syms q1 q2

syms q1dot q2dot

M = [sin(q1) sin(q2) . . .
. . . . .]

Mdot = [diff(M(1,1), q1) * q1dot + diff(M(1,1), q2) * q2dot ...
- - - - -]

C = [... q1dot^2 . . . .
]

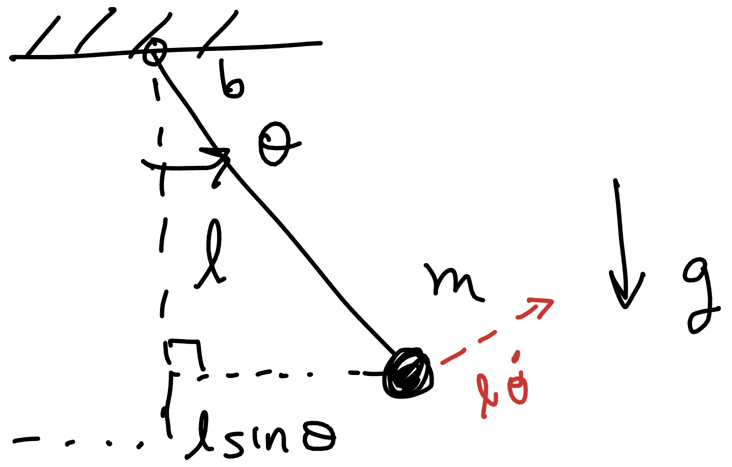
S = Mdot - 2*C;

test = [x1 x2] * S * [x1; x2];

(1x2)

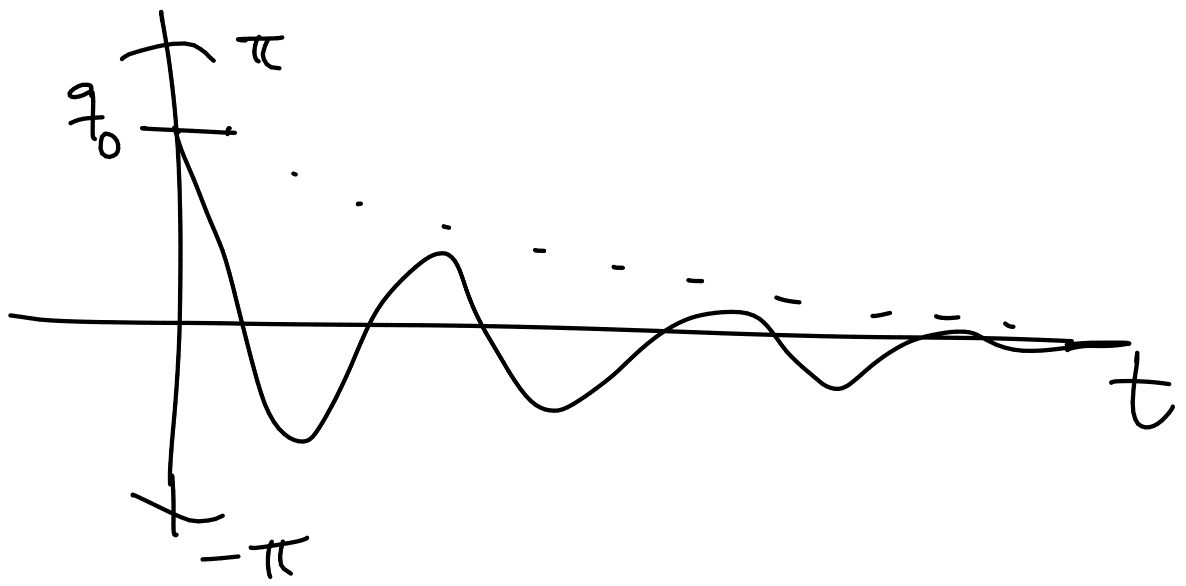
simplify(test)

