

# MCE371: Vibrations

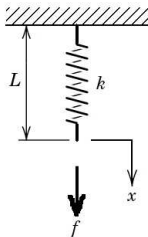
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Department of Mechanical Engineering

Handout 2  
Fall 2017



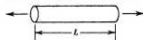
# Spring Law : One End Fixed



- Ideal linear spring law:  $f = kx$ . What are the units of  $k$ ?
- More generally:  $f = F(x)$  **nonlinear spring**
- What physical elements obey  $f = F(x)$ ?

# Physical Elements with Approx. Linear Spring Laws

Rod in compression/tension



$$k = \frac{EA}{L}$$

$A$  = cross-sectional area

Rod in torsion



$$k = \frac{G\pi D^4}{32L}$$

$L$  = length

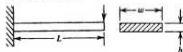
Helical wire coil



$$k = \frac{Gd^4}{64nR^3}$$

$d$  = wire diameter  
 $n$  = number of coils

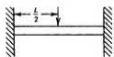
Cantilever beam



$$k = \frac{Ewh^3}{4L^3}$$

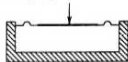
$w$  = beam width  
 $h$  = beam thickness

Doubly clamped beam



$$k = \frac{16Ewh^3}{L^3}$$

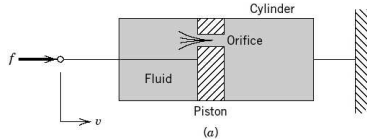
Air spring



$$k = \frac{\gamma P A^2}{V}$$

$A$  = diaphragm area  
 $P, V$  = nominal pressure and volume  
 $\gamma$  = ratio of specific heats

# Damper Law : One End Fixed



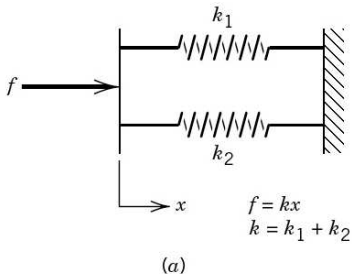
- Ideal linear damper law:  $f = cv$ . What are the units of  $k$ ?
- More generally:  $f = F(v)$  **nonlinear damper**
- What physical elements obey  $f = F(v)$ ?

## Rotational versions of linear element laws

- By “linear spring” we mean that the relationship between force (torque) and displacement (angular displacement) is direct proportionality
- By “linear damper” we mean that the relationship between force (torque) and velocity (angular velocity) is direct proportionality
- Linear and nonlinear springs and dampers can be designed for rectilinear or rotary motion.
- Rotary linear spring law:  $T = k\theta$ . What are the units of  $k$  now?
- Rotary linear damper law:  $T = c\frac{d\theta}{dt} = c\dot{\theta}$ . What are the units of  $c$  now?

## Equivalent Spring Constant: Parallel

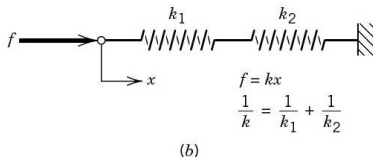
We now derive the equivalent spring constant for the arrangement of Fig.1.3-7 (a) in Palm:



The equivalent spring constant of a parallel spring arrangement (common displacement) is the sum of the individual constants. That is, springs in parallel combine like resistors in series (capacitors in parallel).

## Equivalent Spring Constant: Series

We now derive the equivalent spring constant for the arrangement of Fig.1.3-7 (b) in Palm:



The equivalent spring constant of a series spring arrangement (common force) is the inverse of the sum of the reciprocals of the individual constants. That is, springs in series combine like resistors in parallel (capacitors in series).

# Mechanical-Electrical Analogy

Spring Law:  $f = kx$

Ohm's Law:  $V = Ri$

Capacitor Law:  $V = (1/C)q$

If we interpret force as voltage and displacement as charge, there is an analogy between springs and capacitors: Note that force  $\times$  displacement = work and voltage  $\times$  charge = work.

The power-based analogy is SPRING=CAPACITOR. This is usually done in the field of Mechatronics (MCE503/603/703).

