MCE/EEC 647/747: Robot Dynamics and Control

Lecture 15: Energy Considerations

Energy Regeneration in Robots

Mechanical Engineering
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Mechanical Energy Balance

Consider the dynamic model of a robotic manipulator with torque/force input (no actuator model):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(g) + \mathcal{R}(q,\dot{q}) + J^T(q)F_e(t) = \tau$$

The total mechanical power at the joints is then

$$P = \dot{q}^T\tau = \dot{q}^T M(q)\ddot{q} + \dot{q}^T C(q,\dot{q})\dot{q} + \dot{q}^T g(g) + \dot{q}^T \mathcal{R}(q,\dot{q}) + \dot{q}^T J^T(q)F_e(t)$$

Suppose the robot moves between joint positions q_A and q_B in a time interval $[t_A, t_B]$.
We want to calculate the associated energy and how its distributed.

Mechanical Energy Balance...

Start with the integral of the inertial term:

$$\int_{t_A}^{t_B} \dot{q}^T(t) M(q(t)) \ddot{q}(t) dt$$

Recall that the kinetic energy is

$$KE = \frac{1}{2} \dot{q}^T M(q) \dot{q}$$

and is derivative is

$$dKE = \dot{q}^T M(q) \ddot{q} dt + \frac{1}{2} \dot{q}^T \dot{M} \dot{q} dt$$

Therefore

$$\int_{t_A}^{t_B} \dot{q}^T(t) M(q(t)) \ddot{q}(t) dt = \int_{q_A}^{q_B} dKE - \frac{1}{2} \int_{t_A}^{t_B} \dot{q}^T \dot{M} \dot{q} dt$$

Mechanical Energy Balance...

Now combine the integrals of the inertial term and Coriolis terms:

$$
\begin{aligned}
\int_{t_A}^{t_B} \dot{q}^T(t) M(q(t)) \ddot{q}(t) + \dot{q}^T C(q, \dot{q}) \dot{q} dt &= \\
\Delta(KE)_{AB} + \frac{1}{2} \int_{t_A}^{t_B} \dot{q}^T (2C(q, \dot{q}) - \dot{M}) \dot{q} dt &= \Delta(KE)_{AB}
\end{aligned}
$$

Now continue with the integral of the gravity term:

$$
\int_{t_A}^{t_B} \dot{q}^T(t) g(q(t)) dt = \int_{t_A}^{t_B} g^T(q) dq = \Delta(PE)_{AB}
$$

since

$$
g^T(q) dq = dPE
$$

Mechanical Energy Balance...

Now consider the integral of the Rayleigh dissipation term:

$$\int_{t_A}^{t_B} \dot{q}^T(t)\mathcal{R}(q(t), \dot{q}(t))dt \tag{1}$$

Since $\mathcal{R}(q, \dot{q})$ represents energy dissipation, the above integral is non-negative. We can evaluate it for specific kinds of mechanical friction. For instance, for $\mathcal{R}(q, \dot{q}) = B\dot{q}$ (viscous friction), B must be positive-semidefinite.

We can call this integral Σ_m.

Mechanical Energy Balance...

Finally, consider the integral of the external force term

$$\int_{t_A}^{t_B} \dot{q}^T(t) J^T(q) F_e^T(t) dt \tag{2}$$

The integral represents the work of the external force calculated at the joints. We call this integral W_e. The overall energy balance now reads

$$\int_{t_A}^{t_B} \dot{q}^T(t)\tau(t)dt = \Delta E_{AB} = \Delta(KE)_{AB} + \Delta(PE)_{AB} + \Sigma_m + W_e$$

That is, the power delivered at the joints is used to change the total energy (kinetic+potential), with some amount being lost as mechanical friction and some power delivered to or received from the external force.

Mechanical Energy Balance - Notes

If the linear parameterization (regressor) and parameter vector are available, the energy balance can be calculated as follows:

$$\int_{t_A}^{t_B} \dot{q}^T(t)\tau(t)dt = \int_{t_A}^{t_B} \dot{q}^T(t)Y(q(t), \dot{q}(t), \ddot{q}(t))\Theta dt + W_e$$

This assumes that the parameterization captures the mechanical dissipation effects.