$$[x_1 \;\; x_2]\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \equiv 0$$



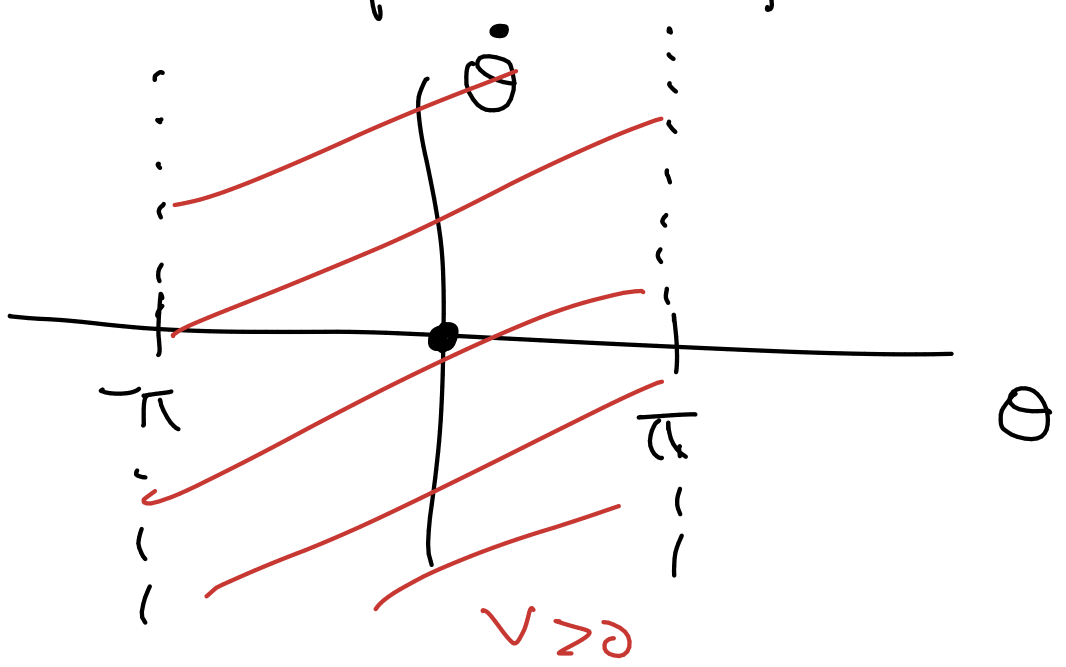
$$-b\dot\theta - \cancel{m}gl\sin\theta = \cancel{m}l^2\ddot\theta$$

$$\ddot\theta + \frac{g}{l}\sin\theta + \frac{b}{l^2 m}\dot\theta = 0$$



$$V = \frac{1}{2}m(l\dot\theta)^2 + mgl(1-\cos\theta)$$

V is positive-definite in $\underbrace{(-\pi,\pi)}_{\theta} \times \underbrace{\mathbb{R}}_{\dot\theta}$



