# Development of a Leg Prosthesis Test Robot

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## 1 Introduction

The development of novel prosthetic devices has progressed at a rapid pace in the last decades. Transfemoral leg prostheses, in particular, have become increasingly sophisticated in terms of the technologies used to modulate knee behavior or even produce push-off forces with battery power. Knee torque may be varied by pneumatic, hydraulic or fluid-rheological means, and embedded microprocessors are typically used for control [1]. The literature shows, however, that none of the available knee prostheses fully restore normal gait [2, 3, 4, 5, 6]. This fact has become a driver for the development of better prostheses, and the recent literature describes some new prosthetic knee programs [7, 8, 9].

In the product development phase, prosthesis testing under normal and hazard conditions is problematic. Informed consent, safety harnesses and lack of repeatability across test subjects inhibit useful data gathering. Robotic testing of prosthesis can remove these obstacles and bring additional benefits. Indeed, robots may be operated under conditions which are deemed unsafe for patients, such as near-fall situations. A robot may be operated continuously for long periods of time, as necessary for real-time optimization of prosthetic control algorithms. Also, robots may be fitted with sensors to measure quantities of interest which are difficult to measure directly in a human subject, such as hip torque and angle. Finally, simultaneous computer control of testing machine and prosthesis allows the real-time integration of musculoskeletal models as part of prosthetic test programs. Real-time sensor data from the machine and the prosthesis provide inputs to these models, which in turn produce outputs which are used to alter the motion profiles generated by the machine, in a feedback process. This test modality is difficult to implement with a patient in the loop.

Although some use of commercial manufacturing robots in prosthesis testing has been reported by the Fraunhofer Institute [10] and the Cleveland Clinic [11], the use or development of a machine aimed at reproducing prosthetic gait has not been reported, to the best of the authors' knowledge. In this paper, robotic testing of transfemoral prosthesis is proposed, and the development, modeling and initial control of a two-degree-of-freedom robot to emulate hip motions is described. It is expected that robotic testing of prostheses will play a major role in extracting the dynamic characteristics of prosthetic knees because the input displacements, velocities, accelerations, torques and forces can be measured with much more accuracy and precision than during human gait trials.

### Methodology

Higher-level design requirements such as the number of degrees of freedom, the shape of the motions to be generated and load capacity requirements leave very few options for the mechanical configuration. Upon fixing the kinematic concept, electromechanical actuation was favored over hydraulics due to the higher achievable positioning accuracy and generally faster control bandwidth of the former. Besides, a hydraulic system requires several ancillary components such as pumps, a tank, valves and filters, resulting in a bulkier and overall less efficient solution for this particular application. Installation of a hydraulic system in a clinical research facility may not be feasible due to noise and cleanliness concerns. The development of the robot comprised several sub-tasks:

- 1. Actuator calculations and final component selection
- 2. Design and strength calculations for the frame
- 3. Detailed mechanical design and fabrication
- 4. Modeling and system identification of servo systems

- 5. Robotic modeling and parameter estimation of a sample passive prosthesis
- 6. Integration of models and development of computer simulation tools
- 7. Controller design and simulation
- 8. Real-time control interfacing and tuning
- 9. Proof-of-concept tests under real-time feedback control

This paper focuses on sub-tasks 4,5,7 and 9, which we believe concentrate the innovative use of technology toward more reliable prosthetic designs. The remainder of the paper is organized as follows: Section 2 describes machine functionality and components; Section 3 first derives separate models for the machine and for the prosthesis, and then integrates them into one, following a robotic manipulator dynamics framework; Section 4 describes various parameter estimation procedures used to populate the dynamic model; Section 5 presents preliminary independent-joint controller that can be used to accurately track motion profiles; Section 6 reports our success in achieving the desired motion profiles and Section 7 offers conclusions and some recommendations for future improvements to this design.

## 2 Machine Design

The robot must produce motions that mimic those of a human hip during walking and running. This design is limited to two degrees of freedom, namely hip vertical displacement and hip swing, which are the minimum required to reproduce two-dimensional gait patterns. Normal hip displacement and swing are periodic oscillations with amplitudes and waveforms that depend primarily on the height of the patient and the walking or running speed. Normal gait profiles used as a guideline for design are a subset of the data collected by van den Bogert [12], which includes walking and running in healthy subjects. The machine is designed for a hip displacement amplitudes of up to 50 mm, with a maximum velocity of 1 m/s. Vertical force capacity is specified at 1200 N, which exceeds the ground force generated by a 78 kg normal subject during fast walk/slow running. The vertical motion stage is comprised of a DC motor, a ballscrew and a linear slide. Overall vertical motion range is 12 inches, of which up to 100 mm are used to accommodate the expected vertical hip motion profiles. The remaining space is used to shift the center of oscillation, as it may be required to test prostheses of various lengths. The center of oscillation may also be changed during real time operation to initiate and regulate contact between the foot and the walking surface. Since the machine has a fixed vertical axis, a treadmill is used as a walking surface.

