## Energy-Oriented Design, Control and Optimization of Robotic Systems Synopsis of Results from NSF Grant #1536035

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## **Trajectory Optimization**

We continue to refer to the internal energy balance equation without external forces:

$$\Delta E_s = \int_{t_1}^{t_2} \sum_{j=1}^e \left( \tau_j^d \dot{q}_j - \frac{R_j}{a_j^2} \left( \tau_j^d \right)^2 \right) dt \tag{1}$$

which gives the change in stored energy in the star configuration. A meaningful problem is to find the virtual control torques  $\tau_j^d$  that maximize this energy, given problem-specific boundary conditions and constraints.

Depending on the approach followed to obtain optimal solutions, open-loop optimal torques  $\tau_j^{*d}$  and corresponding optimal trajectories  $q^*$  may be computed; or a feedback solution may be obtained in special cases, for instance when dynamics are linear (as in a Cartesian robot). In both cases, recall that SVC can be used to calculate the final controls (converter duty cycles) with an algebraic equation which uses storage element voltage feedback.

In [2, 1], the point-to-point trajectory and control optimization of a PUMA robotic manipulator was considered. Numerical optimal control was used offline to determine the open-loop virtual torques and optimal trajectories to transfer the end effector position between two points. Only the lower three joints were used, since the remaining degrees of freedom in this robot constitute the wrist.

The original servo drives in the PUMA were by-passed and replaced by low-cost SyRen regenerative drives manufactured by Dimension Engineering. The star configuration was used, connecting all three joints to a single Maxwell ultracapacitor bank rated at 48V and with capacitance 165 F. A comprehensive instrumentation layout was created, with current sensors installed on the ultracapacitor and motor sides. With these sensors, power flow could be tracked and losses evaluated at key points of the electrical subsystem.

A two-stage control approach was used, whereby the open-loop optimal trajectories were tracked by a suitable robust controller. The robust passivity-based control approach was found to be highly suited for this, and yielded very accurate tracking of optimal trajectories. The results provided firm validation of the modeling framework, the energy balance equations and the practicality and simplicity of SVC.

The optimization problem took the form

$$\max_{\boldsymbol{\tau}^{d} \boldsymbol{q}} J = \int_{0}^{t_{f}} \sum_{j=1}^{e} \left( \tau_{j}^{d} \dot{q}_{j} - \frac{R_{j} \tau_{j}^{d^{2}}}{a_{j}^{2}} \right) dt$$
(2)

subject to

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \mathcal{R}(q,\dot{q}) + g(q) + \mathcal{T} = \tau^d$$
(3a)

$$\frac{-a_j}{R_j} V_{cap} \le \tau_j^d \le \frac{a_j}{R_j} V_{cap} \tag{3b}$$

$$q_{t=0} = q_i \qquad \dot{q}_{t=0} = \dot{q}_i$$
 (3c)

$$q_{t=t_f} = q_f \qquad \dot{q}_{t=t_f} = \dot{q}_f \tag{3d}$$

where it was assumed for simplicity that  $V_{cap}$  was constant during the optimization time horizon. With a large capacitance, the energy loss or gain of the supercapacitor corresponds indeed to very small variations of this voltage.

The problem was solved offline using the IPOPT (interior point optimizer) numerical solver [3] using a refined model of the PUMA 560 manipulator. Details of the efforts taken to arrive at an accurate model which included friction can be consulted in [1].

Several test cases were attempted, some of which produced surprising solutions. For instance, in one case the end effector is required to move between two points lying at the same height. Rather than keeping the end effector in the horizontal plane containing the points and only twisting the waist, the optimal solution first elevated the end effector so that links 2 and 3 were nearly vertical, requiring a smaller gravity compensation torque (and consequent energy consumption) in comparison with holding them near the initial position at a lower height. Elevation consumes some energy, but a portion of it is recovered when descending to reach the initial height. This turned out to be optimal for this case.

Overall, the energy savings introducing by optimal energy regeneration were as high as 13%.

## Simulation code and media

Code is supplied to reproduce the optimal trajectory calculations of [1] for the PUMA 560 using IPOPT, which must be installed for Matlab. Videos of the robot following the optimal trajectories with the robust feedback controller are also available. The animation, video and plots are synchronized and show when regeneration occurs, and in which joint(s).

## References

- [1] P. Khalaf. Design, Control, and Optimization of Robots with Advanced Energy Regenerative Drive Systems. PhD thesis, Cleveland State University, 2019.
- [2] P. Khalaf and H. Richter. Trajectory optimization of robots with regenerative drive systems: Numerical and experimental results. *IEEE Transactions on Robotics*, (doi: 10.1109/TRO.2019.2923920):1–16, 2019.
- [3] Andreas Wächter and Lorenz T Biegler. On the implementation of an interior-point filter linesearch algorithm for large-scale nonlinear programming. *Mathematical programming*, 106(1):25– 57, 2006.