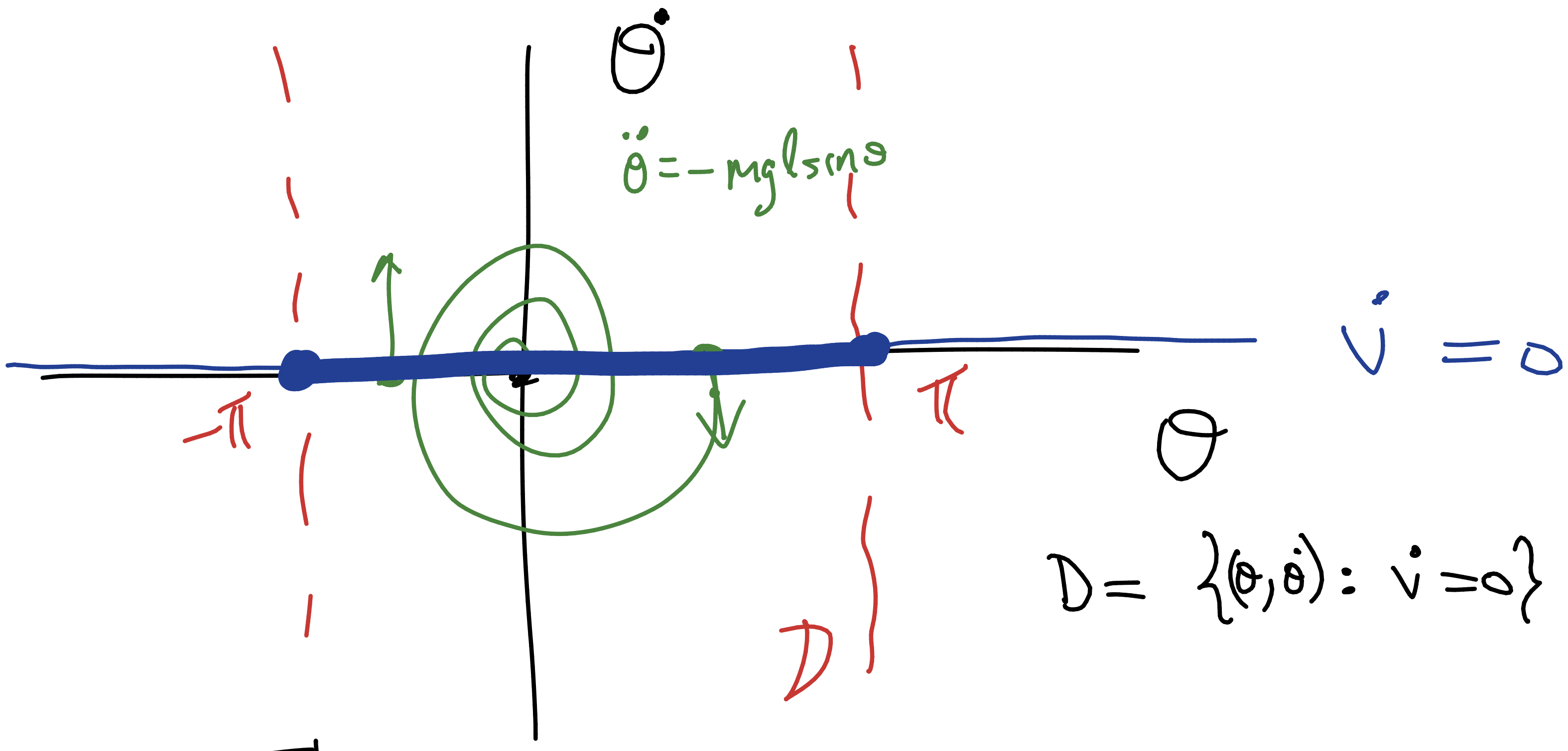
$$\dot V = ml^2\dot\theta\ddot\theta + mgl\sin\theta\,\dot\theta$$

$$\ddot\theta = \boxed{-\frac{g}{l}\sin\theta - \frac{b}{ml^2}\dot\theta}$$

$$\dot V = ml^2\dot\theta\left[-\frac{g}{l}\sin\theta - \frac{b}{ml^2}\dot\theta\right] + mgl\sin\theta\,\dot\theta$$