The rotary motion stage, including motor, is carried by the vertical slide. Prostheses are attached to the rotary plate by means of an adjustable threaded rod, which is secured to a bracket on the plate with two 2.75-inch nuts. The threaded rod, in combination with the adjustable center of oscillation of the vertical stage, offer great flexibility for standoff adjustments. Although thigh angular excursion in the normal gait data does not exceed 50 degrees, the rotary actuator has an unlimited angular range. Following biomechanical data from [12], the design values for thigh angular velocity and torque are is 150 degrees per second and 75 Nm.

A schematic diagram of the robot and its components is shown in Fig. 1, and the finished machine is shown in the photograph of Fig. 2

#### 2.1 Support Structure

The linear slide is attached to a rigid frame built with A500 rectangular steel tubing. Frame elements are welded together, and bolted joints are used for the mounting plates. The frame, the welds and the bolted joints were verified for static loading and fatigue using the SolidWorks software. A treadmill is secured to the frame by bolts, and anchor bolts are used to secure the entire assembly to the lab floor. Overall machine dimensions are  $48'' \times 61'' \times 26''$ .

### 2.2 Servo Systems

The vertical servo system is composed of a ballscrew-driven vertical slide (manufactured by RAF Automation, Solon, Ohio), directly coupled to a brushless DC motor (Mitsubishi HF-KP73). The motor is powered by a torque-mode servo amplifier (Mitsubishi MR-J3-70A). An analog input voltage applied to the servo amplifier results in a proportional torque on the motor shaft and ballscrew. Vertical position (hip displacement) is



Figure 1: Machine Schematic



Figure 2: Overall Robot Installation

measured with a 1000 line/rev incremental encoder rotating synchronously with the motor. An absolute position reference is established with a limit switch mounted on the vertical guides. The ballscrew has a diameter of 1 inch, a lead of 0.5 inches per revolution and the total useful travel of the slide is 12 inches.

The rotary servo system is composed of a brushless DC motor (ElectroCraft RapidPower RP34) coupled to a rotary plate through an inchworm-gear reducer with ratio 80:1 (RM-8-SM-34, manufactured by Newmark Systems, Rancho Santa Margarita, California). Angular position (thigh angle) is measured with an incremental encoder rotating synchronously with the motor. An absolute position reference is established with a limit switch mounted on the rotary plate. Real-time instrumentation and control is handled by a dSPACE DS-1102 system and associated software. The developer may easily convert Matlab/Simulink code into user-friendly real-time operating interfaces.

## 3 Robotic Modeling

The overall machine-and-prosthesis system is best modeled in the standard framework of robotics. Indeed, the system fits the category of a 3-link rigid robot with a prismatic-revolute-revolute (PRR) configuration. When a conventional leg prosthesis is attached, the system is underactuated, since the torque of the knee joint may not be externally controlled. When advanced prototypes featuring actively-controlled knee damping are attached, the system can be regarded as fully-actuated. A general dynamic model [13] for the robot is



Figure 3: Denavit-Hartenberg Coordinate Frame Assignments

given in joint coordinates as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + B(q,\dot{q}) + J_e^T F_e + g(q) = F_a$$
(1)

where  $q^T = [q_1 \ q_2 \ q_3]$  is the vector of joint displacements (in our case  $q_1$  is the vertical displacement,  $q_2$ is the thigh angle and  $q_3$  is the knee angle), D(q) is the inertia matrix,  $C(q, \dot{q})$  is a matrix accounting for centripetal and Coriolis effects,  $B(q, \dot{q})$  is a nonlinear damping matrix (in our case due to the knee damper),  $J_e$  is the kinematic Jacobian relative to the point of application of external forces  $F_e$ , g(q) is the gravity vector and  $F_a$  is a vector of net actuator inputs, including their inertial and frictional effects. The explicit form of  $F_a$  is developed below. Matrices D(q),  $C(q, \dot{q})$  and g(q) are readily obtained using the standard Euler-Lagrange approach. Their entries are listed at the appendix. Matrix  $B(q, \dot{q})$  is specific to the leg prosthesis and discussed in Section 3.3. The external force Jacobian and the external force vector are discussed next.

