MCE/EEC 647/747
Homework 5 - Spring 2015
Due 4/7/15 at the beginning of class

This assignment is entirely based on a Furuta pendulum, a system frequently used to study underactuated control systems. This assignment focuses on modeling, linearization and linear state feedback control of the linearized model. Start by watching the following: `https://www.youtube.com/watch?v=ggVRmfNs_z8`. Note that there is a motor for the arm, but not for the pendulum.

Next, download the paper by Xu, Iwase and Furuta "Time-Optimal Swing-Up Control of Single Pendulum", ASME Journal of Dynamic Systems, Measurements and Control, v. 123, N.3, p.518-527, 2001. You can access the paper through our library.

Focus on Section 2, Fig.1 and the description of model parameters. The authors use certain method to derive the dynamic equation (Eq.10). You will use the methods of this course to obtain essentially the same equation.

1: Assign DH frames: Use the same world frame as in Fig.1, placing the origin on the intersection of the arm centerline and the cylinder's axis. Call $q_1 = \theta_1$ and correct the positive direction according to the right-hand rule. Do the same for the axis of rotation of the pendulum (keep $q_2 = 0$ as the vertical position).

2: Prepare all necessary kinematic transformations and Jacobians: Note that the arm only rotates, its CM location does not matter.

3: Find the mass, Coriolis and gravity matrices: Use the methods developed in the course (use symbolic computation). To do this, ignore all friction. Compare your equation to Eq.10. Aside from friction, you should get the same result.

4: Small-signal linearization: Convert Eq.10 to state-space form, with position-velocity states. Linearize the state-space model relative to the zero equilibrium point as done in class. Convert the linearized model back to a set of two 2nd-order differential equations as in Eq. 11 and compare.

5: Linear controller design: Use the linearized model to design an LQ regulator as done in Section 4.2. Use the parameters of Table 1. Simulate the performance of the LQ regulator applied to the actual nonlinear model (use initial conditions which are close to the equilibrium point). Use Simulink.

6: Quadratic Lyapunov function: Find a state-space representation (just an "A" matrix) of the linearized model under LQ state feedback. Find a quadratic Lyapunov equation by solving Lyapunov's equation for some $Q > 0$.

7: Doctoral students only: Add an online computation of the above Lyapunov function to the Simulink model. Run a simulation using some initial condition within the region of attraction of the closed-loop system. Does the Lyapunov function decrease monotonically? Did you expect this to happen?

8: Regressor and parameters: Find a symbolic linear parameterization of the dynamic model of part 3. Give a minimal parameter vector and the corresponding regressor. Check that the regressor and parameters work (they reproduce the original equations).