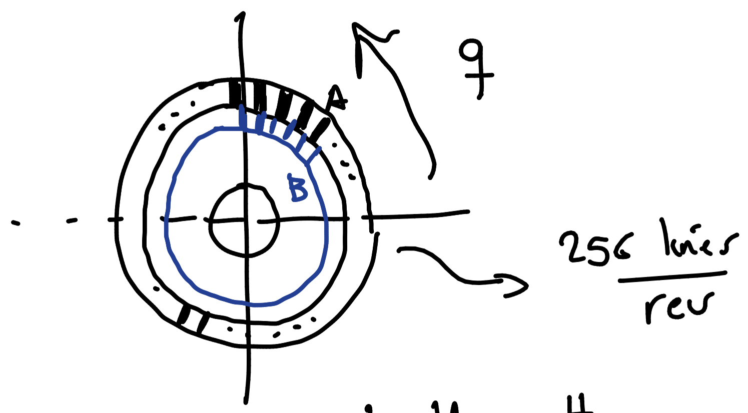
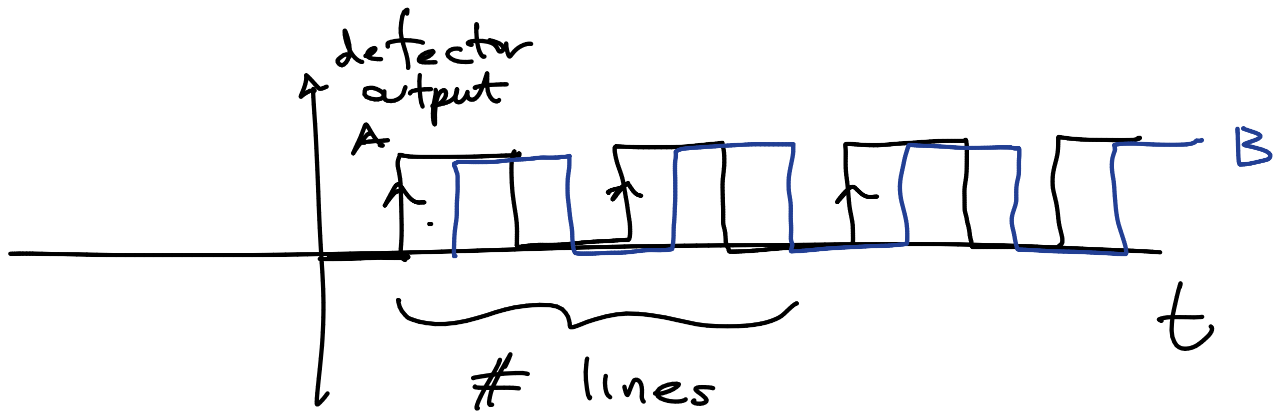
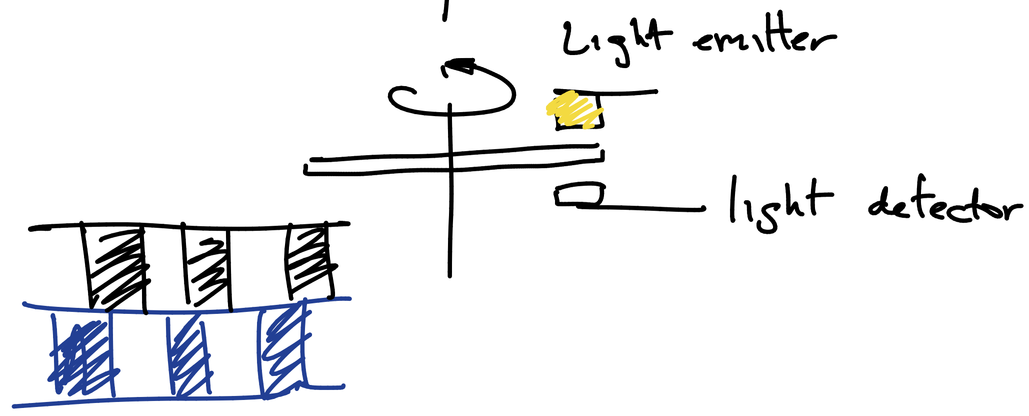
Quadrature optical encoder