#### **3.1** Forward Kinematics

A set of reference frames is defined to establish a basis for dynamic model derivations and to keep track of leg geometry during analysis, simulation and real-time operation. The frame assignments shown in Fig. 3 follow the standard Denavit-Hartenberg convention [14]. The frame-to-frame transformations matrices are expressed in terms of standard primitives [13] as:

$$A_1^0 = (\operatorname{Trans}_{z,q_1})(\operatorname{Rot}_{x,90}) \tag{2}$$

$$A_2^1 = (\operatorname{Rot}_{z,q_2})(\operatorname{Trans}_{z,d_0})(\operatorname{Trans}_{x,l_2})$$
(3)

$$A_3^2 = (\operatorname{Rot}_{z,q_3})(\operatorname{Trans}_{x,c_3}) \tag{4}$$

where  $l_2$  is the length of link 2 (thigh),  $d_0$  is the offset of link 2 and  $c_3$  is the distance between the knee joint and the center of mass of link 3. The world-frame coordinates of points of interest can be readily computed using the above transformation matrices, assuming q is known. The world position of the foot-mounted load cell is particularly useful when designing force feedback controls. In frame 3 coordinates, the load cell is located at  $[l_{cx} - l_{cy} 0]^T$ . Using the composite transformation from frame 3 to the world frame, the vertical coordinate of the load cell is found as

$$Z_{LC} = q_1 - l_{cy}\cos(q_2 + q_3) + (c_3 + l_{cx})\sin(q_2 + q_3) + l_2\sin(q_2)$$
(5)

In the modeling and simulation stage, this coordinate may be compared to the treadmill's standoff height to determine belt deflection and estimate the vertical component of ground reaction force,  $F_{GV}$ . In a simplified

point-foot model, the belt is regarded as a stiffness, returning a force proportional to deflection. Also, side forces (in the world Y-direction) are ignored. The friction force due to non-slip contact between belt and foot in the world X-direction is denoted by  $F_{GH}$ . This input affects knee angle, but its effect on thigh angle is negligible due to non back-driveability of the rotary actuator. Likewise,  $F_{GH}$  has no effect on the vertical actuator. As explained in Section 3.5, a kinematic constraint associated to the foot-treadmill contact eliminates the need to compute  $F_{GH}$  during simulation. In this paper, no external torques are considered at the location of the load cell, but they can certainly be incorporated if  $J_e$  is appended with the appropriate angular velocity Jacobian. Thus, we consider a force vector of the form  $F_e^T = [0 \ 0 \ -F_{GV}]$ . The velocity Jacobian at the load cell location is listed in the appendix.

### 3.2 Servo System Models

The servo systems are modeled as current-driven DC machines with inertias attached to their shafts by a mechanical transmission. For the linear actuator, we consider Coulomb friction on the carriage guides and neglect viscous damping effects. For the rotary actuator, we consider viscous damping in the rotating gear.

#### Vertical Stage

The torque balance equation on the ballscrew is  $T_{m_1} - T_r = J_b \ddot{\theta}$ , where  $T_{m_1}$  is the torque applied by the motor,  $T_r$  is the torque due to the interaction between ballscrew and carriage nut,  $J_b$  is the moment of inertia of the ballscrew and  $\theta$  is the rotation angle. A transmission screw of pitch radius r and lead l (in units of length per revolution) exhibits a linear torque-force relationship [15] of the form:

$$F = \frac{1}{\gamma}T_r \tag{6}$$

where  $\gamma = \frac{l}{2\pi\eta}$ , F is the thrust force and  $\eta$  is an efficiency figure, typically between 0.8 and 0.9 for ballscrews. Using the ballscrew lead, we have  $\theta = q_1/l$ . Also, we define and calibrate the servo amplifier constant through the relationship  $T_{m_1} = \gamma k_1 u_1$ , where  $u_1$  is the analog control voltage and  $k_1$  is a constant. The force exerted by the ballscrew on the carriage becomes:

$$F = k_1 u_1 - m_0 \ddot{q}_1 \tag{7}$$

where  $m_0 = J_b/(\gamma l)$  represents an inertial parameter which is not subject to gravity.

### **Rotary Stage**

The rotary servo system can be modeled by the transfer function

$$G_2(s) = \frac{W_2(s)}{U_2(s)} = \frac{k_2}{(J_o + i^2 J_m)s + b_r}$$
(8)

where  $W_2(s)$  and  $U_2(s)$  are the Laplace transforms of the thigh angular speed and rotary actuator control voltage, respectively,  $k_2$  is the servo amplifier gain (in N-m per volt),  $J_o$  is the rotary inertia associated with the actuator gear, mounting plate and accessories, i = 80 is the gear ratio,  $J_m$  is the moment of inertia of the worm gear and motor armature, and  $b_r$  is a viscous damping coefficient associated with the rotation of the gear (friction on the motor side is small in comparison, and can be safely neglected).

