MCE/EEC 647/747: Robot Dynamics and Control

Lecture 13: Output Feedback and Observers
Part I: Linear Observers

Mechanical Engineering

Hanz Richter, PhD

Output Feedback Control

- Basic state-space control methods assume that all states are available for measurement, so that the control law can be calculated.
- In practice, this rarely happens. Only a set of outputs, assumed linear functions of the states, are measured and controlled. The output equation has the form

$$y = Cx$$

- Since C is generally not an invertible matrix, x cannot be obtained from y.
- *Output feedback* refers to a control law calculation based on y only. **Output feedback stabilization is possible when the plant is at least *detectable*.**
- That is, the unobservable states decay naturally, while y contains enough information on the remaining states.

Observers

- A state estimator or *observer* is an artificial dynamic system that evolves simultaneously with the plant. Its function is to estimate the state of the plant using input and output information.
- The observer has two inputs: u and y from the plant, and its output is the estimate of the plant state vector $\hat{x}$
- How are observers built? Why not just

$$\dot{\hat{x}} = A\hat{x} + Bu$$

- The Luenberger observer solves the initial condition mismatch problem by introducing feedback.
- Add an error-correcting term $H(y - \hat{y}) = H(y - C\hat{x})$ to obtain

$$\dot{\hat{x}} = (A - HC)\hat{x} + Bu + Hy$$

- Matrix H is the *observer gain*. It must be chosen so that $A - HC$ has eigenvalues on the left half of the complex plane for $\hat{x}$ to converge to x.

Separation Principle and Dynamic Compensator

- What happens when the state estimates $\hat{x}$ are used in the computation of the state feedback law ($u = -K\hat{x}$)?
- The closed-loop system can be reduced to

$$\dot{x} = (A - BK)x + BK\tilde{x}$$

$$\dot{\tilde{x}} = (A - HC)\tilde{x}$$

  where $\tilde{x} = x - \hat{x}$ is the estimation error.
- Note that the convergence of the estimator and the stability of the plant can be enforced *separately*, by placing the poles of $(A - BK)$ and $(A - HC)$ on the left half of the complex plane.
- It is good practice to tune the observer to converge faster than the plant, by choosing faster poler in the observer.
- The transfer function of the observer-controller combination can be shown to be $K(s) = \frac{U(s)}{Y(s)} = -K[(sI - A + BK + HC)^{-1}]H$

Example

Design an observer-based state feedback controller to stabilize the double integrator plant. Use LQR tuning for both the observer and the controller. Simulate.