1: The quadratic equation shown below represents a four-dimensional ellipsoidal surface.

\[ 2x_1^2 - 2x_1x_2 + 2x_1x_3 + x_2^2 - 2x_2x_3 + 3x_3^2 + 3x_4^2 = 1 \]

Use the matrix methods learned in this course to find:

1. The lengths of the principal semi-axes
2. The four-dimensional unit vectors aligned with the principal axes

2: Determine the Jacobian (velocity and angular velocity) for the cylindrical manipulator of HW2 in symbolic form, referred to wrist center. Symbolic solution with Matlab highly suggested.

3: For the 3x3 linear velocity Jacobian above, find all singularities and provide a graphical interpretation. What is the rank of the 3x3 angular velocity Jacobian? Provide an interpretation.

4: For the 2-link planar manipulator, prove that Yoshikawa’s manipulability measure is independent of the first joint coordinate (consider the 2x2 linear velocity Jacobian only).

5: For the 2-link planar manipulator, plot the manipulability ellipses (planar velocity only) at a few values of \( q_2 \) for \( q_1 = 0 \). Take link lengths equal to 1. Determine the “best” value of \( q_2 \) visually.

6: Assuming the statement in Problem 4 is true, find the value of \( q_2 \) maximizing \( \mu \) analytically. What is \( \mu \)'s maximum value?