### 3.3 Prosthesis Model

A passive prosthetic leg was attached to the machine to test the modeling approach and operate the machine in real-time with a preliminary control system. The Mauch MicroLite S [16] leg consists of two rigid links connected by a hinge. A damper is connected between the links, as shown in Fig. 1, to stabilize the knee during the stance phase and limit knee angle during the swing phase. Typically, higher damping is required during the stance phase than in the swing phase. For this reason, the damper features two adjustment dials that can be used to set separate damping coefficients for knee flexion ( $q_3$  increasing) and knee extension ( $q_3$ decreasing).



Figure 4: Damper extension geometry

The prosthesis, therefore, is regarded as links 2 and 3 of the overall robot. Joint 3 is subject to an internal torque due to damper action. This nonlinear damping torque can be calculated as  $\partial \mathcal{R}/\partial \dot{x}_d$ , where  $\mathcal{R}$  is the Rayleigh dissipation function:

$$\mathcal{R} = \frac{1}{2} b_k \dot{x}_d^2 \tag{9}$$

where  $b_k$  is the direction-dependent damping coefficient and  $\dot{x}_d$  is the expansion rate of the damper. The expansion rate is readily found by considering the geometry of the damper attachment and using differentiation of the cosine law. The nonlinear damping term of Eq. 1 is thus found to be:

$$B^{T}(q,\dot{q}) = \begin{bmatrix} 0 & 0 & -\frac{b_{k}o_{d}^{2}r_{d}^{2}\cos^{2}(q_{3})\dot{q}_{3}}{l_{d}^{2}} \end{bmatrix}$$
(10)

where  $o_d$ ,  $r_d$  and  $l_d$  are the damper offset, swing radius and instantaneous damper length, as shown in Fig. 4. In the absence of manufacturer's data, a custom experimental procedure described in Section 4 was used to estimate  $b_k$ .

#### 3.4 Overall Swing-Mode Model

With a passive knee, the actuator input term  $F_a$  in Eq. 1 has the form  $F_a^T = [(F - f_f) T 0]$ , where  $f_f$  is the Coulomb friction force on the linear guides and T is the net torque of the rotary stage. The Coulomb friction force is assumed to have the ideal form  $f_f = f$  sign  $(\dot{q}_1)$ , where f is the magnitude of the force. From Eq. 7, the first component of  $F_a$  in Eq. 1 becomes

$$F_a(1) = F - f_f = k_1 u_1 - f \operatorname{sign} (\dot{q}_1) - m_o \ddot{q}_1$$
(11)

To integrate the rotary actuator model into the robotic model we note that a net torque balance on the motor and worm gear axis yields  $T_{m_2} - T_L = J_m \ddot{q}_{m_2}$ , where  $T_{m_2} = k_2 u_2/i$  is the motor torque reflected to the output, and  $T_L$  is the load torque on the same axis. On the link axis, this torque is multiplied by *i* to obtain *T*, the net torque acting on link 2:

$$T = iT_L = iT_{m_2} - i^2 J_m \ddot{q}_2 = k_2 u_2 - i^2 J_m \ddot{q}_2$$
(12)

When  $F_a$  is substituted into Eq. 1, a re-arrangement of terms yields the final dynamic model:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B(q,\dot{q}) + F_f(\dot{q}) + J_e^T F_e + g(q) = Ku$$
(13)

where  $F_f^T(\dot{q}) = [f \text{ sign } (\dot{q}_1) \ 0 \ 0]$ ,  $K = \text{diag}\{k_1, k_2, 0\}$  is the matrix of servo amplifier constants,  $u = [u_1 \ u_2 \ 0]$  is the vector of control voltages and the inertial contributions from the robotic links and the servo system elements are combined into the mass matrix as follows:

$$M(q) = D(q) + \operatorname{diag}\{m_0, \ i^2 J_m, \ 0\}$$
(14)

where  $m_0$  is the inertial (not subject to gravity) contribution of the vertical motor armature and ballscrew, discussed in Sect. 3.2, *i* is the gear ratio of the rotary actuator,  $J_m$  is the armature inertia of the rotary actuator motor and D(q) is the inertia matrix associated with the three robotic links. Equation 13 is expressed in state-space form as:

$$\dot{z} = w \tag{15}$$

$$\dot{w} = M^{-1}(Ku - B - Cz_2 - F_f - J_e^T F_e - g(q))$$
(16)



Figure 5: Swing/Stance Model as a Hybrid Dynamic System

where z is the vector of joint coordinates and w is the vector of joint velocities. The vertical component of ground force is calculated as  $F_{GV} = k_b |s_z - Z_{LC}|$  when  $Z_{LC} > s_z$  and otherwise  $F_{GV} = 0$ , where the constant  $s_z$  is the treadmill standoff (vertical distance between the origin of coordinates and the belt) and  $k_b$  is the belt stiffness.

