MCE/EEC 647/747
Homework 4 - Spring 2017
Due 3/23/17

A magnetic levitation system manufactured consists of an electromagnet and an optical sensor to detect the vertical position of a steel ball. The objective is to establish a feedback loop to maintain the ball floating in mid-air, between the sensor post and the electromagnet end surface. A system schematic is shown in Fig. 1. Assume that we can control the current

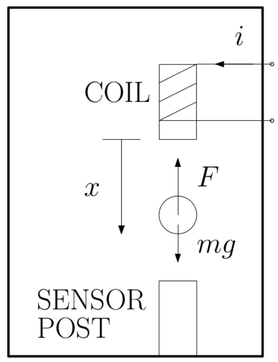

Figure 1: Schematic of Maglev System

using an amplifier capable of delivering any requested current between -2.5 and 2.5 A. The upward force exerted by the electromagnet on the ball is

$$F = \frac{ki^2}{2x^2}$$

where $k = 6.5308 \times 10^{-5}$ N-m^2/A^2, i is the current on the coil and x is the gap between the top of the ball and the electromagnet flat end. The mass of the ball is $m = 0.068$ kg and the acceleration of gravity is $g = 9.81$ m/s^2.

Task 1

1. Find the differential equation relating i (input) to x (output), accounting for gravity and electromagnetic force.

2. Show that the current needed to maintain the ball in equlibrium at an air gap value of x_{eq} is

$$i_{eq} = \sqrt{\frac{2mg}{k}} x_{eq} = k_{ff} x_{eq} \tag{1}$$

3. Obtain a linearized differential equation model for operation near i_{eq} and x_{eq}. For this, use a 2-variable Taylor expansion formula for $mg - F$ at the equilibrium point.

4. Use the above to show that the linearized transfer function from Δi to Δx is

$$G(s) = \frac{\Delta X(s)}{\Delta I(s)} = \frac{-2g/i_{eq}}{s^2 - 2g/x_{eq}} \tag{2}$$

5. Obtain a state-space representation of the linearized system at the equilibrium point.

1 Control Design by Loopshaping

The transfer function from incremental current to incremental position is given by Eq. 2. Design a compensator to meet the following specifications:

- Zero steady-state error to step inputs
- Settling time to a step input of approx. 1 second
- Overshoot as small as you can.
- The range of motion is limited between $x = 0$ and $x = 0.014$ m. The current must be kept within ± 2.5 A.

1. Transfer the design specifications to the frequency domain (except for the fourth).
2. Find i_{eq} corresponding to $x_{eq} = 0.006$ m from Eq. 1. Then obtain the numerical values for the TF of Eq. 2 and the value of feedforward gain k_{ff}.
3. Test for a stable response using Simulink file. Use the *linearized* plant and a step input of 1 mm. The settling time should be near 1 second, but the overshoot may be high (100%). The actual system will use a gentle ramp, so don't be concerned about the overshoot.
4. Check that the current spike stays within ± 1 A for a step input of 1mm.
5. Prepare a Simulink model to test the compensator against the actual nonlinear plant, as done in class (you have to account for offsets, in this case the bias current must be injected in a feedforward fashion). Instead of a step input, use a ramp to transfer the position from rest at the post (14mm) to mid-air, at 6mm from the flat end of the electromagnet in 0.8 seconds. Check that the current remains within bounds.

2 Control Design by LQR

Use a state-space representation to design a linear state feedback controller for regulation to a setpoint with the same specifications as above. Use LQR to find the feedback gain. Test the controller against the nonlinear plant using Simulink.