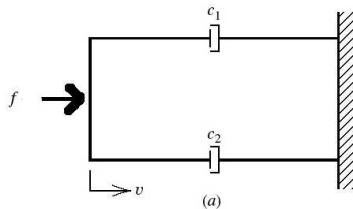


# Examples and Exercises

- 1 Read and understand Example 1.3-3 on your own.
- 2 In class: we explain/solve Ex. 1.3-5 and 1.3-7.
- 3 Recommended: 1.21, 1.23, 1.28, 1.36 and 1.41c.
- 4 ~~Homework 1: 1.24, 1.25, 1.41b.~~

## Equivalent Damper Constant: Parallel

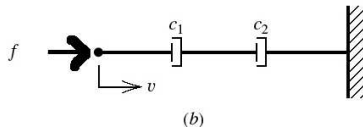
We now derive the equivalent damper constant for the arrangement of Fig.1.4-14 (a) in Palm:



The equivalent damping constant of a parallel damper arrangement (common displacement) is the sum of the individual constants.

## Equivalent Damper Constant: Series

We now derive the equivalent damper constant for the arrangement of Fig.1.4-14 (b) in Palm:



The equivalent damping constant of a series damper arrangement (common force) is the inverse of the sum of the reciprocals of the individual constants.

# Mechanical-Electrical Analogies

Damper Law:  $f = c\dot{x}$

Ohm's Law:  $V = Ri$

Capacitor Law:  $V = (1/C)q$

If we interpret force as voltage and velocity as current, there is an analogy between dampers and resistors: Note that force  $\times$  velocity = power and voltage  $\times$  current = power.

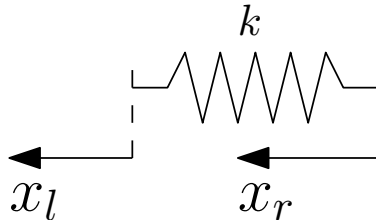
The power-based analogy is DAMPER=RESISTOR. This is usually done in the field of Mechatronics (MCE503/603/703). Also note that capacitors and springs conserve energy, while dampers and resistors dissipate it.

# Examples and Exercises

- 1 Recommended: 1.47a.
- 2 ~~Homework 1: 1.47b.~~

# Methodology for Springs/Dampers with 2 Moving Ends

Springs:



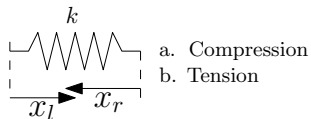
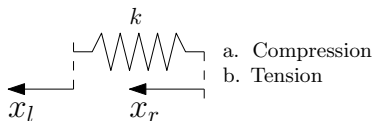
- 1 Draw a pair of arrows for the displacements of the ends. Label them with two variables like  $x_r$  and  $x_l$ . It doesn't mean the spring always moves like that. It just says that when it does, the variables have positive values. If a calculation gave a negative value for a variable, you can interpret.
- 2 The directions of the arrows can be drawn as you please, but they are usually dictated by whatever is connected to the spring. Don't complicate yourself with weird choices. We'll do that in class once.

# Springs with 2 Moving Ends ...

- 1 Assume that the spring is in tension or compression (arbitrary choice).
- 2 Now you're committed to your earlier choices: Figure out which variable is larger than the other to match your tension/compression assumption.
- 3 Draw a pair of forces on the spring to match your assumption.
- 4 Write the expression for the force in the spring so that a positive quantity results for the force.

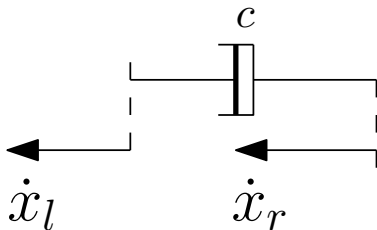
# Examples

Write the force equation for the following schematics





# Dampers with 2 Moving Ends



- 1 Draw a pair of arrows for the velocities of the ends. Label them with two variables like  $\dot{x}_r$  and  $\dot{x}_l$ . It doesn't mean the damper always moves like that. It just says that when it does, the variables have positive values. If a calculation gave a negative value for a variable, you can interpret.
- 2 The directions of the arrows can be drawn as you please, but they are usually dictated by whatever is connected to the damper.

# Warm-Up Example: 2 Masses and 2 Springs

Choose a positive direction for  $x_2$ . Make the tension/compression assumption and draw a free-body diagram accordingly. Apply Newton's Law to mass 2.