This form is useful to find the total energy (not the distribution) if the trajectories are known functions of time.

Also note that if the trajectory is cyclic $(q(t_A) = q(t_B)$ and $\dot{q}(t_A) = \dot{q}(t_B))$, then $\Delta(KE) + \Delta(PE) = 0$.

Actuator Energy Balance : Voltage-Driven

Consider a voltage-driven DC motor for a specific joint with coordinate q_i. The dynamics of the actuator are given by two equations:

$$\begin{aligned} V_i - iR - a\dot{q}_i &= 0 \\ ai - B_t\dot{q}_i - \tau_i - J_t\ddot{q}_i &= 0 \end{aligned}$$

where J_t and B_t are inertial and mechanical damping parameters reflected to the link side. Also, $a = n\alpha$, where n is the gear ratio and α is the torque constant. The armature resistance is R.

To find an electric energy balance, multiply the mechanical equation by $\dot{q}_i$ and the electric equation by i. Combining:

$$P_e = iV = J_t\ddot{q}_i\dot{q}_i + B_t\dot{q}_i^2 + \tau_i\dot{q}_i + i^2R$$

Electric Energy Balance : Voltage-Driven

Integrating and adding electrical power across all actuators (using $\tau\dot{q}$ from the robot's energy balance):

$$\Delta E_{AB} = \Delta(KE)_{act} + \Sigma_{act} + \Delta(KE)_{AB} + \Delta(PE)_{AB} + \Sigma_m + W_e + \Sigma_e$$

where Σ_e is the sum of the integrals of the i^2R terms of all joints (total Joule losses). We can combine the mechanical energies and dissipation of the robot and actuator:

$$\Delta E_{AB} = \Delta(KE)_{AB} + \Delta(PE)_{AB} + \Sigma_m + W_e + \Sigma_e$$

Bidirectional Power Transfer

Suppose a robotic joint accelerates in the positive direction. Then it demands power according to $\tau_i \dot{q}_i$, and both τ_i and $\dot{q}_i$ are positive. By convention we call this positive power (from the actuator to the robot link).

If the robot decelerates, still with $\dot{q}_i > 0$, then $\tau_i < 0$ (a braking torque). Power is negative (from the robot link to the actuator).

There are 4 combinations of signs of τ_i and $\dot{q}_i$, 2 giving positive power and 2 giving negative power. A *back-drivable* mechanism can operate with all four combinations of torque and velocity.

A car jack and a guitar tuning peg should not be back-drivable.
The drive system of an elevator and a hybrid car transmission should be back-drivable (why)?

Energy Regeneration

If a robot has a back-drivable actuator, there's an opportunity to store some of the energy associated with link deceleration (negative power) instead of wasting it in friction brakes or braking resistors.

An electric robot drive system may be powered by the supply line, a battery (mobile robots) or innovative energy storage and return devices (supercapacitors).

All three options have some ability to convert negative mechanical power back to electric power. Some industrial drives are able to return power to the grid. The other two options involve storing the energy in the same medium used to deliver positive power.

Power Density vs. Energy Density

Energy density is the ratio of energy storing capacity to volume. Power density is the ratio of rate-of-energy capacity to volume.

Similar definitions exist for specific energy and power (using mass instead of volume).

Batteries have very large specific energies but low specific power:

- Lead-acid: about 40 W-h/kg and 180 W/kg
- Lithium-Ion: about 200 W-h/kg and 300 W/kg.

Batteries can be recharged, but not fast. Exercise: calculate the regenerative current that a 12V battery must handle in an electric vehicle coming to a stop from 30 km/h at constant acceleration, in 15 sec. Assume the car has a mass of 1500 kg.

