Energy-Oriented Design, Control and Optimization of Robotic Systems

Synopsis of Results from NSF Grant #1536035

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Parametric Optimization

Recall that the internal energy balance equation without external forces is

$$\Delta E_s = \int_{t_1}^{t_2} \sum_{j=1}^{e} \left(\tau_j^d \dot{q}_j - \frac{R_j}{a_j^2} \left(\tau_j^d \right)^2 \right) dt \tag{1}$$

This equation gives the change in electric energy stored between any two times. Using the linear parameterization property (regressor-parameter form), the augmented robot dynamics can be expressed as

$$\tau_j = Y_j(q^d, \dot{q}^d, \ddot{q}^d)\Theta \tag{2}$$

where $Y_{n\times p}$ is the regressor of the augmented manipulator evaluated along reference trajectories, Y_j is the j-th row of the regressor, and $\theta_{p\times 1}$ is the parameter vector which combines individual physical parameters of to the robotic manipulator (i.e. link lengths, link masses, etc.).

This equation can be used to reformulate the EEB in vector form and conclude some equivalencies between the star and distributed configurations. For instance, the change in stored energy for a single semiactive joint in the distributed configuration is

$$\Delta E_{sj} = -\frac{R_j}{a_j^2}\theta^T \mathcal{G}\theta + \left(\mathcal{H} - \frac{2R_j}{a_j^2} I \right)^T \theta \tag{3}$$

where vector $\mathcal{H}$ is defined as

$$\mathcal{H} = \left[\int_{t_1}^{t_2} Y_{j1}\dot{q}_j^d \, dt, \quad \int_{t_1}^{t_2} Y_{j2}\dot{q}_j^d \, dt, \quad \cdots, \quad \int_{t_1}^{t_2} Y_{jp}\dot{q}_j^d \, dt \right]^T \tag{4}$$

vector I as

$$I = \begin{bmatrix} \int_{t_1}^{t_2} \left(m_j n_j^2 \ddot{q}_j + \left(b_j n_j^2 + \frac{a_j^2}{R_j} \right) \dot{q}_j^d + \mathcal{T}_j \right) Y_{j1} \, dt \\ \int_{t_1}^{t_2} \left(m_j n_j^2 \ddot{q}_j + \left(b_j n_j^2 + \frac{a_j^2}{R_j} \right) \dot{q}_j^d + \mathcal{T}_j \right) Y_{j2} \, dt \\ \vdots \\ \int_{t_1}^{t_2} \left(m_j n_j^2 \ddot{q}_j + \left(b_j n_j^2 + \frac{a_j^2}{R_j} \right) \dot{q}_j^d + \mathcal{T}_j \right) Y_{jp} \, dt \end{bmatrix} \tag{5}$$

and matrix $\mathcal{G}$ as

$$\mathcal{G} = \begin{bmatrix} \int_{t_1}^{t_2} Y_{j1}^2 \, dt & \cdots & \int_{t_1}^{t_2} Y_{j1}Y_{jp} \, dt \\ \vdots & \ddots & \vdots \\ \int_{t_1}^{t_2} Y_{jp}Y_{j1} \, dt & \cdots & \int_{t_1}^{t_2} Y_{jp}^2 \, dt \end{bmatrix} \tag{6}$$

We consider the problem of optimally selected the parameter vector Θ (reflecting the robot's design) to maximize the stored energy, for given robot joint trajectories.

A remarkable finding here is that ΔE_{sj} is quadratic in θ, and hence the optimization problem admits a unique global maximum, provided $\mathcal{G}$ is positive definite. In [2, 1], we show that $\mathcal{G}$ is a *Gram* matrix, and therefore automatically positive-semidefinite. Further, $\mathcal{G}$ is always positive-definite and invertible as long as the columns of the regressor are linearly independent.

With this, a closed-form, global expression is available for the optimal parameter vector:

$$\theta^* = \frac{a_j^2}{2R_j}\mathcal{G}^{-1}\left(\mathcal{H} - \frac{2R_j}{a_j^2}I\right) \tag{7}$$

Substitution gives the maximum attainable energy regeneration

$$\Delta E_{sj}^* = \frac{a_j^2}{4R_j}\left(\mathcal{H} - \frac{2R_j}{a_j^2}I\right)^T \mathcal{G}^{-1}\left(\mathcal{H} - \frac{2R_j}{a_j^2}I\right) \tag{8}$$

It was shown that optimizing ΔE_{sj} in the distributed configuration is the same as optimizing the contribution of the j-th semiactive joint to ΔE_s of the common storage in the star configuration. Also, optimizing $\sum \Delta E_{sj}$ in the distributed configuration is the same as optimizing ΔE_s in the star configuration.

Results were also developed to optimize part of the parameter vector (some robot features may be considered as not subject to design), to optimize the *fundamental electromechanical parameter* a_j/R_j and to find the optimal gear ratio. All problems admit closed-form, global solutions under mild conditions.

0.1 Limitations

The parameter optimization problems have some shortcomings:

1. They assume a given trajectory is followed by the robot, and the optimal parameters and energies are tailored to these trajectories. The robot may become very energy-inefficienty when different trajectories are used.

2. The solution for Θ^* may be difficult to link to valid selections of the corresponding individual parameters, such as masses, lengths, etc. Direct optimization of individual parameters is no longer analytically tractable, however. In [2], it is argued that Θ^* is still informative, as it provides the direction in which parameters must be changed to approach the optimum.

References

[1] Poya Khalaf and Hanz Richter. Parametric optimization of stored energy in robots with regenerative drive systems. In *2016 IEEE International Conference on Advanced Intelligent Mechatronics (AIM)*, pages 1424–1429. IEEE, 2016.

[2] Poya Khalaf and Hanz Richter. On global, closed-form solutions to parametric optimization problems for robots with energy regeneration. *Journal of Dynamic Systems, Measurement, and Control*, 140(3):031003, 2018.