A	B
1	0
1	1
0	1
0	0

raw counts A, B $\longrightarrow$ $\boxed{\text{quad. decoder}}$ $\longrightarrow$ position

$$\dot{x} \approx \frac{x_{k+1} - x_k}{\Delta t}$$

Order of error $\sim \Delta t$

central diff:

$$\underbrace{k \qquad k+1}_{\Delta t} \qquad \underbrace{k+1 \qquad k+2}_{\Delta t}$$

$\dot{x}_1 \qquad \dot{x}_2$

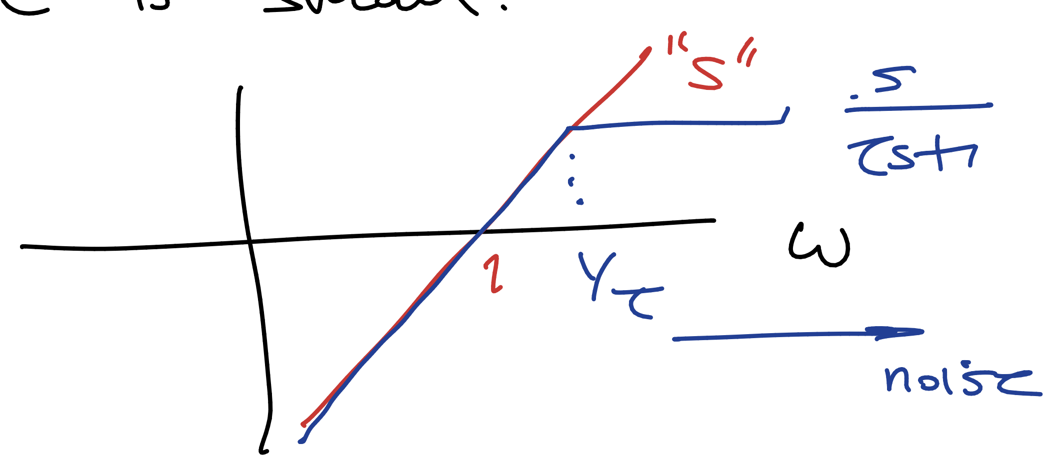
$$\underbrace{\text{average} \quad \frac{\dot{x}_1 + \dot{x}_2}{2}}_{\text{central diff.}} \qquad O(\Delta t^2)$$

real-time

$$\xrightarrow{q(t)} \boxed{\frac{s}{\tau s + 1}} \longrightarrow \approx \dot{q}$$

band-limited "dirty" derivative

τ is small.



state estimator (observer)