Ultracapacitors

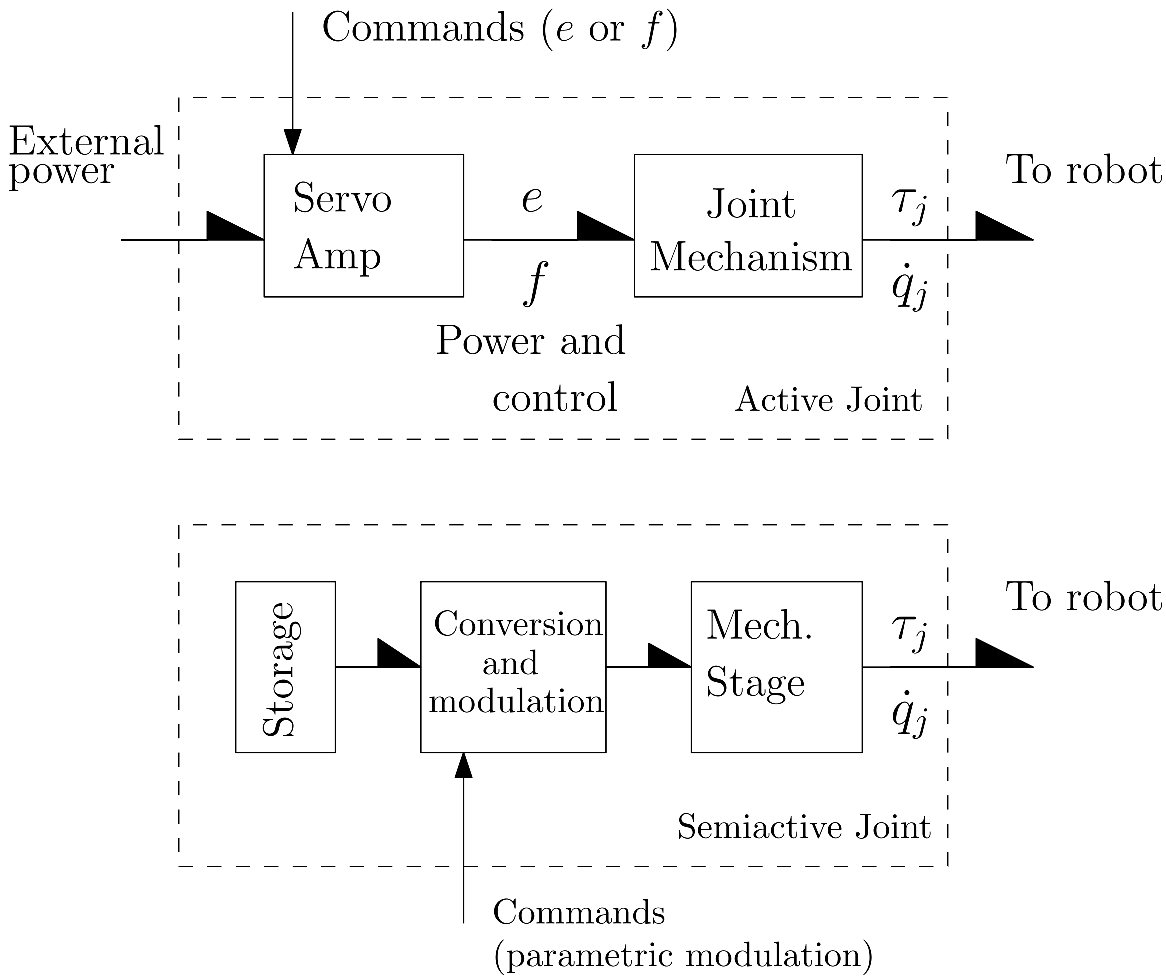
Ultracapacitors (also called supercapacitors) are a new storage technology becoming widespread across motion applications. Ultracapacitors have smaller energy densities than batteries, but much larger power densities (10 kW/kg). The capacitance can be very large (thousands of F), and supercapacitors allow much higher charge and discharge cycles than batteries.

Unlike batteries, ultracapacitors can deliver and accept very large currents, making them ideal for regenerative applications.