#### 3.5 Hybrid Swing-Stance Model

Although the model derived in the previous sections accounts for the external ground reaction force, it does not incorporate any constraint due to the interaction of the foot and the moving surface of the treadmill. However, tests are conducted with a ground force which is sufficiently large to prevent slippage between the foot and the treadmill belt. Therefore, the horizontal component of foot velocity is constrained to match that of the treadmill belt during the stance phase. The foot is still allowed to deflect the belt in the vertical direction, giving rise to the vertical component of the contact force. As done earlier, it is assumed that this component is a purely elastic effect. The horizontal velocity of the load cell point on the foot can be readily obtained from the Jacobian listed at the appendix as:

$$\dot{x}_{lc} = -(\dot{q}_3 + \dot{q}_2) \left[ (c_3 + l_{cx}) \sin(q_2 + q_3) - l_{cy} \cos(q_2 + q_3) \right] - l_2 \dot{q}_2 \sin(q_2) \tag{17}$$

To enforce the constraint,  $\dot{x}_{lc}$  is equated to  $-V_H$ , the treadmill speed. Then Eq. 17 is solved for  $\dot{q}_3$ , yielding the desired algebraic equation:

$$\dot{q}_3 = h_{VH}(q_2, q_3, \dot{q}_2) \tag{18}$$

Equation 18 is integrated during simulation to obtain  $q_3$ . That is, knee velocity is not calculated dynamically but algebraically during stance, from the kinematic constraint. Knee velocity is the integrated to give knee angle as before. This implies that only 5 states are required during stance  $(q_1, q_2 \text{ and their derivatives, and} q_3)$ , while 6 are needed in the swing phase. A discrete variable  $i_g$  which governs the transitions between swing and stance completes the description of the swing-stance model as a *hybrid dynamical system*. We define the *gait phase index*  $i_g$  as zero whenever the vertical reaction force is positive (stance phase), and as 1 if the force is zero (swing phase). Define the components of the state vectors as  $z = [z_1 \ z_2 \ z_3]^T = [q_1 \ q_2 \ q_3]^T$ and  $w = [\dot{q}_1 \ \dot{q}_2 \ \dot{q}_3]^T$  and let

$$M^{-1}(Ku - B - Cz_2 - F_f - J_e^T F_e - g(q)) = [p_1(z, w) \ p_2(z, w) \ p_3(z, w)]^T$$
(19)

In swing mode, z is integrated from w, which in turn is integrated from the accelerations  $p_i(z, w)$  for i = 1, 2, 3. In the stance mode,  $z_1$  and  $z_2$  are integrated from  $w_1$  and  $w_2$ , but  $z_3$  is integrated from the kinematic constraint of Eq. 18, and  $w_3$  becomes unnecessary. The other two angular velocities are integrated from their corresponding accelerations, as in the swing phase. Fig. 5 is a graphical representation of the hybrid model.

## 4 Parameter Estimation Procedures

### 4.1 Ballscrew Torque/Thrust Constant and Friction Force

A custom fixture was built to measure parameter  $\gamma$  in Eq. 6. The ballscrew was loaded statically by pressing the carriage against a load cell, recording thrust and torque (with a separate torque cell). The value of  $\gamma$  was

measured by linear regression to be 0.0024. This corresponds to an efficiency of 0.84 as calculated from Eq. 6, which falls in the expected range. With zero load and the slide in a horizontal position, a breaking torque must be applied to the ballscrew in order to initiate carriage motion. We regard this torque to correspond to static friction torque on the linear guides only. The breaking torque was measured repeatedly using the torque cell, yielding an average value of 0.2 Nm. From Eq. 6, the static friction force on the linear guides can be estimated as f = 83.33 N. Note that f is likely to decrease once the carriage starts moving. Since the main purpose of the model is to facilitate controller designs, we regard the magnitude of the friction force to be constant under static or dynamic conditions, a conservative assumption.

### 4.2 Vertical Actuator

The vertical slide is back-driveable, i.e., the carriage will move and turn the ballscrew when a force parallel to its axis is applied. This feature can be exploited to estimate various parameters by decoupling the motor and allowing the carriage to move downward under the action of gravity while recording its velocity. Two tests are conducted: one with the carriage and mounting plates included in link 1, and a second test where an additional mass  $\Delta m$  of known value is attached to the carriage. The governing equation for downward motion in the free-fall test is:

$$\left(m + \frac{J_b}{\gamma l}\right)\dot{v} = mg - f \tag{20}$$

where v is the velocity of the carriage (positive downward), f is the Coulomb friction force and m can equal the baseline mass  $m_1$  or  $m_1 + \Delta m$ , depending on the test. The downward accelerations  $\dot{v}_o$  and  $\dot{v}$  for the baseline and added-mass test are obtained by numerical differentiation of the captured velocity data. The ballscrew inertia is estimated with a standard formula [17] as  $J_b = 0.0015$  kg-m<sup>2</sup>. Since f has been already calculated, the linearly-moving mass can be calculated from Eq. 20. Using the data corresponding to the baseline mass test, we obtain  $m_1 = 43.28$  kg, while the data corresponding to the added-mass test yields a total mass of 61.67 kg. The true added mass was accurately measured to be  $\Delta m = 21.35$  kg, which predicts  $m_1 = 40.32$  kg from the calculated total mass. Although only two data points are available, a conservative estimate of the uncertainty in  $m_1$  can be taken as  $\pm 1.5$  kg or 3.6%.

