# MCE/EEC 647/747
# Homework 3 - Spring 2017
**Due 2/28/17**

**Problem:** Consider the RPR robot shown in Fig. 1, with the DH parameters of Table 1, with $d > 0$.

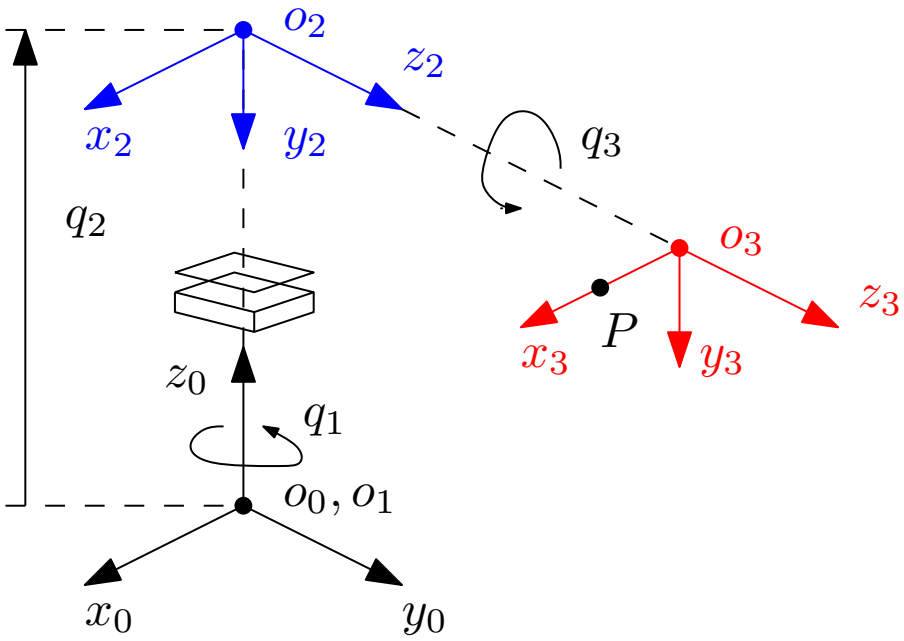


Figure 1: 3-dof robot

Point P has coordinates $P^1 = [l; 0; 0]^T$, where $l > 0$ is assumed.

1. Find the linear and angular velocity Jacobians relative to $P$, in symbolic form.

2. Use the velocity Jacobian to find the world velocity of $P$ when $q_1 = 0, q_3 = -\pi/2, q_2 = 1, \dot{q}_1 = \dot{q}_2 = 0$ and $\dot{q}_3 = -1$. Sketch the robot configuration and verify that the calculated velocity makes sense. Repeat for $q_1 = \pi/2, q_3 = \pi/2, q_2 = 1, \dot{q}_1 = \dot{q}_2 = 0$ and $\dot{q}_3 = 1$.

3. Find all singular configurations for the linear velocity and angular velocity as separate cases. Provide a convincing explanation for the results. Bonus 10 points: present your explanation to the class in 3 minutes or less.

4. Find the parameters of the manipulability ellipsoid/ellipse (principal axes directions and lengths) for $q_1 = \pi/4, q_2 = 1, q_3 = \pi/2, l = 1$ and $d = 1$. Repeat for $q_1 = \pi/2, q_2 = 1, q_3 = -\pi/4, l = 1$ and $d = 1$. Carefully sketch the ellipsoid or ellipse for each case (computer graphics will give you 5 bonus points).

5. Find the Yoshikawa manipulability measure (based on linear velocity only) in symbolic form and find the angle(s) at which it is maximum.

6. Find the Yoshikawa manipulability measure (based on the stacked velocity and angular velocity Jacobian) in symbolic form to show that it only depends on $q_3$.

7. Doctoral students: Find the analytical solution for the angles $q_3$ that maximize the above manipulability.

8. For $l = 1$ and $d = 1$, find the angles $q_3$ that result in maximum manipulability.

| Link | $\theta$ | $d$ | $\alpha$ | $a$ |
|---|---|---|---|---|
| 1 | $q_1^*$ | 0 | 0 | 0 |
| 2 | 0 | $q_2^*$ | $-\frac{\pi}{2}$ | 0 |
| 3 | $q_3^*$ | d | 0 | 0 |

Table 1: DH parameters for RPR robot