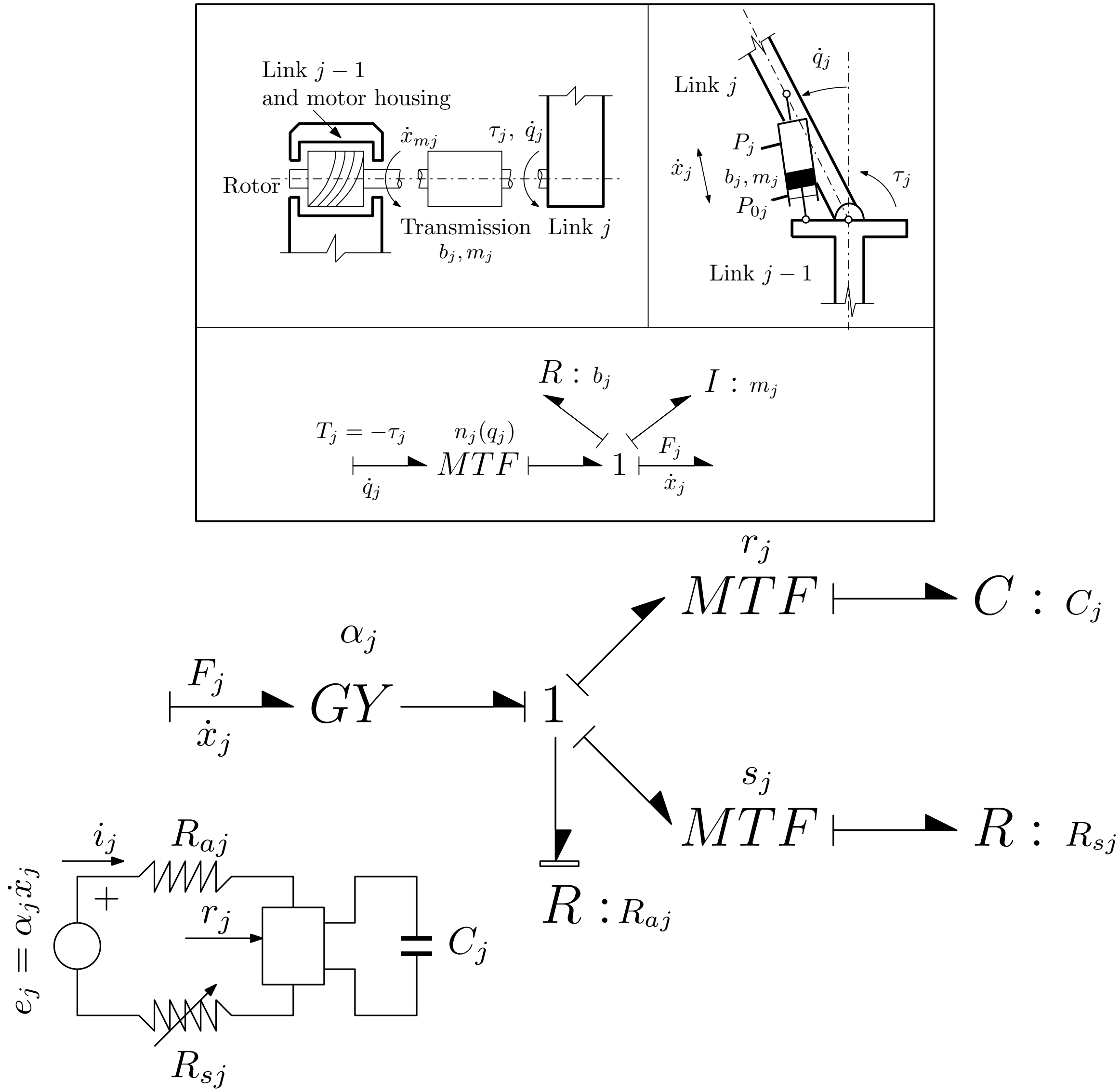
New nanotechnology research is producing ultracapacitors with specific energies in the same order of magnitude as batteries.

Ultracapacitor-Based Drive System

Research at CSU has considered two types of actuation systems: active (conventional) and semi-active (with an ultracapacitor as energy storage element):



Ultracapacitor-Based Drive System...



Semiactive Control

The control input for joint j is r_j, the ratio of motor voltage to capacitor voltage. This control can be set to any desired value between -1 and 1. Regenerative servo amplifiers with the assumed control mode are available. It can be shown[a] that the dynamics of the robot with this actuation system become:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + \mathcal{R} + g(q) = u$$

where the components of u are

$$u_j = \frac{a_j r_j}{C_j R_j} y_j$$

and y_j is the charge of the ultracapacitor.