Once the mass and friction parameters are known,  $k_1$  may be estimated by applying a constant analog input voltage  $u_1$  and extracting the resulting initial acceleration. Since the mass and friction force are known, the net force  $k_1u_1$  can be calculated, yielding an estimated value of  $k_1$ . Repeating the procedure with various values of  $u_1$ , it was found that the values of  $k_1$  varied in a range between 224 and 672 N/V. Alternatively, the carriage can be loaded statically against a load cell in a horizontal position, recording the applied voltage and the resulting force. This method yielded a value of  $k_1=375$  N/V from linear regression. This value is adopted for the nominal model used in controller design. Subsequent tests show that the controller is robust enough to accommodate the uncertainty in this amplifier gain.

#### 4.3 Rotary Actuator

The transfer function of Eq. 8 was measured by applying a swept-sinewave signal  $u_2(t)$  and recording the resulting output  $\dot{q}_2(t)$ , followed by a standard system identification technique. The measured transfer function was

$$G_2(s) = \frac{1}{0.1s + 0.65} \tag{21}$$

Note that the above data determines only the ratios  $(J_o + i^2 J_m)/k_2$  and  $b_r/k_2$ . However,  $k_2$  was measured directly by attaching a bar to the actuator and loading it statically against a load cell, measuring the applied voltage and corresponding torque. The servo amplifier was found to have an approximately linear torque-voltage curve, with an average sensitivity of  $k_2 = 15$  N-m/V. This value is then used to find  $b_r$  and  $J_o$ , since  $J_m$  is known from the motor manufacturer data.

#### 4.4 **Prosthesis Parameters**

Link lengths  $l_2$  and  $l_3$  and masses  $m_2$  and  $m_3$  were measured directly, while the locations of the centers of mass were determined by balancing the disassembled links on a knife edge. Since the moment of inertia and mass of the rotary actuator's gear, mounting plate and mounting rod are much larger than those of the prosthetic thigh, the moment of inertia of link 2 at the center of mass was assumed to coincide with the moment of inertia at the axis of rotation:  $I_{2z} = J_o + J_r$ , were  $J_r$  is the moment of inertia of the connecting

Parameter	Value	Units	Parameter	Value	Units
$\frac{J_s}{\gamma l}$	317.5	kg	$l_2$	0.425	m
$m_1$	43.28	kg	$l_3$	0.527	m
$m_2$	8.57	kg	$c_2$	-0.339	m
$m_3$	2.33	kg	$c_3$	0.320	m
$I_{2z}$	0.435	kg-m <sup>2</sup>	$r_d$	0.190	m
$I_{3z}$	0.062	kg-m <sup>2</sup>	$o_d$	0.029	m
$J_m$	$1.82 \times 10^{-4}$	kg-m <sup>2</sup>	$b_r$	9.75	N-m-s
f	83.33	Ν	$b_k$	427.7-2776.6	N-s/m

Table 1: Key Model Parameters

rod and nuts, measured in SolidWorks as 0.105 kg-m<sup>2</sup>. To determine the moment of inertia  $I_{3z}$ , the link was suspended and allowed to oscillate in a compound pendulum fashion. The period of oscillation and the distance between the suspension point and the center of mass were used to calculate the moments of inertia using the well-known period formula. The values of damper constant were determined for each dial setting using a custom fixture. The damper was fitted with a load cell to measure axial force, and a laser sensor was used to capture displacement. Damper extension and force were recorded as the damper was manually extended and compressed. Numerical differentiation was used to obtain velocity histories, which were linked to force by linear regressions. This was done for positive and negative velocities separately, yielding two sets of values for  $b_k$ . Key model parameters and their values have been listed in Table 1.

### 4.5 Model Validation

The model is evaluated according to its usefulness in control design, rather than its open-loop prediction abilities. This is because model parameters are subject to uncertainty, and prediction quality is strongly dependent on the choice of parameters. The model, with nominal parameters, was used to design and simulate a closed-loop controller, as described in Section 5. The controller was then deployed in real time and plant outputs were compared with the corresponding simulation outputs. As it can be seen in Figs. 6, 7 and 8, model predictions are more accurate for actively-controlled outputs than for knee angle.

