CIS 492/593 Lab 5 Deutsch and Deutsch-Jozsa Algorithms Fall 2023

The purpose of this hands-on lab is to familiarize yourself with:

- the Deutsch algorithm
- the Deutsch-Jozsa algorithm

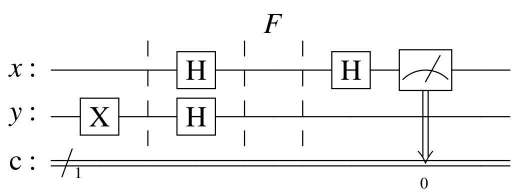
Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

```
cd QC
jupyter notebook
```

Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. `# Experiment 1`) in the top of the code.

Experiment 1: Deutsch – the Constant 0 function

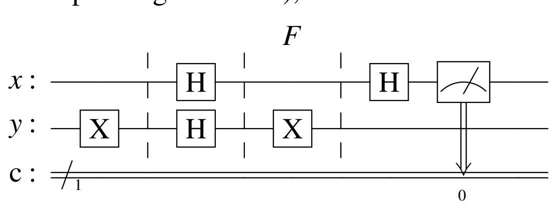
- In the cell, type
  ```
  %load ~cis492s/pub/DeutschConst0.py
  ```
  and click "Run" to load a program in which the black box function F is a constant 0 function (i.e. f(0) = 0, and f(1) = 0). Instead of using the default name 'q' for the qubits, we call the function `QuantumRegister()` twice to label the top qubit as $'x'$ and the output qubit as $'y'$, respectively. The corresponding circuit diagram is like below:

  Note that because $f(x) = 0$, the output of the bottom qubit becomes $y \oplus f(x) = y \oplus 0 = y$. Hence, there are only two straight wires in the black box F and there is no connection between the qubit x and the qubit y.
- Click "Run" to execute the code. What is the output of the top qubit x? Refer the textbook page 149 about how to determine the the black box whether it is a constant or a balanced function. Explain the result.

Experiment 2: Deutsch – the Constant 1 function

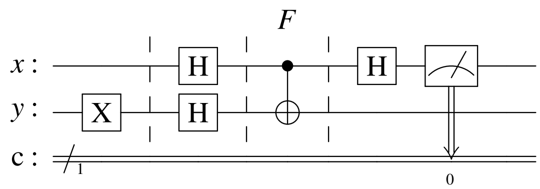
- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is a constant 1 function (i.e. f(0) = 1, and f(1) = 1). Hence, $y \oplus f(x) = y \oplus 1 = \bar{y}$. That is, you need to modify the code (i.e. putting a NOT gate on the qubit y and changing the corresponding comment), like below:

- Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 3: Deutsch – the Identity function (balanced)

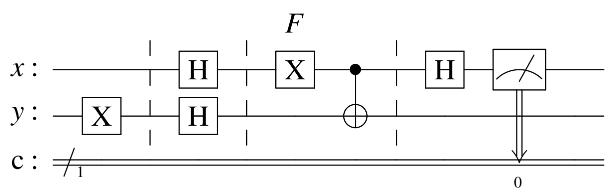
- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is the identity function (i.e. f(0) = 0, and f(1) = 1). To realize $y \oplus f(x)$, you need to modify the code (i.e. adding a CNOT gate with the qubit x as the control while the qubit y as the target, and changing the corresponding comment), just like below:



- Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 4: Deutsch – the Inversion function (balanced)

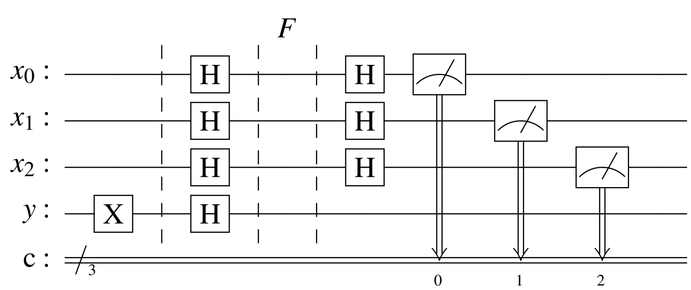
- Click the cell which contains the code done in Experiment 3. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is the inversion function (i.e. f(0) = 1, and f(1) = 0). You need to modify the code (i.e. adding a NOT gate to the qubit x, and changing the corresponding comment), like the figure below:



- Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 5: Deutsch-Jozsa – the Constant function

- Using a new and empty cell, type
  `%load ~cis492s/pub/DJ-Const0.py`
  and click "Run" to load a program in which the black box function F is a constant 0 function with three parameters x_0, x_1, and x_2. The corresponding circuit diagram is like below:



- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see any output string other than '000' during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).
- Click the cell which contains the code done above. Click the "Edit" button and select "Copy Cells".

Click "Edit" again and choose "Paste Cells Below".

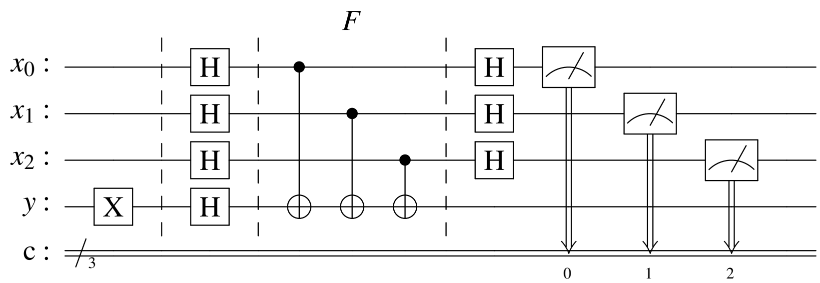
- Modify the code to make the black box F a constant 1 function. *Hint: see Experiment 2.*
- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see any output string other than '000' during the 1000 times of executions?

Experiment 6: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$

- Construct the truth table for $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$ to verify it is a balanced function.
- Using a new and empty cell, type

```
%load ~cis492s/pub/DJ-Const0.py
```

  and click "Run" to load a template program.
- Modify the code to implement $x_0 \oplus x_1 \oplus x_2$, like the circuit below:



- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see '000' appeared during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).

Experiment 7: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0 x_1 \oplus x_2$

- Construct the truth table for $f(x_0, x_1, x_2) = x_0 x_1 \oplus x_2$ to verify it is a balanced function.
- Modify the code to implement the balanced function $f(x_0, x_1, x_2) = x_0 x_1 \oplus x_2$.
- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see '000' appeared during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).

Click "File" and choose "Save as". Type "lab5" in the entry box and click "Save" to save your work today into the file `lab5.ipynb`.

Open a terminal and type

```
ssh grail
```

to login to the server grail. Then, type

```
cd QC
turnin -c cis492s -p lab5 lab5.ipynb
```

to electronically turn in your file `lab5.ipynb`.

Shutdown the jupyter notebook and logout the workstation.
Hand in your lab report before your leave.

Questions:

Below are some questions which will be included in the Homework 3 (Due: Dec. 5). You may start working on these questions now.

- If the black box is a constant function as in the Experiments 1, 2, and 5,

  a) Is there any wire connection between the qubit(s) x and qubit y ?
  b) What is the initial value for the x qubit(s)?
  c) Explain why the measured value for each x qubit(s) is 0.
     ($Hint$: How do the gates applied on each x qubit(s) work? Review the Experiment 2 in Lab2)

- If the black box is a balanced function as in the Experiments 3, 4, 6 and 7,

  d) Is there any connection via the CNOT gate between the qubit(s) x and qubit y ?
  e) If the answer above is Yes, what happen if we put an H-gate for the corresponding x qubit and the qubit y? ($Hint$: Review the Experiment 7 in Lab3)
  f) What is the initial value for the y qubit before applying the H-gate?
  g) Explain why the measured value for each x qubit(s) is not 0 (as in the Experiments 3 and 4) or not '000' (as in the Experiments 6 and 7).