## Energy-Oriented Design, Control and Optimization of Robotic Systems Synopsis of Results from NSF Grant #1536035

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## SVC and Energy Optimization with Brushless DC Machines

Most of our findings concerning simultaneous motion control and energy optimization used a DC motor/generator model. However, many robots are powered by brushless DC motors, which really are multiphase AC machines controlled to behave similarly to their DC counterparts.

One of the conventional and most well-known methods used to control these motors involves the use of a three-phase inverter, a PID current controller and a constant voltage supply as a power source [2]. The inverter is an arrangement of 6 solid-state switches such as MOSFETs. The inverter is intrinsically capable of bidirectional power flow and it produces the current waveforms necessary to produce the commanded motor torque. The PID controller drives the inverter using voltages as control outputs and the error between the actual and reference currents as inputs. Reference currents are commonly obtained from the desired motor torque (in turn produced by an outer loop controller) with the equation

$$\tau_{ind} = \frac{3P}{4} (\lambda_f i_q + (L_d - L_q) i_d i_q)$$

where  $i_d$  and  $i_q$  are current variables obtained by transformation of phase currents to a rotating reference frame. This is known as Park transformation.

There are two degrees of freedom to produce the desired  $\tau_{ind}$ , and  $i_d = 0$  is commonly used, since it maintains orthogonality between the stator and rotor fields and maximizing the strength of the induced torque. However, when energy optimization is considered, this choice is not optimal.

In Amin Ghorbanpour's work [1], the inverter is assumed to be connected to a supercapacitor, and the problem of finding the reference currents that maximize energy regeneration (or equivalently, minimize energy consumption) is posed and solved.

For this, the PID controller is assumed to achieve accurate tracking of reference currents and the inverter is assumed to have no losses. The internal energy balance equation was found to be

$$\Delta E_s = \frac{y^2(t_2) - y^2(t_1)}{2C} = \frac{3}{2} \int_{t_1}^{t_2} \{-R_s(i_q^{*2} + i_d^{*2}) - \omega_r \lambda_f i_q^* - (L_d - L_q)\omega_r i_q^* i_d^* - L_q i_q^* \frac{di_q^*}{dt} - L_d i_d^* \frac{di_d^*}{dt} \} dt$$

where the last two terms represent inductive storage, which is subsequently neglected for relatively small BLDC motors.

The problem of maximizing  $\Delta E_s$  by choosing currents  $i_d$  and  $i_q$  under the constraint given by the desired  $\tau_{ind}$  becomes a constrained minimization of a functional, solved in our work by introducing a Lagrange multiplier. A quartic equation is obtained for  $i^d$ , which is then solved analytically in the paper by invoking a 1922 paper (!!) describing the nature of the roots of such equations.

Interestingly, when the d and q frame inductances  $L_d$  and  $L_q$  are assumed equal, the solution would be suboptimal and lead to the well-known condition  $i_d = 0$ .

## Simulation code and media

Code is supplied to reproduce the simulations of [1] for a two-link planar robot driven actively in the first joint and semiactively in the second joint, using the model of the paper. Arbitrary sinusoidal trajectories were chosen for each joint, and the desired  $\tau_{ind}$  is mapped to inverter inputs (PWM duty ratios) using the instantaneous supercapacitor voltage as feedback. This effectively implements SVC for BLDC motors.

Simulations confirm that a reduction of energy consumption is obtained by using the optimal control strategy.

## References

- Amin Ghorbanpour and Hanz Richter. Control with optimal energy regeneration in robot manipulators driven by brushless dc motors. In ASME 2018 Dynamic Systems and Control Conference, pages V001T04A003–V001T04A003. American Society of Mechanical Engineers, 2018.
- [2] Iqbal Husain. Electric and hybrid vehicles: design fundamentals. CRC press, 2011.