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[a] Richter, H. IFAC 2014, JDSMC 2015

Semiactive Virtual Control

The idea for *semiactive virtual control* (SVC) is to regard u_j as a virtual control input and specify a motion control law τ^d for the robot.
Then the actual control is simply found as

$$r_j = \frac{\tau_j^d R_j}{a_j \frac{y_j}{C_j}}$$

Note that the denominator is the instantaneous voltage of the capacitor, used for feedback. We can calculate this control as long as there's some charge left in the capacitor.

The above calculation is called *virtual matching*.

Internal Dynamics and Energy Balance

When virtual matching is exact, the virtual control becomes effective and the motion task is accomplished. But there are internal dynamics associated to y_j. It can be shown that under SVC, the internal dynamics can be expressed as an energy balance equation:

$$\Delta E_{sj} = -\int (Y_j \Theta \dot{q}_j + \frac{R_j}{a_j^2} \Theta^T Y_j^T Y_j \Theta) dt$$

where ΔE_{sj} is the change of energy stored in the capacitor of joint j and Y_j is the j-th row of the regressor.

Internal Dynamics and Energy Balance

Many research problems arise from the above model, control and energy balance equation. Several of these problems are simplified because ΔE_{sj} is quadratic in Θ.

1. Optimal robot parameters: Find design parameters Θ that maximize the stored energy given a pre-defined motion trajectory.

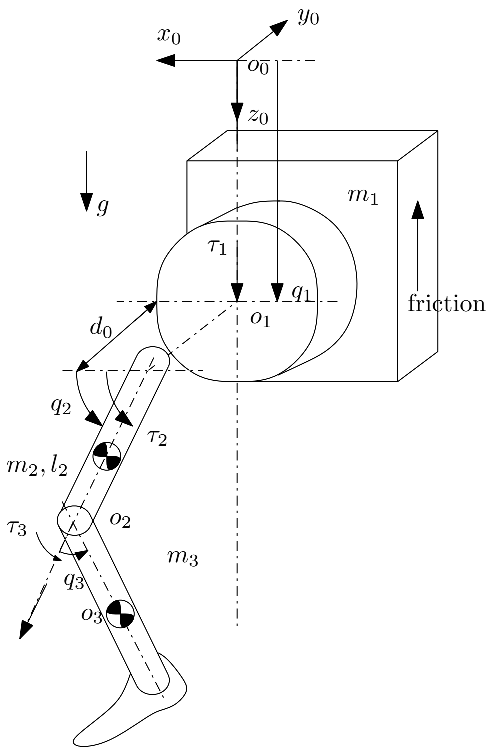
2. Optimal gear ratio: Find the gear ratio n that maximizes the stored energy given a pre-defined motion trajectory.

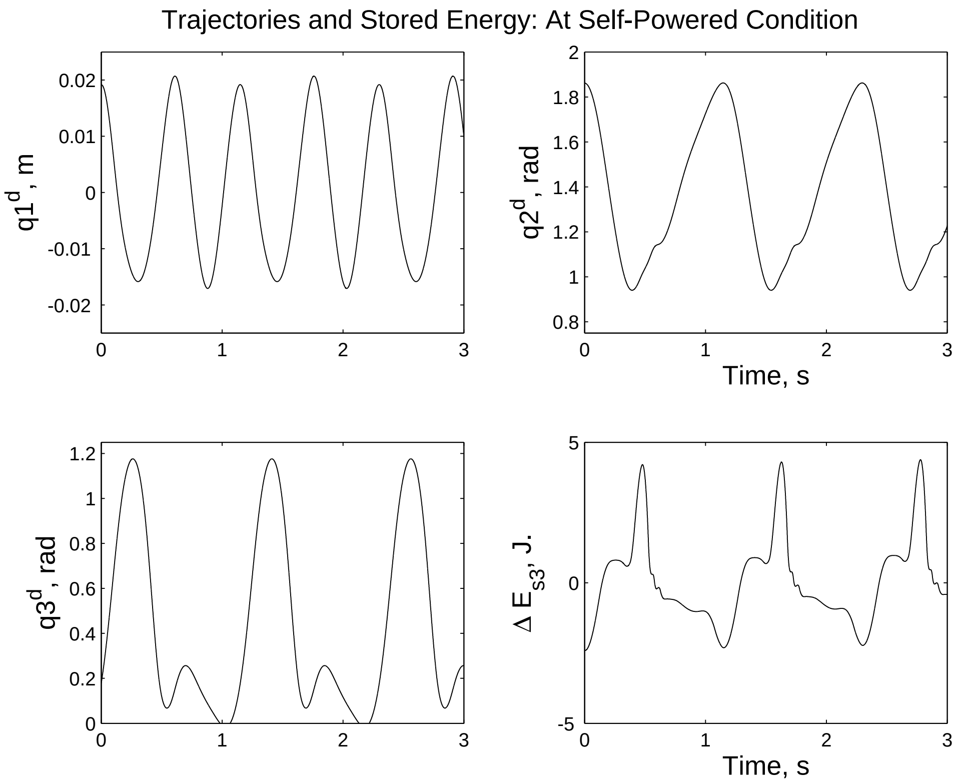
3. Optimal trajectory between two points: For fixed Θ and n, find the best trajectory to follow to maximize energy storage.

