MCE/EEC 647/747: Robot Dynamics and Control

Lecture 13: Output Feedback and Observers
Part II: Nonlinear Observers

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The Extended Observer

Consider the nonlinear system:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned}$$

An extended observer imitates the structure of the Luenberger observer used for linear systems:

$$\dot{\hat{x}} = f(\hat{x},u) + \kappa(\hat{x}, y - h(\hat{x}))$$

where κ is a function to be determined. The dynamics of the estimation error are

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = f(x,u) - f(x-e,u) + \kappa(x-e, h(x) - h(x-e))$$

The Extended Observer...

Note that $\kappa(x, 0)$ needs to be zero for any x for $e = 0$ to be an equilibrium point of the error dynamics. The extended observer will be successful in tracking the state if $e = 0$ can be made an asymptotically stable equilibrium point.

A condition called *exponential detectability* guarantees that we can find a function κ with the required characteristics. For details, see Kwatny and Blankenship, *Nonlinear Control and Analytical Mechanics*, Birkhäuser 2000.

Example

Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= -x_1^3 + x_1 + u \\ y_1 &= x_1^3 + x_2 \end{aligned}$$

Develop an extended observer using a linear function for κ. Determine the allowable ranges for the coefficients of κ. Simulate. Is convergence global?

A High-Gain Observer for Manipulators

This observer structure was first proposed by Nicosia and Tomei in 1990. Recall the state-space version of the robot dynamic equation:

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= -M^{-1}Cz_2 - M^{-1}g + M^{-1}\tau \\
y &= z_1
\end{aligned}
$$

So it has been assumed that only z_1 (joint position) is available as a measurement. The observer is given by

$$
\begin{aligned}
\dot{\hat{z}}_1 &= \hat{z}_2 + \frac{1}{\varepsilon}H_p(y - \hat{z}_1) \\
\dot{\hat{z}}_2 &= \frac{1}{\varepsilon^2}H_v(y - \hat{z}_1)
\end{aligned}
$$

where ε is a small number (hence the name “high-gain observer”. H_p and H_v need to satisfy a restriction.

High-Gain Observer...

Matrices H_p and H_v need to be positive-definite and such that

$$
H = \begin{bmatrix} -H_p & I \\ -H_v & 0 \end{bmatrix}
$$

is Hurwitz. As an example, the observer is tested on the two-link RP manipulator used earlier.

Sliding Mode Observer

The use of switching terms for control and/or estimation in robotics started with Slotine and Li in the late 1980's. The robust inverse dynamics and passivity-based robust approaches studied earlier include a switching term in the control input.

Canudas de Wit and Fixot proposed a kind of sliding mode observer in the early 1990's intended to produce a joint velocity estimate to be used in conjunction with the passivity-based adaptive control studied in this course. The observer is given by

$$\begin{aligned}\dot{\hat{z}}_1 &= \hat{z}_2 - \Gamma_1 \tilde{z}_1 - \Lambda_1 \text{ sign } (\tilde{z}_1) \\ \dot{\hat{z}}_2 &= -\Lambda_2 \text{ sign } (\tilde{z}_1) - W(z_1, v, \hat{\Theta})(r' - \Lambda_1 \text{ sign } (\tilde{z}_1)) + \nu\end{aligned}$$

where $W(z_1, v, \hat{\Theta}) = -\hat{M}(z_1) + \hat{C}(z_1, v) - K$, Λ_1 and Λ_2 are positive-definite matrices and r' is similar to the definition of r used for the passivity-adaptive controller, but replacing $\dot{q}$ by its estimate:

$$r' = \hat{z}_2 - v$$

$$v = \dot{q}_d - \Lambda \tilde{q}$$

Sliding Mode Observer...

The switching term ν is defined as

$$\nu = \begin{cases} -\frac{\psi(\hat{z}_2, \tau)}{\lambda_1} \Lambda_1 \text{ sign } (\tilde{z}_1) & \text{, if} ||\Lambda_1 \text{ sign } (\tilde{z}_1)|| \neq 0 \\ 0 & \text{, otherwise} \end{cases}$$

The function $\psi(.)$ provides a bound for the joint velocities and it is defined as

$$\psi(\hat{z}_2, \tau) = \sigma_0 ||\hat{z}_2||^2 + \sigma_0 \lambda_1 + \lambda_1^2 + \sigma_2 ||\tau||$$

The values of σ_0, σ_1 and σ_2 are obtained from:

$$-M^{-1}(z_1) C(z_1, z_2) z_2 - M^{-1}(z_1) g(z_1) + M^{-1}(z_1) \tau \leq \psi(\hat{z}_2, \tau)$$

Recall that the inertia matrix and its inverse are norm bounded and so is $g(z_1)$:

$$\begin{aligned}||M(q)|| &\leq \bar{M} \\ ||M^{-1}(q)|| &\leq \underline{M} \\ ||g(q)|| &\leq \bar{G}\end{aligned}$$

Sliding Mode Observer...

The second term of the robot dynamic equation is also bounded as

$$||C(z_1, z_2)z_2|| \leq \bar{C}||z_2||^2$$

These bounds can be calculated to find the values of σ_0, σ_1 and σ_2. An analytical calculation of these bounds may be difficult. We will use a numerical search for these values.

Also note that λ_1 arises from assuming $\Lambda_1 = \lambda_1 I$ for simplicity. Note that this observer cannot be used separately from a special form of control law due to its dependence on τ.

Overall Observer-Based Adaptive Scheme

The observer defined above is proved to produce local stability when used in conjunction with specific control and adaptation laws.

Adaptation Law:

$$\dot{\hat{\Theta}} = -\Gamma^{-1}Y^T(z_1, \hat{z}_2 - \Lambda_1 \text{ sign } (\tilde{z}_1), v, a' + \Lambda\Lambda_1 \text{ sign } (\tilde{z}_1))(r' - \Lambda_1 \text{ sign } (\tilde{z}_1))$$

Here a' is a special version of the a used with the passivity-adaptive controller studied before:

$$a' = \ddot{q}_d - \Lambda(\hat{z}_2 - \ddot{q}_d)$$

Note that the regressor $Y(q, \dot{q}, \ddot{q})$ must be manipulated to have the required form.

Overall Observer-Based Adaptive Scheme...

Control Law:

$$\tau = \tau_0 - W(z_1, v, \hat{\Theta})\Lambda_1 \text{ sign } (\tilde{z}_1)$$

τ_0 is the standard adaotive-passivity control law with $\hat{z}_2$ in place of z_2:

$$\tau_0 = \hat{M}(z_1)a' + \hat{C}(z_1, \hat{z}_2)v + \hat{g}(z_1) - Kr'$$

As an example, we apply the overall scheme to the PR manipulator.