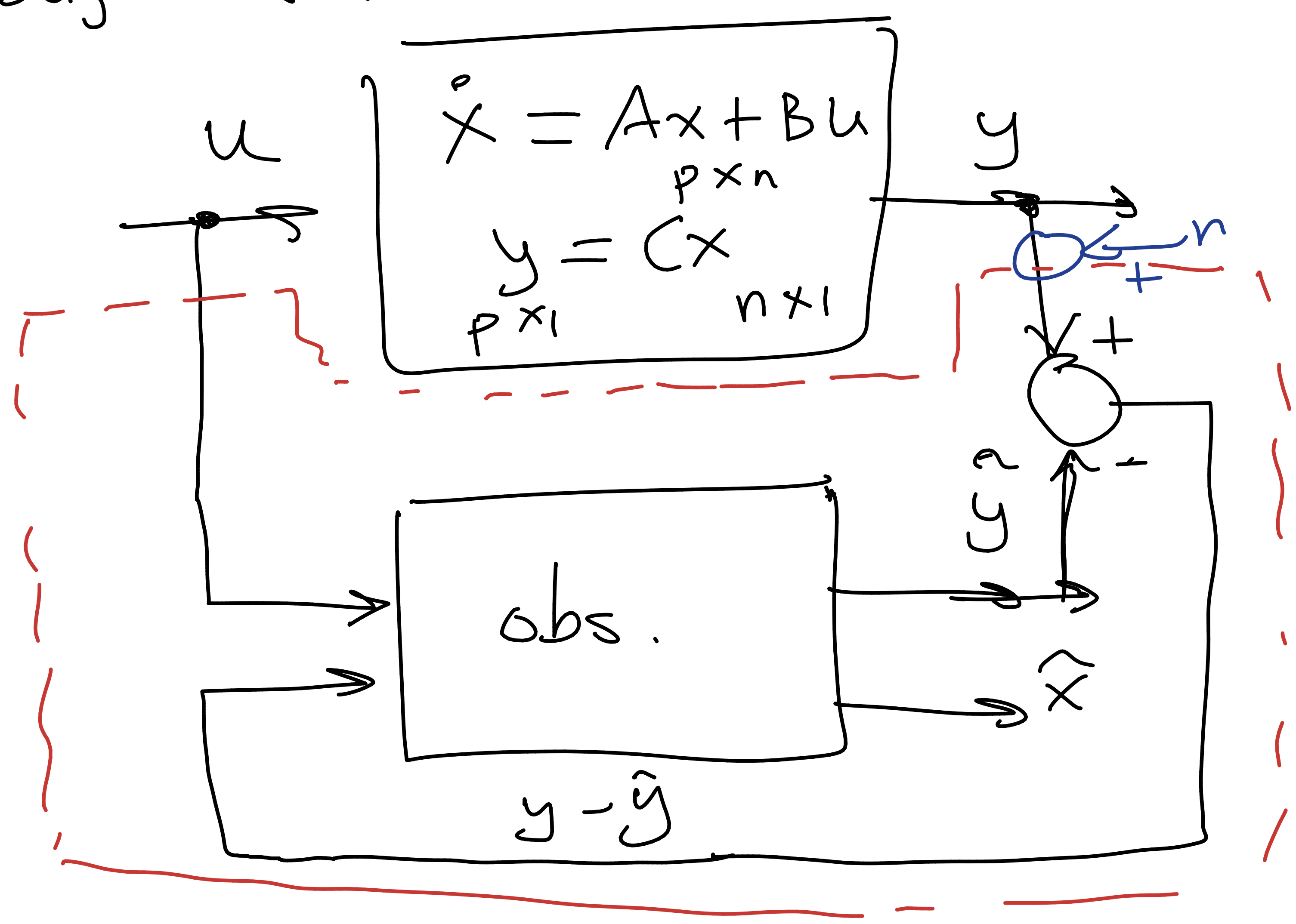
Linear:

$$\dot{x} = Ax + Bu$$

$$\left.\dot{\hat{x}} = A\hat{x} + Bu\ \right\}.$$

run as a real-time simulation driven by u

Luenberger observer



$$\underset{n\times 1}{\dot{\hat{x}}} = A\hat{x} + Bu + \underset{n\times p}{H}\,\underbrace{(y - \hat{y})}_{\text{output estimation error}} \quad p\times 1$$