4. Optimal control: Find the optimal control and trajectories for stored energy maximization.

These problems have been studied (some solved) by CSU researchers Richter, Khalaf, Warner, van den Bogert, Khademi and Simon.

Example: Prosthesis Test Robot





An critical gear ratio of $n = 23.26$ results in $\Delta E_3 = 0$ for the knee (semiactive). The hip and thigh were fully-active.

Example: Prosthesis Test Robot

For gear ratios lower and higher than the critical value, the energy would decrease or build up in the capacitor:

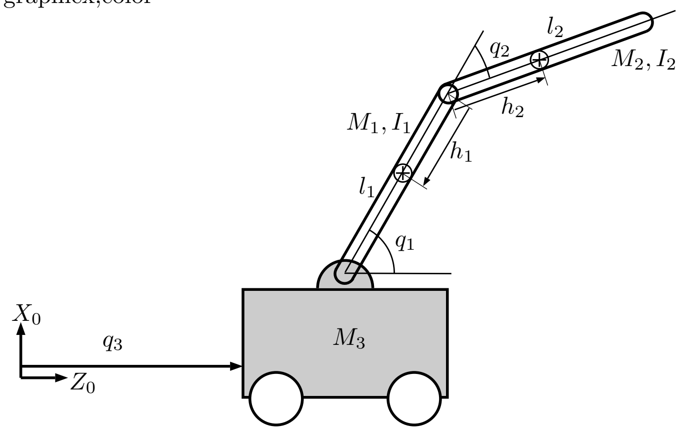
Table 1: Energy Balance Figures (Joules)

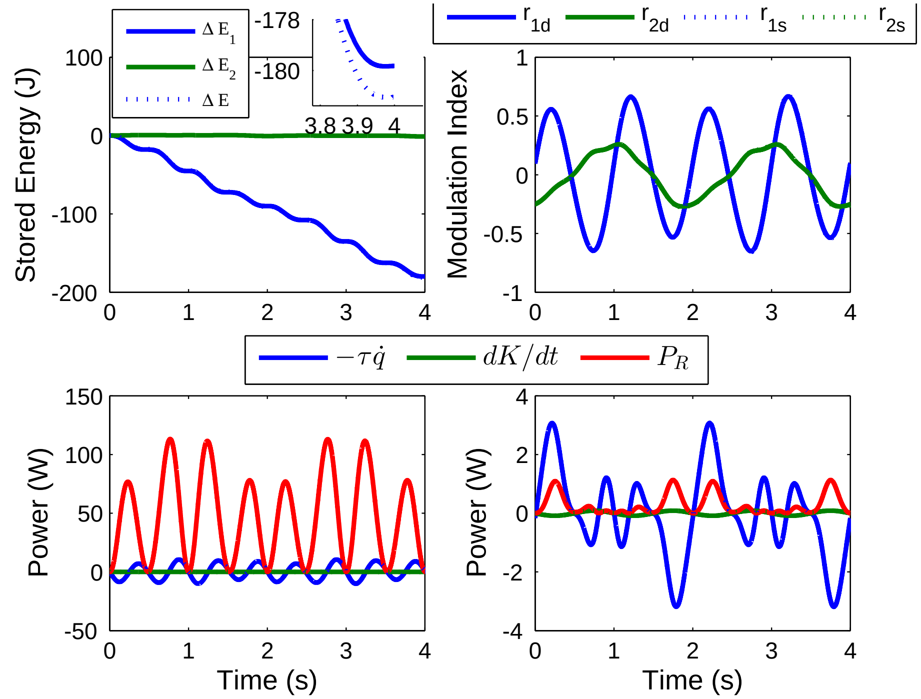
n_3	W_{ACT}	ΔE_{s3}	σ_m^T	σ_e^T	ΔE_m^T
20	65.666	-5.6136	50.6722	20.5989	0.0091
30	65.666	5.8303	50.6722	9.1551	0.0091