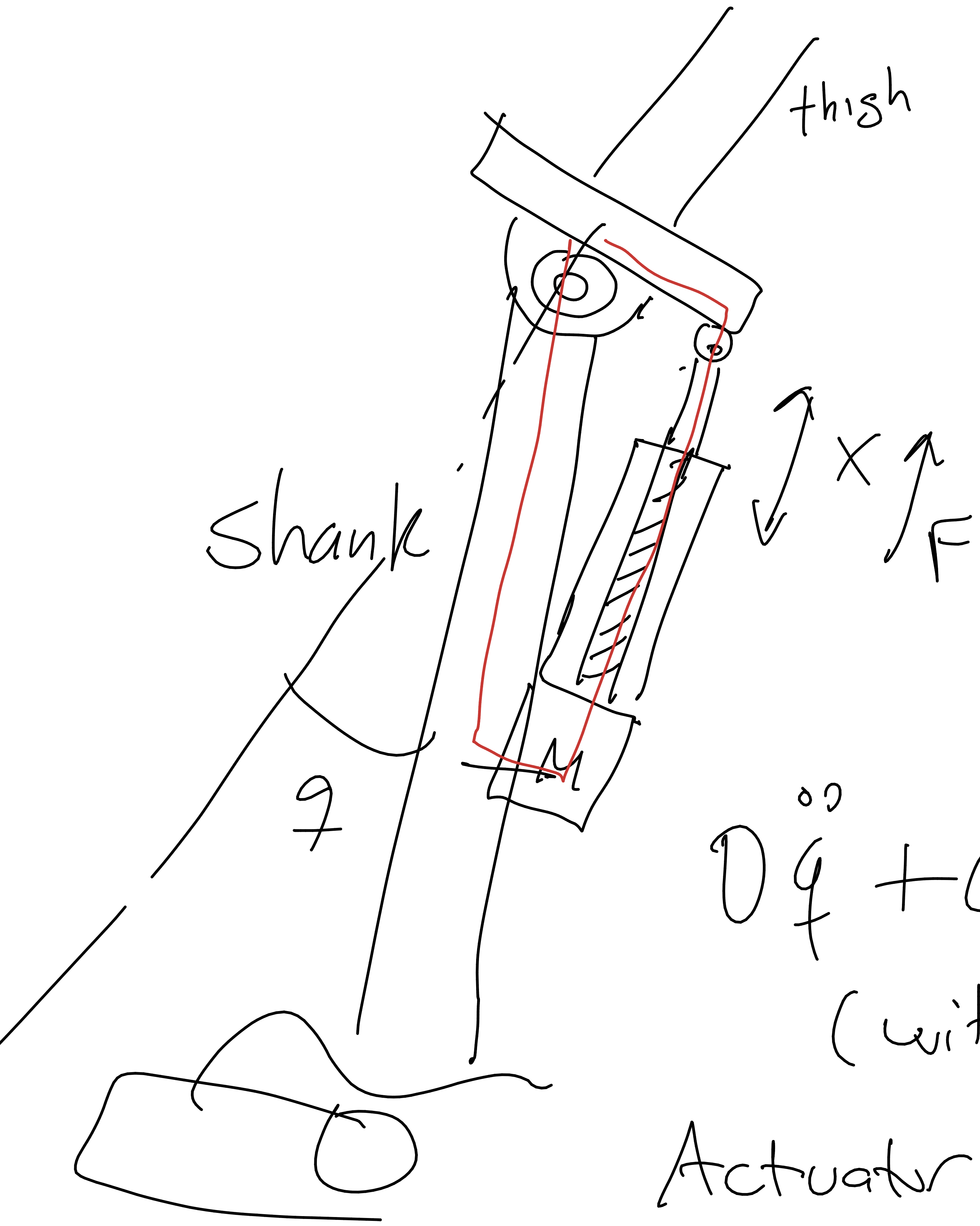
$$= \cancel{-mgl\sin\theta\dot\theta} - b\dot\theta^2 +$$

$$\dot V = -b\dot\theta^2$$



$D = \{(\theta, \dot{\theta}) : \dot{v} = 0\}$

The only invariant set contained in D is $(0,0)$



$\dot{x} = \dot{x}(q, \dot{q})$

$x = x(q)$

$F = F(q, \dot{q})$

$D\ddot{q} + C\dot{q} + g = \tau$

(without actuator)

Actuator:

$\tau = \tau(x)$

$M\ddot{q} + C\dot{q} + g = u$

- $M\ddot{q} + C\dot{q} = u$ (for PD, assume $B=0$, $g=0$)
- $u = -K_p\tilde{q} - K_D\dot{\tilde{q}}$, $\tilde{q} = q - q^d$, q^d: constant setpoint

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_p \tilde{q}$$

$$\dot{V} = \frac{1}{2}\left[\ddot{q}^T M(q)\dot{q} + \dot{q}^T(\dot{M}\dot{q} + M\ddot{q}) + \dot{\tilde{q}}^T K_p \tilde{q} + \tilde{q}^T K_p \dot{\tilde{q}}\right]$$