$$\hat{y} = C\hat{x}$$

$$e = \hat{x} - x$$

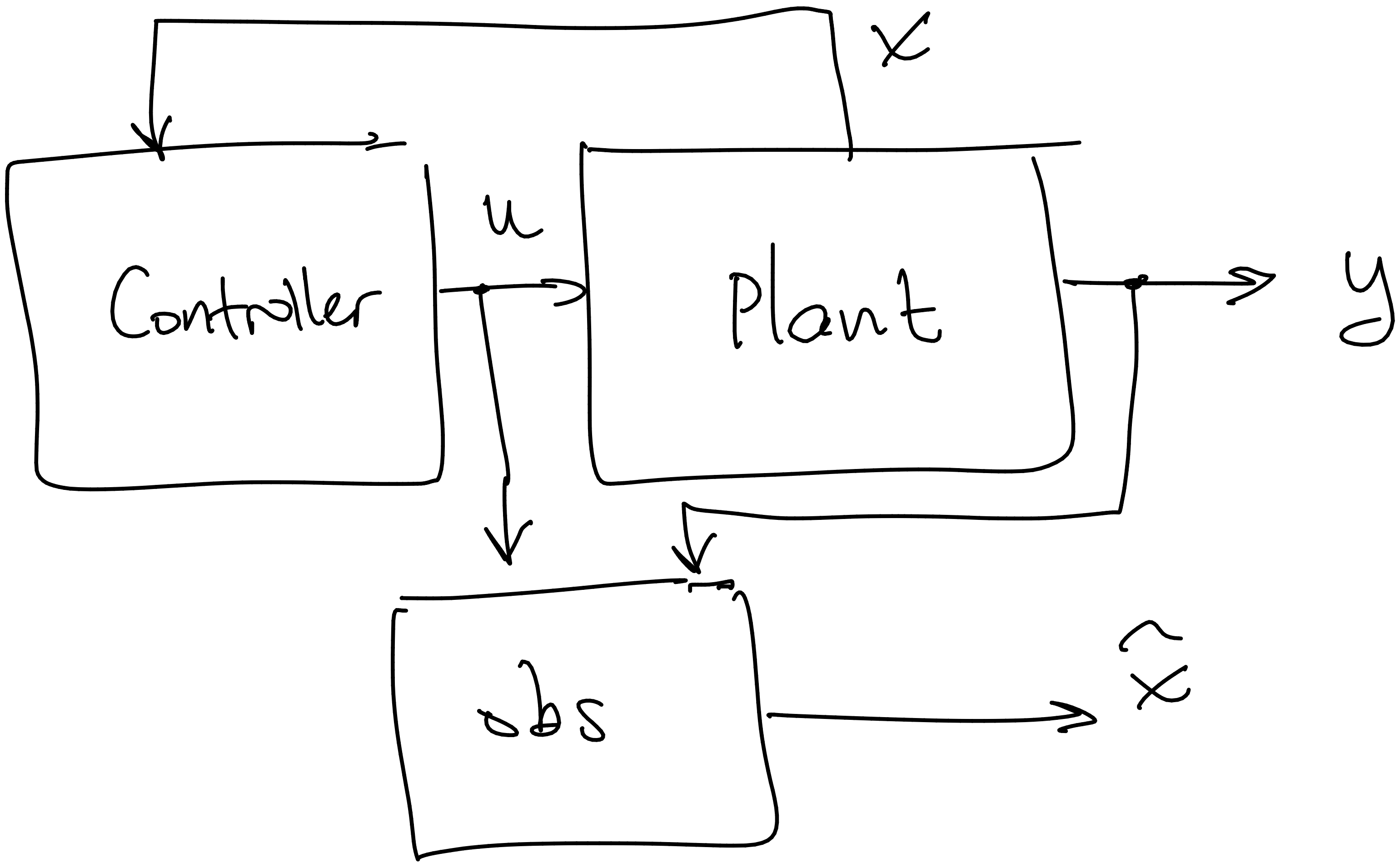
$$\dot{e} = \dot{\hat{x}} - \dot{x} = A\hat{x} + \cancel{Bu} + H(Cx - C\hat{x}) - Ax - \cancel{Bu}$$