The difference is in the Joule dissipation alone, showing the importance of an optimal gear ratio selection.

Example: Optimal Parameter Selection

graphicx,color

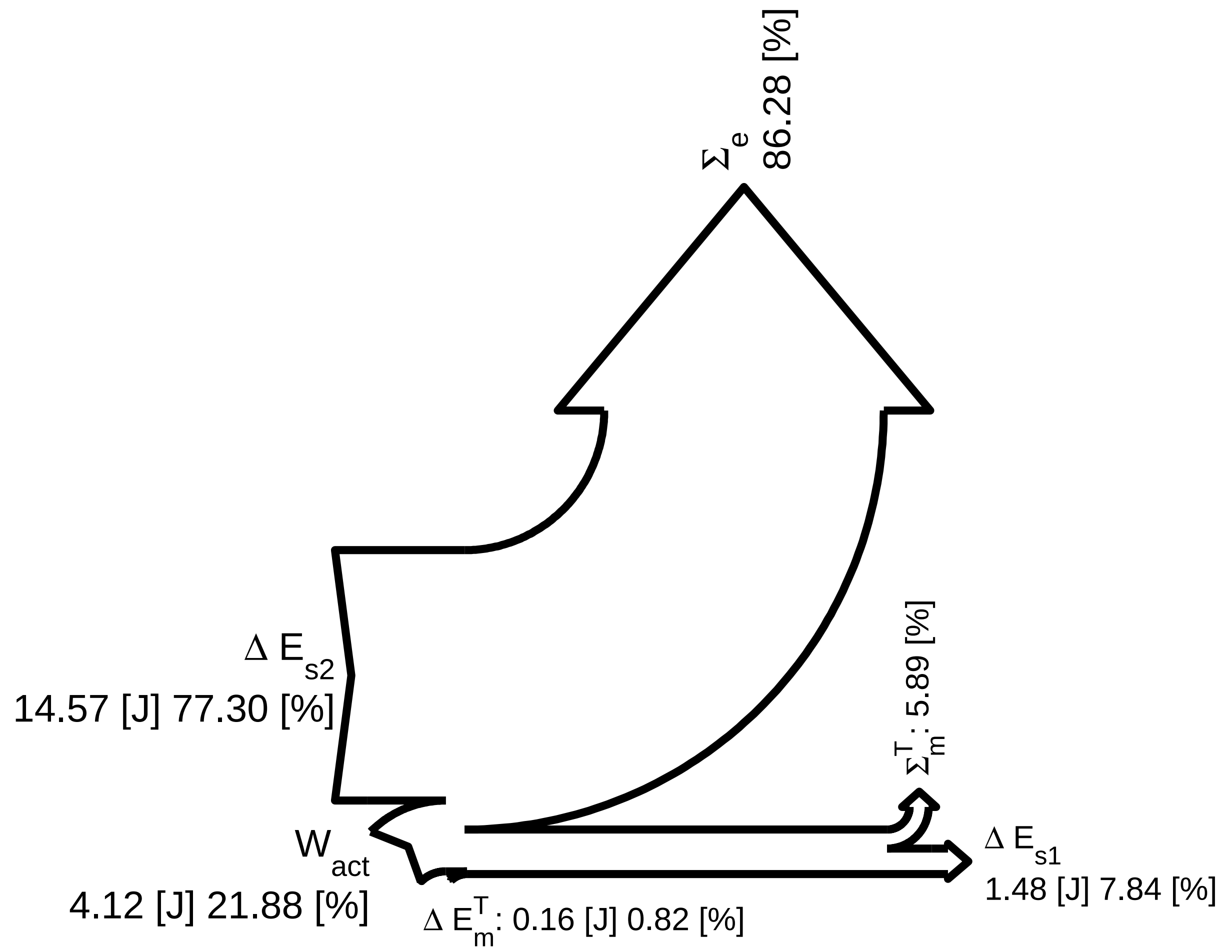




The cart is active and the two upper links are semiactive. With initial parameters and a given set of trajectories, Joule losses are large.