## 5 Independent-Joint Control System

The overall prosthetic testing concept is demonstrated by using a preliminary control system to track realistic motion profiles. Since the rotary actuator is non back-driveable, the inertial coupling due to vertical motion and leg swinging can be safely ignored. This implies that the rotary actuator can be controlled using a local, single-input single-output (SISO) servo loop. Similarly, leg swinging is deemed to produce small inertial forces on the vertical servo system, therefore a sufficiently robust SISO controller may be used to track vertical hip displacement profiles.

Sliding Mode Control (SMC) was chosen as a development controller due to its good robustness properties and straightforward implementation [18, 19, 20]. Neglecting inertial coupling, each joint is assumed to follow a single-axis linear electromechanical model of the form

$$J\theta + b\theta = ku + \tau_d \tag{22}$$

where  $\theta(t)$  is the controlled position variable, u(t) is the control voltage (assumed proportional to motor torque), k is a constant reflecting a combination of servo amplifier gain, motor torque constant and rotary/linear motion conversion, J is the inertia of the load and motor and b is a viscous damping coefficient. Variable  $\tau_d$  represents an uncertain torque input consisting of actual external torque disturbances, unmodeled dynamics, parametric uncertainties and unmodeled static effects such as friction torque. Unmodeled dynamics include gravity torque and inertial coupling. An example of a parametric uncertainty is a value of J subject to uncertainty  $\Delta J$ . The uncertainty term can capture this error as  $-(\Delta J)\ddot{\theta}$ . SMC laws employ the concept of a performance surface, commonly referred to as *sliding manifold*. The sliding manifold is a function s chosen so that desirable tracking performance and reduced or no sensitivity to uncertainties is obtained when the system state is forced to remain in the set s = 0. In terms of the tracking error  $e = \theta^d - \theta$ , the system of Eq. 22 admits a sliding function of the form  $s = \dot{e} + \lambda e$ , with  $\lambda > 0$  a tunable constant. Note



Figure 6: Comparison between model and experimental outputs: hip displacement Thigh Angle Outputs



Figure 7: Comparison between model and experimental outputs: thigh angle



Figure 8: Comparison between model and experimental outputs: knee angle

that if s(t) = 0 for all t after some reaching time  $t_r$ , ideal first-order decay is achieved for the tracking error. A frequently-used SMC law capable of achieving and maintaining s = 0 in finite time despite the presence of the uncertain term  $\tau_d$  has the form:

$$u = \frac{J}{k} \left[ (\ddot{\theta}^d + \lambda \dot{\theta}^d) + (\frac{b}{J} - \lambda) \dot{\theta} + \eta \operatorname{sign} (s) \right]$$
(23)

where  $\eta > 0$  is chosen according to an assumed bound for  $\tau_d$ . Note that the motion profiles to be tracked enter the control law through the feedforward term  $\ddot{\theta}^d + \lambda \dot{\theta}^d$ . Motion profiles used for prosthetic testing are available as data sets, which may be readily differentiated offline to generate the required feedforward term. Implementation of this control law is simpler than that of a PID controller, since no online integration or differentiation are needed, and only position and velocity measurements are required, which are available from optical encoders.

The control law of Eq. 23 was developed using simplified actuator models. Its applicability in actual robot and prosthesis test conditions must be evaluated by simulation studies. Such controller validation was conducted by simulating the controller against the overall model of Section 3.4. Simulation results indicated that the controller would perform well, and that actuator limits would not be surpassed when tracking the planned motion profiles.

## 6 Real-Time Control Tests

Tests were conducted under pure motion feedback control, using the independent SMCs of Section 5. The robot is initially operated without ground contact, to verify the ability of the control system to track motion profiles without force disturbances. Then , manual bias control on the vertical stage was used to gradually "land" the foot on the treadmill, monitoring the force sensed by the load cell. The speed of the treadmill was manually synchronized to the horizontal velocity of the foot during contact, preventing slippage. This was done to evaluate the ability of the control system to operate in the presence of large force disturbances. Figures 9 and 10, show the hip displacement and hip swing tracking performance achieved by the independent-joint SMC, as well as the measured knee angle and control voltage histories. In this test, manual biasing was applied until a force peak of 450 N was observed. Figure 11 shows the ground force as measured by the load cell. As expected, the ground forces act as uncompensated disturbances, preventing exact tracking of vertical hip displacement and thigh angle during ground contact. Although knee angle is not being controlled, the prosthesis damping mechanism maintains it between 0 and 65 degrees, a range compatible with normal human walking. Note also that the control voltages are within the servo amplifier



Figure 9: Hip displacement and thigh angle tracking performance (with ground contact)



Figure 10: Knee angle and control voltage histories (with ground contact)

limits of  $\pm 8$  V for the vertical stage and  $\pm 10$  V for the rotary stage. Note that the force history shows a double peak, where the higher peak corresponds to flexion of the forefoot. With initial foot contact, ground forces are distributed over a relatively large area, only partly occupied by the load cell. As the foot flexes, the load cell is exposed to most of the distributed force, thus producing a higher reading. Although normal human gait also features a double-peak force profile, our tests have shown that the location of the peaks and their relative magnitudes will be largely influenced by load cell mounting and the phase of gait at the time of ground contact. If force feedback is to be implemented as part of a test, it would be convenient to install a force sensor on the ankle link, converting ground force requirements to ankle force demands.