$$\dot{e} = A(\hat{x} - x) - HC(\hat{x} - x)$$

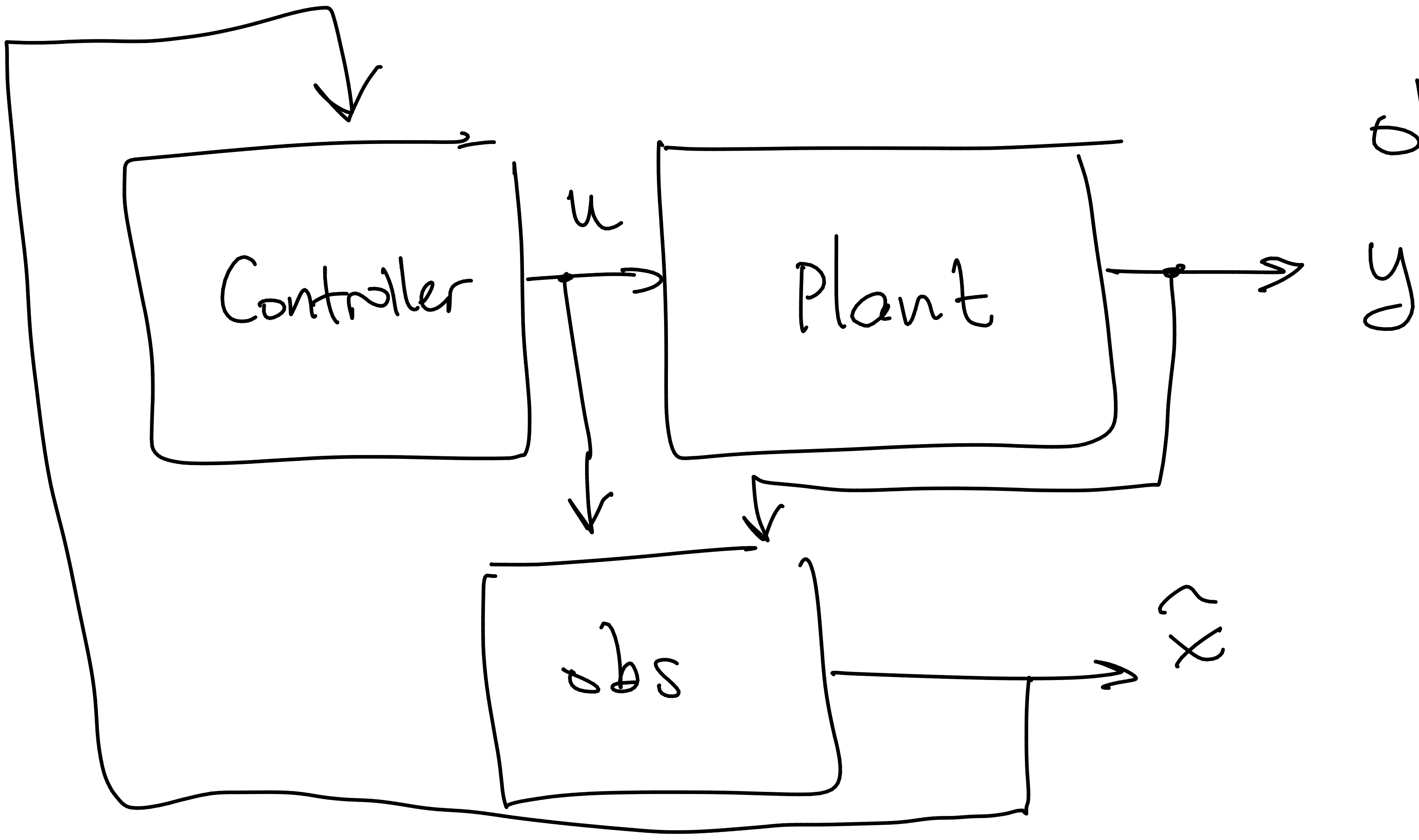
$$\boxed{\dot{e} = (\underset{n\times n}{A} - \underset{n\times p}{H}\underset{p\times n}{C})\underset{n\times 1}{e}}$$

To have $e \rightarrow 0$, make $A - HC$ stable by selecting H.

Kalman filter



state feedback control



observer-based control

$$
\begin{cases}
\dot{x} = Ax + Bu \\
u = -K\hat{x}\ , \quad y = Cx \\
\dot{\hat{x}} = A\hat{x} + Bu \\
\qquad + M(y - \hat{y}) \\
\hat{y} = C\hat{x}
\end{cases}
$$