$\ddot{q} = M^{-1}(u - C\dot{q})$ (from robot dynamics)

$$2\dot{V} = \left[(u^T - \dot{q}^T C^T)\underbrace{(M^{-1})^T}_{M=M^T>0} M\dot{q} + \dot{q}^T\dot{M}\dot{q} + \dot{q}^T \cancel{M}\cancel{M^{-1}}(u - C\dot{q})\right) + \dot{\tilde{q}}^T K_p\tilde{q} + \tilde{q}^T K_p \dot{\tilde{q}}\Big]$$

$$2\dot{V} = \left[(u^T - \dot{q}^T C^T)\dot{q} + \dot{q}^T\dot{M}\dot{q} + \dot{q}^T(u - C\dot{q}) + \dot{\tilde{q}}^T K_p\tilde{q} + \tilde{q}^T K_p\dot{\tilde{q}}\right]$$

$$2\dot{V} = \Big[\underbrace{u^T\dot{q} + \dot{q}^T u}_{\text{scalars!}} - \underbrace{\dot{q}^T C^T\dot{q} - \dot{q}^T C\dot{q}}_{=(\,)^T} + \dot{q}^T\dot{M}\dot{q} + \dot{\tilde{q}}^T K_p\tilde{q} + \tilde{q}^T K_p\dot{\tilde{q}}\Big]$$

$$2\dot{V} = \left[2u^T\dot{q} - 2\dot{q}^T C\dot{q} + \dot{q}\dot{M}\dot{q} + \dot{\tilde{q}}^T K_p\tilde{q} + \tilde{q}^T K_p\dot{\tilde{q}}\right]$$

Zero because $\dot{M} - 2C$ is sk. symm.

$$2\dot{V} = \left[2u^T\dot{q} + \cancel{\dot{q}^T(\dot{M} - 2C)\dot{q}} + \ldots\ldots\right]$$

$$2\dot{V} = \Big[2\big[-\underbrace{\tilde{q}^T K_p}_{K_p,\,K_D \text{ are diagonal}} - \dot{\tilde{q}}^T K_D\big]\dot{q} + \underbrace{\dot{\tilde{q}}^T K_p\tilde{q} + \tilde{q}^T K_p\dot{\tilde{q}}}_{\text{scalars!}}\Big]$$

(q^d constant)

$$2\dot{V} = \left[\cancel{-2\tilde{q}^T K_p\dot{\tilde{q}}} - 2\dot{\tilde{q}}^T K_D\dot{\tilde{q}} + \cancel{2\dot{\tilde{q}}^T K_p\tilde{q}}\right]$$

$$\boxed{\dot{V} = -\dot{\tilde{q}}^T K_D\dot{\tilde{q}}}$$

$\dot{V}$ is neg. semidefinite

where is $\dot{V} = 0$?

Suppose $\dot{V} = 0 \implies \dot{\tilde{q}} = 0 \quad (\implies \dot{q} = 0)$

then $u = -K_p \tilde{q}$

$$M\ddot{q} + \cancel{C\dot{q}} = -K_p \tilde{q}$$

if $\tilde{q} \neq 0 \to$ robot accelerates away from $(\dot{q} = 0)$

by Lasalle: $q(t) \longrightarrow q^d$ as $t \to \infty$

PD + gravity compensation:

$$u = -K_p \tilde{q} - K_D \dot{\tilde{q}} + \underbrace{g(q)}_{\text{cross-coupled term.}}$$

$$M\ddot{q} + C\dot{q} + \cancel{g(q)} = -K_p \tilde{q} - K_D \dot{\tilde{q}} + \cancel{g(q)}$$

cross-coupled term.

$$M\cancel{\ddot{q}} + C\cancel{\dot{q}} + g(q) = -K_p \tilde{q} - \cancel{K_D \dot{\tilde{q}}}$$

w/o. grav. comp.

stable: $\ddot{q} = 0 \quad \dot{q} = 0$