Preliminary steps

1. Determine the slight difference between Matlab’s `atan2` and the function shown in Appendix A of SHV.

2. Read Sections 2.5.1 and 2.5.2 of SHV to develop a detailed understanding of the inverse orientation solutions using Euler.

3. Read Section 3.3 to understand the geometric approach to the inverse position solutions for common robot configurations.

**Problem 1:** Consider the PRP robot with spherical wrist shown in Fig. 1. Consider all 3 d.o.f. of the spherical wrist to be concentric (zero lengths between joints).

![Figure 1: 6-dof robot with spherical wrist.](image)

1. Assign a complete set of coordinate frames following the D-H convention and include a summary table. Sketch all frames.

2. For a point $p$ with local coordinates $p^6 = (0, 0, d_3)$, determine the forward kinematic transformation $H_{0}^6$ (write code that can calculate the transformation for any given joint coordinate vector $q$ and parameters $d_1, d_2, d_3$; don’t try to show the symbolic expression!).

3. Use the following numerical values: $d_1 = d_2 = 1, d_3 = 0.25$. Suppose that the joints are moved according to the following functions:
   - $q_1(t) = q_3(t) = 0.5 + 0.25 \sin(t)$
   - $q_2(t) = q_4(t) = 0.5 \sin(2t)$
\[ q_5(t) = \cos(t) \]
\[ q_6(t) = 2t \]

4. Make a 3D plot of the world position of \( p \) for \( t \) from zero to 10 time units.

5. Plot the \( x \), \( y \) and \( z \) world coordinates of \( p \) for the same time interval.

**Problem 2:** Consider the 4-axis SCARA robot shown in Fig. 2. The spindle at the end of the second link can move vertically and also rotate through 360 degrees, providing two degrees of freedom that can be independently actuated.

![Figure 2: 4-dof SCARA robot.](image)

A welding electrode is attached horizontally at the bottom end of the spindle (see Fig. 3). The electrode rotates and moves up and down with the spindle. The robot will be used to weld a helical seam on a circular-base cylinder located along the reference position shown in the figure. The cylinder has radius 50 mm, is 100 mm tall and is supported 75 mm above the base of the robot. The electrode must be normal to the circumference of the workpiece during the welding operation. For simplicity, assume that the electrode will not change length as it is consumed.

The seam must start 10 mm from the base of the cylinder, at the point nearest to the spindle. The helix should have 4 turns and stop 40 mm above the starting point. The seam must be generated at a rate of 4 seconds per revolution.

1. Assign coordinate frames starting from the information given in Fig. 3 and the dimensions at the end of this document. Use the D-H convention.

2. Develop the necessary forward kinematics as Matlab code.

3. Consult SHV for the solutions to the inverse kinematics for this robot and make necessary adjustments or simplifications as you see fit.
4. Find the mathematical expressions for the space curve to be followed, as well as the end frame orientation and write code to generate them numerically, using the required welding speed.

5. Use exact inverse kinematic formulas to find the four required joint angle histories.

6. Verify by feeding your solutions back into the forward kinematics.

7. Attempt a direct numerical solution using Corke’s Robotics Toolbox (work in meters to prevent numerical issues).

Doctoral students only (747):
Repeat the problem, but consider that the electrode is wearing out at a rate of 2 mm per revolution.
External dimensions

c = 150mm

d = 225mm

e = 760mm
$a = 325 \text{ mm}$

$b = 225 \text{ mm}$