$$D^T(q) = D(q) > 0 \quad \forall q$$

$\Longrightarrow$ there is a transformation

$$\underbrace{T^{-1}(q)\, D(q)\, T(q)}_{D_d(q)} \Longrightarrow \begin{bmatrix} \lambda_1(q) & & & \\ & \lambda_2(q) & & \\ & & \ddots & \\ & & & \lambda_n(q) \end{bmatrix}$$

$$0 < \lambda_1(q) \leq \lambda_n(q)$$

$$0 < \lambda_2(q) \leq \lambda_n(q)$$

$$\vdots$$

$$\Longrightarrow \quad D_d(q) \leq \lambda_n I = \lambda_n T^{-1}(q) T(q)$$

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cart pendulum – parameterization.

$$D(q) = \begin{bmatrix} \underbrace{m_1 + m_2}_{\theta_1} & -\underbrace{h\, m_2}_{\theta_2} \sin(q_2) \\ & \underbrace{m_2 h^2 + I_{2z}}_{\theta_3} \end{bmatrix}$$

$$D(q) = \begin{bmatrix} \theta_1 & -\theta_2 \sin q_2 \\ & \theta_3 \end{bmatrix}$$

$$C(q,\dot q) = \begin{bmatrix} 0 & -\theta_2 \sin q_2 \\ 0 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} 0 \\ -g\,\theta_2 \cos q_2 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 & -\theta_2 \sin q_2 \\ \cdot & \theta_3 \end{bmatrix}\begin{bmatrix} \ddot q_1 \\ \ddot q_2 \end{bmatrix} + \begin{bmatrix} 0 & -\theta_2 \sin q_2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -g\,\theta_2 \cos q_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$\theta_1 \ddot q_1 - \theta_2 \sin q_2 \ddot q_2 - \theta_2 \sin q_2 \dot q_2 = \tau_1$$

$$-\theta_2 \sin q_2 \ddot q_1 + \theta_3 \ddot q_2 - g\,\theta_2 \cos q_2 = \tau_2$$

$$\underbrace{\left[\begin{array}{c|c} \ddot q_1 & -\sin q_2 \ddot q_2 - \sin q_2 \dot q_2 \\ \hline 0 & -\sin q_2 - g \cos q_2 \end{array} \;\middle|\; \begin{array}{c} 0 \\ \hline \ddot q_2 \end{array}\right]}_{2\times 3} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$Y(q,\dot q,\ddot q) \qquad \theta$$