Figure 11: Vertical ground force

## 7 Conclusions

Robot dynamic modeling was successfully used to model both machine and prosthesis, leading to a simulation model than can be used to test control concepts prior to real-time deployment. Other leg prostheses, including those including knee actuation and ankle force generation can be easily interfaced to the machine model. The robot will also prove useful in making objective comparison studies among competing prosthetic designs, since test conditions can be made accurately uniform.

Hazards testing can be conducted to some degree with this design, or with simple modifications. For instance, the inclination of the treadmill can be varied to simulate walking on ascending slopes. The mounting screws on the treadmill may also be adjusted to simulate a banked surface. Steps and surface irregularities can be simulated by replacing the treadmill with a reciprocating horizontal slide, on whose carriage a step can be mounted.

A higher level of realism in robotic testing of transfermoral prostheses may be achieved by adding rotational degrees of freedom to the mechanism. Rotations in the coronal and transverse planes are very small in comparison to hip swing ranges. A spherical mount may be inserted between the vertical carriage and the rotary actuator of this design to enable the additional degrees of freedom. This simple modification would allow testing at various fixed values of the two additional angles. Dynamic changes are likely to require a major re-design, since co-located actuation of several degrees of freedom is problematic. A hexapod platform driven by piezoelectric actuators could be used, if one existed with the required load capacities.

No attempt was made to implement force feedback for the stance phase during the preliminary tests, and therefore, no claim is made regarding the realism of the sensed force profiles in relation to those arising during normal gait. Although motion tracking was obtained even with ground contact, the results motivate the development of advanced controllers. The results strongly indicate that the robot will be capable of attaining combined motion-force tracking when such control algorithms are implemented.

## Appendix: Kinematic and Dynamic Parameters

Velocity Jacobian at Load Cell Location:

$$\begin{aligned} J_e(1,1) &= 0 \\ J_e(1,2) &= -(c_3 + l_{cx})\sin(q_2 + q_3) + l_{cy}\cos(q_2 + q_3) - l_2\sin(q_2) \\ J_e(1,3) &= -(c_3 + l_{cx})\sin(q_2 + q_3) + l_{cy}\cos(q_2 + q_3) \\ J_e(2,1) &= J_e(2,2) = J_e(2,3) = 0 \\ J_e(3,1) &= 1 \\ J_e(3,2) &= (c_3 + l_{cx})\cos(q_2 + q_3) + l_{cy}\sin(q_2 + q_3) + l_2\cos(q_2) \\ J_e(3,3) &= (c_3 + l_{cx})\cos(q_2 + q_3) + l_{cy}\sin(q_2 + q_3) \end{aligned}$$

Inertia Matrix:

Note: When integrating the actuator model into the robotic model,  $I_{2z} = J_2 + r^2 J_m$  is used, where  $J_2$  is the moment of inertia of the gears and link 2,  $J_m$  is the moment of inertia of the motor and r is the gear ratio. Coriolis/Centripetal Matrix:

$$\begin{array}{rcl} C(1,1) &=& 0\\ C(1,2) &=& -\dot{q}_2(l_2m_3 + m_2(c_2 + l2))\sin(q_2) - c_3m_3(\dot{q}_2 + \dot{q}_3)\sin(q_2 + q_3)\\ C(1,3) &=& -c_3m_3\sin(q_2 + q_3)(\dot{q}_2 + \dot{q}_3)\\ C(2,1) &=& 0\\ C(2,2) &=& -c_3l_2m_3\dot{q}_3\sin(q_3)\\ C(2,3) &=& -c_3l_2m_3\sin(q_3)(\dot{q}_2 + \dot{q}_3)\\ C(3,1) &=& 0\\ C(3,2) &=& c_3l_2m_3\dot{q}_2\sin(q_3)\\ C(3,3) &=& 0 \end{array}$$

Gravity Vector:

$$g = \begin{bmatrix} -g(m_1 + m_2 + m_3) \\ -c_3gm_3\cos(q_2 + q_3) - g(m_2(c_2 + l_2) + l_2m_3)\cos(q_2) \\ -c_3gm_3\cos(q_2 + q_3) \end{bmatrix}$$

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