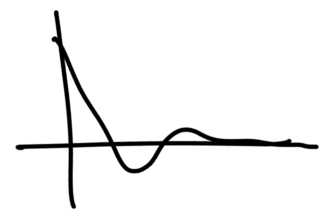
$$\begin{cases} \dot{x}_1 = x_2 + u \\ \dot{x}_2 = \sin(x_1) - x_3 \\ \dot{x}_3 = x_1 x_2 \\ y = x_2 \end{cases}$$

$$\ddot{y} = \cos x_1 (x_2 + u) - x_1 x_2 \equiv v$$

$$\hookrightarrow u = \frac{x_2(x_1 - \cos x_1)}{\cos x_1}$$

choose for example $v = -\dot{y} - y \Longrightarrow \ddot{y} + \dot{y} + y = 0$



Restrict the dynamics to $y = 0$

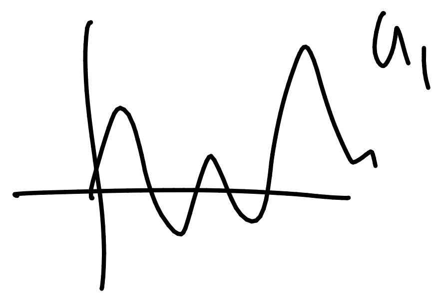
$x_2 = 0$

$\longrightarrow \dot{x}_3 = 0$

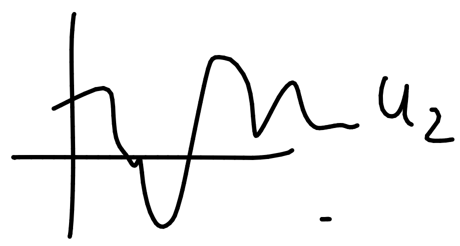
$u \to 0$ as $y \to 0$

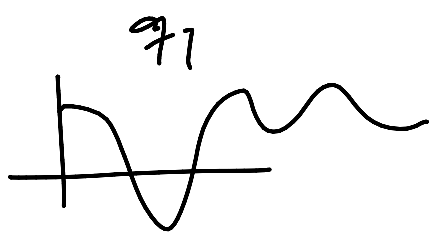
$\longrightarrow \dot{x}_1 = 0$

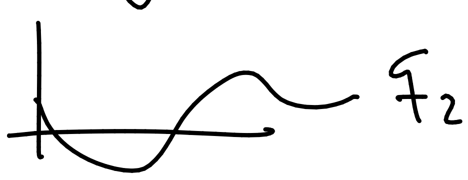
$$Y(q, \dot{q}, \ddot{q})\Theta = u$$



Apply $u_1(t)$, $u_2(t)$ ... $u_n(t)$







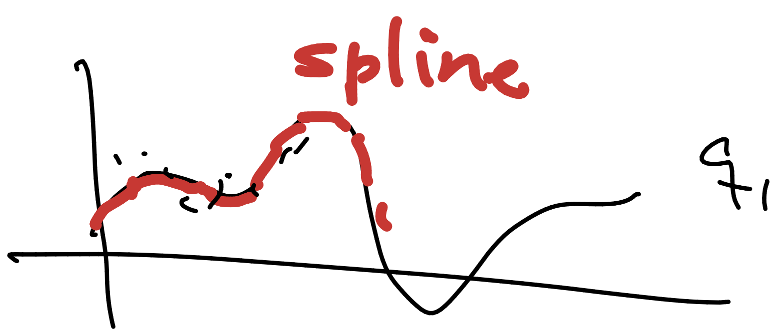
Measure $q_1(t)$, $q_2(t)$ ...

maybe measure $\dot{q}_1(t)$, $\dot{q}_i(t)$ ...

(if not, use numerical differentiation

$$\left\{ \begin{array}{l} q(t) \\ \dot{q}(t), \ \ddot{q}(t) \end{array} \right.$$

$$\{ \ u(t)$$



$N \times p \quad Y(q(t_1), \dot{q}(t_1), \ddot{q}(t_1))\,\theta = u(t_1)$

$N \times p \quad Y(q(t_2), \dot{q}(t_2), \ddot{q}(t_2))\,\theta = u(t_2)$

$\vdots \qquad\qquad\qquad \vdots$

$t_m \qquad\qquad\qquad t_m$

θ : $p \times 1$

N : # links

m : # time points

$$\underbrace{\begin{bmatrix} Y(t_1) \\ \vdots \\ Y(t_m) \end{bmatrix}}_{Nm \times p} \underset{p \times 1}{\theta} = \underbrace{\begin{bmatrix} u(t_1) \\ \vdots \\ u(t_m) \end{bmatrix}}_{Nm \times 1}$$

Y_{data} $u(t_1) \to n \times 1$ U_{data}

$$\boxed{\hat{\theta} = Y_{data}^{*}\, U_{data}}$$

least-squares estimate of θ