MCE/EEC 647/747
Homework 5 - Spring 2017
Due 4/11/17

1: Consider the RPR robot used for HW3 (Fig. 1), with the DH parameters of Table 1, where $d > 0$.

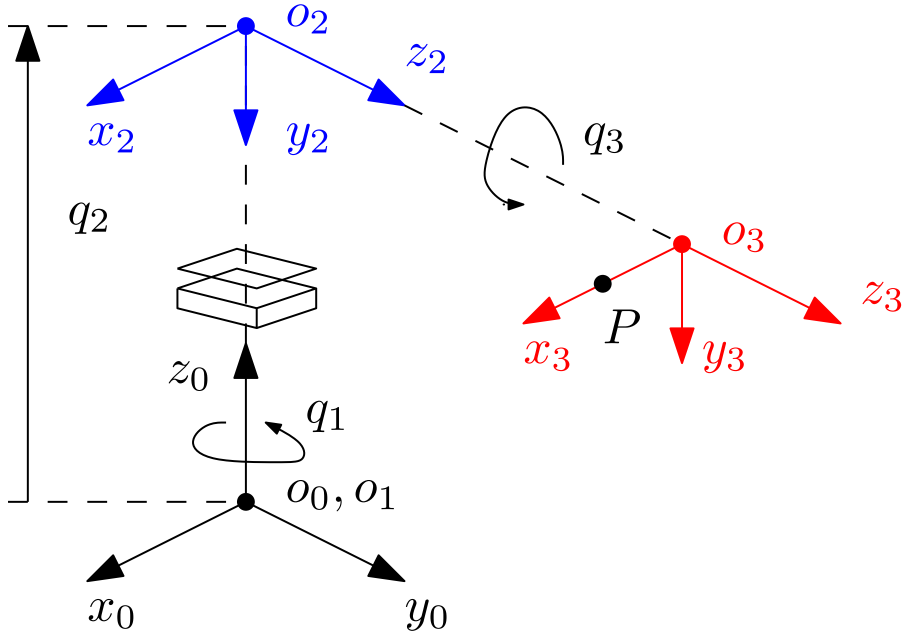

Figure 1: 3-dof robot

Point P has coordinates $P^3 = [l\ 0\ 0]^T$, where $l > 0$ is assumed. Consider the following:

- Link 1 has moment of inertia I_{z1} about the z_0 axis
- Link 2 has mass m_2 and center of mass at o_2
- Link 3 has a moment of inertia matrix $I = \text{diag}\,[I_{x3}, I_{y3}, I_{z3}]$ relative to a frame parallel to frame 3 but centered at the center of mass (halfway between o_2 and o_3). The mass of the link is m_3.

Link	θ	d	α	a
1	q_1^*	0	0	0
2	0	q_2^*	$-\frac{\pi}{2}$	0
3	q_3^*	d	0	0

Table 1: DH parameters for RPR robot

Do the following:

1. Find the mass matrix $M(q)$, Coriolis matrix $C(q, \dot{q})$ and gravity vector $g(q)$ for this robot using symbolic computer algebra. Write clearly commented code and report on the results.
2. Use symbolic computing to verify that M is symmetric and $\dot{M} - 2C$ is skew-symmetric.
3. Find a minimal linear parameter representation $Y(q, \dot{q}, \ddot{q})\Theta$ for this robot. Verify the results as done in class.

2:

- Let x be a column vector and let $f(x) = (Ax + b)^T C(Dx + e)$, where A, C and D are matrices and b and e vectors of compatible dimensions. A formula[1] for the differential of f with respect to x is
$$df(x) = ((Ax + b)^T CD + (Dx + e)^T C^T A)dx$$
Assume that x has 2 elements and that all matrices are 2-by-2. Verify the formula by using symbolic computer algebra (find $f(x)$ and differentiate explictly).
- Let X be a matrix and let $f(X) = X^{-1}$. A formula for the differential of f with respect to X is
$$d(X^{-1}) = -X^{-1} dX X^{-1}$$
Assume that X is 2-by-2 and verify the formula as above.

3: Consider the multi-input nonlinear control system
$$\dot{x} = A(x)x + (1 + x^T x)Bu$$
where $A(x)$ is n-by-n and B is n-by-m and constant.

- Use the quadratic Lyapunov function
$$V(x) = x^T P x$$
with $P = P^T > 0$ to show that the control law
$$u = -(PB)^* \frac{(\Gamma + PA(x) + A^T(x)P)x}{1 + x^T x}$$
with $\Gamma = \Gamma^T > 0$ results in asymptotic stability of the origin, where * denotes the Moore-Penrose pseudoinverse and PB is assumed full-rank.
- Take
$$A = \begin{bmatrix} \sin(x_1)\cos(x_2) & x_1 - x_2 \\ 0 & x_1^2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \tag{1}$$

Select $P = P^T > 0$ such that PB is full-rank and simulate the control system using initial condition $x_0^T = [1 \ -2]$. Plot the time histories of each state, the control and a state-space trajectory (x_1 vs. x_2). Also plot the Lyapunov function value and its derivative as a function of time.

[1]Matrix Reference Manual, Imperial College: `http://www.ee.ic.ac.uk/hp/staff/dmb/matrix/calculus.html`