Passivity-based control

$$M\ddot{q} + C\dot{q} + g = u \quad \ldots\ldots (1)$$

$$u = Ma + Cv + g - Kr \quad \ldots\ldots (2)$$

$$\begin{cases} v = \dot{q}^d - \Lambda\tilde{q} & \tilde{q} = q - q^d \\ a = \dot{v} = \ddot{q}^d - \Lambda\dot{\tilde{q}} & \\ r = \dot{q} - v = \dot{\tilde{q}} + \Lambda\tilde{q} & \end{cases}$$

Λ, K : diagonal p. def. (gains)

Substitute u, r, a, v into (1)

$$\ldots\ldots \quad M\dot{r} + Cr + Kr = 0 \quad \ldots\ldots (3)$$

Choose $V = \frac{1}{2} r^T M r + \tilde{q}^T \Lambda K \tilde{q}$

$$\dot{V} = \frac{1}{2}\dot{r}^T M r + \frac{1}{2} r^T \dot{M} r + \frac{1}{2} r^T M \dot{r} + \underbrace{\dot{\tilde{q}}^T \Lambda K \tilde{q} + \tilde{q}^T \Lambda K \dot{\tilde{q}}}_{\text{scalar}}$$

$$\dot{V} = r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r + 2\tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$\dot{V} = r^T M\left[-M^{-1}Kr - M^{-1}Cr\right] + \frac{1}{2} r^T \dot{M} r + 2\tilde{q}^T \Lambda K \dot{\tilde{q}}$$

use (3)

$$\dot{V} = -r^T K r + \overbrace{\frac{1}{2} r^T (\dot{M} - 2C) r}^{0} + 2\tilde{q}^T \Lambda K \dot{\tilde{q}}$$

$$\dot{V} = -r^T K r + 2\tilde{q}^T \Lambda K \dot{\tilde{q}}$$

use the definition of $r = \dot{\tilde{q}} + \Lambda\tilde{q}$

$$\vdots$$

$$\dot{V} = -\tilde{q}^T \Lambda K \Lambda \tilde{q} - \dot{\tilde{q}}^T K \dot{\tilde{q}}$$

$$\dot{V} = -\tilde{q}^T \Lambda K \Lambda \tilde{q} - \dot{\tilde{q}}^T K \dot{\tilde{q}}$$

call $e = \left[\begin{array}{c} \tilde{q} \\ \hline \dot{\tilde{q}} \end{array}\right]$

$$\dot{V} = -e^T Q e, \quad \text{where} \quad Q = \left[\begin{array}{c|c} \Lambda K \Lambda & 0 \\ \hline 0 & K \end{array}\right]$$

$\dot{V}$ is neg. def.

V is p. def, radially unbounded

$\longrightarrow$ closed-loop syst. is globally asymptotically stable.

Regressor implementation:

$$u = \underbrace{Ma + Cv + g}_{Y(q, \dot{q}, v, a)\,\theta} - Kr$$

$$\underbrace{Y(q, \dot{q}, v, a)}_{\text{"}Y_{av}\text{" control regressor}}\,\theta - Kr$$

Robust Passivity-based control

$$M\ddot{q} + C\dot{q} + g = u \quad \cdots\cdots (1)$$

$$u = \hat{M}a + \hat{C}v + \hat{g} - Kr \quad \cdots\cdots (2)$$

$$\begin{cases} v = \dot{q}^d - \Lambda\tilde{q} & \tilde{q} = q - q^d \\ a = \dot{v} = \ddot{q}^d - \Lambda\dot{\tilde{q}} & \\ r = \dot{q} - v = \dot{\tilde{q}} + \Lambda\tilde{q} & \end{cases}$$

Λ, K : diagonal p. def. (gains)

By linear parameterization :

$$\hat{M}a + \hat{C}v + \hat{g} = Y_{av}\hat{\theta} \quad --- (3)$$

Substitute (2), v, a, r into (1)

(add and subtract $M(\ddot{q}^d - \Lambda\dot{\tilde{q}}) + C(\dot{q}^d - \Lambda\tilde{q})$)

$\vdots$

$$M\dot{r} + Cr + Kr = Y_{av}(\hat{\theta} - \theta) \quad --- (4)$$

Pick $\hat{\theta} = \underbrace{\theta_0}_{\text{nominal}} + \underbrace{\delta\theta}_{\text{switching term}}$

nominal → constants

true, unknown

$$M\dot{r} + Cr + Kr = Y_{av}(\theta_0 + \delta\theta - \theta)$$
$$= Y_{av}(\tilde{\theta} + \delta\theta)$$

$\tilde{\theta} = \theta_0 - \theta$

constant

Suppose $\|\tilde{\theta}\| \leq \rho$: a known bound.

