Problems

P1.7 Assume that the cannon fires initially at exactly 5:00 p.m. We have a positive feedback system. Denote by $\Delta t$ the time lost per day, and the net time error by $E_T$. Then the following relationships hold:

$$\Delta t = 4/3 \text{ min.} + 3 \text{ min.} = 13/3 \text{ min.}$$

and

$$E_T = 12 \text{ days} \times 13/3 \text{ min./day}.$$  

Therefore, the net time error after 15 days is

$$E_T = 52 \text{ min.}.$$  

P1.8 The student-teacher learning process:

P1.9 A heart-rate control system:
P1.19  A control system to keep a car at a given relative position offset from a lead car:

P1.20  A control system for a high-performance car with an adjustable wing:

P1.21  A control system for a twin-lift helicopter system:
E1.5  Tacking a sailboat as the wind shifts:

E1.6  An automated highway control system merging two lanes of traffic:

E1.7  Using the speedometer, the driver calculates the difference between the measured speed and the desired speed. The driver throttle knob or the brakes as necessary to adjust the speed. If the current speed is not too much over the desired speed, the driver may let friction and gravity slow the motorcycle down.
Use loop currents to avoid excessive variables.

\[ V_i - i_1 R_1 = V_x \quad \cdots \quad (1) \]
\[ V_x - i_2 R_2 = V_0 \quad \cdots \quad (3) \]
\[ i_1 - i_2 = \frac{C}{V_x} \quad \cdots \quad (2) \]
\[ V_0 = L \frac{di_2}{dt} \quad \cdots \quad (4) \]

Eliminate \( V_x, i_1, i_2 \).

Differentiate (3):
\[ V_x = V_0 + R_2 \frac{di_2}{dt} = V_0 + \frac{R_2}{L} V_0 \]

Combine with (2) to solve for \( i_1 \):
\[ V_x = V_0 + \frac{R_2}{L} \frac{di_2}{dt} = \frac{i_1 - i_2}{C} \]

Then
\[ i_1 = CV_0 + R_2 CV_0 + \frac{i_2}{L} \]

Substitute into (1):
\[ V_i - R_1 \left( CV_0 + R_2 CV_0 + \frac{i_2}{L} \right) = V_x = V_0 + \frac{i_2 R_2}{L} \]

Differentiate:
\[ V_i - R_1 CV_0 - \frac{R_1 R_2}{L} V_0 - R_1 \frac{di_2}{dt} = V_0 + \frac{R_2}{L} \frac{di_2}{dt} \]

Finally, use (4):
\[ \frac{di_2}{dt} = \frac{V_0}{L} \]

and substitute;
\[ V_i - R_1 C V_0 - \frac{R_1 R_2 C}{L} V_0 - \frac{R_1 V_0}{L} = V_0 + \frac{R_2 V_0}{L} \]

Re-arranging:

\[(R_1 C) V_0 + \left(1 + \frac{R_1 R_2 C}{L}\right) V_0 + \left(\frac{R_2}{L} + \frac{R_1}{L}\right) V_0 = V_i \]

\[3) \quad \text{Bar:} \]

\[\text{small displ.: (a+b)sin}\theta_1 \propto (a+b)\theta_1 \]

\[f_d \quad \text{Assume bar horizontal displacement} \quad > x \quad \text{(damper in compression)} \]

Since the bar has negligible mass, torques must balance:

\[f_d(a+b) = faa \quad \cdots (1) \]

\[\text{Damper:} \quad (a+b)\theta_1 \quad \text{assumed} \]

\[f_d = B_1 \left((a+b)\theta_1 - x\right) \quad \cdots (2) \]

\[\text{Block:} \quad \text{assume spring in compression} \]

\[f_d - f_5 = M\ddot{x} \quad \cdots (3) \]
Spring: \[ f_s \quad \begin{array}{c} \xrightarrow{x} \quad \xrightarrow{y} \quad \xleftarrow{f_s} \end{array} \quad \text{from lower point of right bar} \]

Right bar:

\[ z = d \sin \theta_2 = d \theta_2 \]
\[ y = c \sin \theta_2 = c \theta_2 \]

\[ f_s = K(x-y) \quad \text{(since compression \( x > y \) was assumed)} \]

\[ f_s = K(x - c \theta_2) \quad \text{--- (4)} \]

\[ \ddot{z} = d \ddot{\theta}_2 \]

\[ f_{d_2} = B_2 \ddot{z} = B_2 d \ddot{\theta}_2 \quad \text{--- (5)} \]

Since the bar has negligible mass, torques must balance:

\[ df_{d_2} = cf_s \quad \text{--- (6)} \]

From (1):

\[ f_{d_1} = \frac{af_a}{a+b} \]

From (5) and (6):

\[ f_s = \frac{d f_{d_2}}{C} = \frac{d}{C} B_2 \ddot{\theta}_2 = \frac{d^2 B_2}{C} \ddot{\theta}_2 \]
Substituting into (3):

\[
\frac{a f_a}{a+b} - \frac{d^2 B_2 \Theta_2}{c} = M \ddot{x} - \ddot{\Theta}_2 - (\star)
\]

From (4) \( f_s = K (X - c \Theta_2) = \frac{d^2 B_2 \Theta_2}{c} - (\star\star) \)

Equations (\star) and (\star\star) are a system of two differential equations in \( x \) and \( \Theta_2 \) with input \( f_a \).

To eliminate \( \Theta_2 \), solve for \( \dot{\Theta}_2 \) from (\star\star):

\[
\dot{\Theta}_2 = \frac{c}{d^2 B_2} \left[ \frac{a f_a}{a+b} - M \ddot{x} \right]
\]

Differentiate (\star\star) and substitute the expression for \( \dot{\Theta}_2 \) (and \( \ddot{\Theta}_2 \)):

\[
K (X - c \dot{\Theta}_2) = \frac{d^2 B_2 \dot{\Theta}_2}{c}
\]

\[
K \left[ \dot{x} - \frac{c^2}{d^2 B_2} \left( \frac{a f_a}{a+b} - M \ddot{x} \right) \right] = \frac{d^2 B_2}{c} \cdot \frac{c}{d^2 B_2} \left[ \frac{a f_a}{a+b} - M \ddot{x} \right]
\]

Re-arrange:

\[
M \dddot{x} + \frac{K c^2 M \dddot{x}}{d^2 B_2} + K \dddot{x} = \frac{a f_a}{a+b} + \frac{K c^2 a f_a}{d^2 B_2 (a+b)}
\]