5 free parameters $\xi, j_b, j_s, \eta_b, \eta_s$ to match with the 5 stylized facts: (i) the total debt to income ratio of 1.32 ; (ii) the total housing asset to income ratio of about 2.45; (iii) the saver's housing to borrower's housing ratio of 1; (iv) the borrower's steady state labor of 1/3; and (v) the saver's steady state labor of 1/3. The parameters are uniquely determined by these facts.

Appendix C. New steady state with housing

Suppose the debt-to-value ratio changes permanently from ξ to ξ'. Given the parameters calibrated in the previous section, I am able to compute the new steady state corresponding to the new debt-to-value ratio as described below:

$$\pi' = \pi = \pi^* = 0.005$$

$$\text{Eq.(A.10)}: w' = w = \frac{\varepsilon - 1}{\varepsilon} + \frac{\phi_P (1-\theta) \pi^* (1+\pi^*) (1-\beta_s)}{\varepsilon}$$

$$\text{Eq.(A.8)}: i' = i = R(1+\pi^*)$$

$$\text{Eq.(A.2)}: N_b' = \left(\frac{w C_b'^{-\gamma}}{\eta_b}\right)^{\frac{1}{\phi}} \tag{C.1}$$

$$\text{Eq.(A.7)} : N_s' = \left(\frac{wC_s'^{-\gamma}}{\eta_s}\right)^{\frac{1}{\phi}} \tag{C.2}$$

$$\text{Eq.(A.11)} : Y' = \chi_b N_b' + \chi_s N_s' \tag{C.3}$$

$$\text{Eq.(A.5)} : D_b' = \frac{\xi' q' H_b'}{R} \tag{C.4}$$

$$\text{Eq.(A.3)} : \phi_b' = C_b'^{-\gamma} \left(\beta_s - \beta_b\right) \tag{C.5}$$

$$\text{Eq.(A.1)} : C_b' = wN_b' - rD_b' \tag{C.6}$$

$$\text{Eq.(A.6)}, \text{Eq.(A.1)} : C_s' = \frac{1}{\chi_s} Y' \left(1 - \frac{\phi_P}{2} \left(\pi - \theta\pi^*\right)^2\right) - \frac{\chi_b}{\chi_s} C_b' \tag{C.7}$$

$$\text{Eq.(A.4)} : j_b H_b'^{-\psi} = C_b'^{-\gamma} q' \left(1 - \xi \left(\beta_s - \beta_b\right) - \beta_b\right) \tag{C.8}$$

$$\text{Eq.(A.9)} : \left(\frac{\overline{H} - \chi_b H_b'}{\chi_s}\right)^{-\psi} = \frac{C_s'^{-\gamma} q' \left(1 - \beta_s\right)}{j_s} \tag{C.9}$$

The algorithm is as below:

- Step 1: Initialize $X^0 = \left(C_b^0, H_b^0, C_s^0, q^0\right)$.
- Step 2: Compute $\left(N_b', N_s', Y', D_b', \phi_b'\right)$ using Eq.(C.1),..., Eq.(C.5).
- Step 3: Compute $X^{(n)} = \left(C_b, H_b, C_s, q\right)$ that solves the system of nonlinear equations Eq.(C.6),...,Eq.(C.9).

- Step 4: Compute $||X^{(n)} - X^{(n-1)}||$, go to Step 5 if the norm is less than the tolerant value. Otherwise go back to Step 2.

- Step 5: Report the solution$X = X^{(n)}$

Appendix D. Algorithm to solve the models.

I use the shooting method to solve the housing model. The same method can be used to solve the model without housing. First, I find the initial and the new steady state. Then I determine, T, the number of periods that is long enough to make sure the economy will converge to the steady state after T periods. Then I follow the algorithm below:

- Step 1: Initialize $X_t^0 = \{C_{b,t}^0, N_{b,t}^0, D_{b,t}^0, H_{b,t}^0, C_{s,t}^0, N_{s,t}^0, i_t^0, \pi_t^0, q_t^0, w_t^0, \phi_{b,t}^0, y_t^0\}$ for $t = 1, ..., T-1$.

- Step 2: Using the initial steady state and the new steady state value for $X_0^{(n)}$ and $X_T^{(n)}$, compute $X_t^{(n)}$ that solves the system of equibrium equations Eq.(A.1),...,Eq.(A.12). for $t \in (1, ..., T-1)$.

- Step 3: Compute $||X^{(n)} - X^{(n-1)}||$, go to Step 4 if the norm is less than the tolerant value. Otherwise go back to Step 2.

- Step 4: Report the solution$X = X^{(n)}$.

Note that we can also use this algorithm to solve the models under the timing assumption of Eggertsson and Krugman (2012).