Take $V = \frac{1}{2}r^T M r + \tilde{q}^T \Lambda K \tilde{q}$

$$\dot{V} = -r^T K r - r^T Y_{av}(\tilde{\theta} + \delta\theta) + \frac{1}{2}r^T(\dot{M} - 2C)r$$
$$+ 2\tilde{q}^T \Lambda K \dot{\tilde{q}}$$

0

Use the definition of r

$$\dot{V} = -e^T Q e + \underbrace{r^T Y_{av}(\tilde{\theta} + \delta\theta)}$$

$$Q = \left[\begin{array}{c|c} \Lambda K \Lambda & 0 \\ \hline 0 & K \end{array}\right], \quad e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$$

force term to be non-positive with $\delta\theta$

$$\delta\theta = \begin{cases} -\rho \dfrac{Y_{av}^T r}{\| Y_{av}^T r \|} & \text{if } \quad Y^T r \neq 0 \\ 0 & \text{if } \quad Y^T r = 0 \end{cases}$$

- Suppose $Y_i^T r = 0 \longrightarrow \dot{V} = -e^T Q e$ ✓ (done)
- Suppose $Y^T r \neq 0$

$$r^T Y_{av}(\tilde{\theta} + \delta\theta) = r^T Y_{av}\left(\tilde{\theta} - \frac{\rho Y_{av}^T r}{\| Y^T r \|}\right) =$$

$$= r^T Y_{av} \tilde{\theta} - \rho \| Y_{av}^T r \|$$

but $\quad |r^T Y_{av} \tilde{\theta}| \leq \| r^T Y_{av} \| \| \tilde{\theta} \| \leq \| r^T Y_{av} \| \rho$

$|a^T b| \leq \|a\| \|b\|$ (Cauchy-Schwarz ineq)

$$r^T Y_{av} \tilde{\theta} \geq 0$$

$$|r^T Y_{av} \tilde{\theta}| = r^T Y_{av} \tilde{\theta} \leq \rho \| r^T Y_{av} \|$$

so $\quad r^T Y_{av} \tilde{\theta} - \rho \| r^T Y_{av} \| \leq 0$

$\longrightarrow \dot{V}$ is n. def

$\searrow r^T Y_{av}\tilde{\theta} < 0 \quad \longrightarrow \quad r^T Y \hat{\theta} - \rho \| Y_{av}^T r \| < 0$

$\longrightarrow \quad \dot{V}$ is n. def.

$\longrightarrow$ global asymptotic stability follows.

Adaptive passivity-based control

Again use $\quad u = \hat{M} a + \hat{C} v + \hat{g} - K r \quad$ ---(2)

Still : $\quad M\dot{r} + C r + K r = Y \tilde{\theta}$

Use $\quad V = \frac{1}{2} r^T M r + \tilde{q}^T \Lambda K \tilde{q} + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta}$

$$\Gamma = \Gamma^T > 0$$

$$\dot{V} = r^T Y \tilde{\theta} + \cancel{\tfrac{1}{2} r^T (\dot{M} - 2C) r} - r^T K r + 2 \tilde{q}^T \Lambda K \dot{\tilde{q}} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}$$

use the definition of r, and note $\quad \dot{\tilde{\theta}} = \dot{\hat{\theta}}$

$$\tilde{\theta} = \underbrace{-\theta}_{\text{actual, constant}} + \hat{\theta}$$

$$\dot{V} = -\tilde{q}^T \Lambda K \Lambda \tilde{q} - \dot{\tilde{q}}^T K \dot{\tilde{q}} + \tilde{\theta}^T \Big[\underbrace{\Gamma \dot{\hat{\theta}} + Y^T r}_{0} \Big]$$

$$\dot{\hat{\theta}} = -\Gamma^{-1} Y^T r$$

adaptation law

$$\dot{V} = -e^T Q e, \quad \text{where}$$

$$Q = \left[\begin{array}{c|c} \Lambda K \Lambda & 0 \\ \hline 0 & K \end{array}\right] \qquad e = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$$

$\dot{V}$ is n.s.d.

(need Barbalat's lemma to conclude $\tilde{q} \to 0$)