Example from Khalaf, P. and Richter, H., ASME DSCC 2015 and ASME JDSMC 2017.

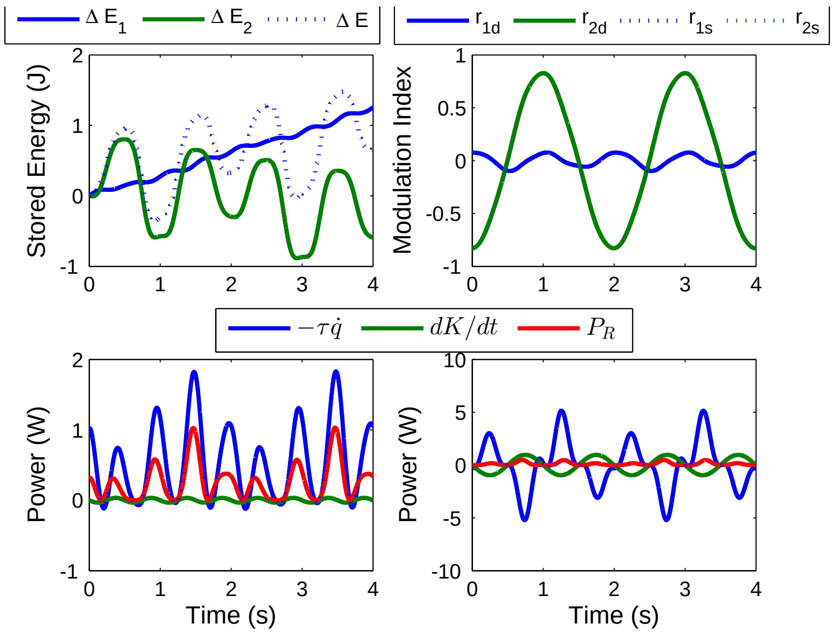
Example: Optimal Parameter Selection...

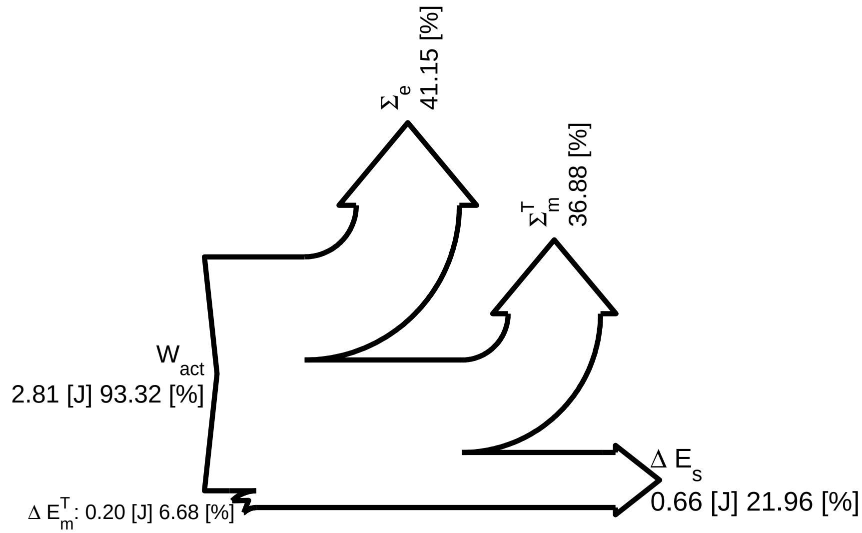


Sankey diagrams (used also in thermal systems) are very useful to describe energy balances. Before optimization, most energy is lost as heat.

Example from Khalaf, P. and Richter, H., ASME DSCC 2015 and ASME JDSMC 2017.

Example: Optimal Parameter Selection...





After redesigning robot parameters, the same control and trajectory results in an improved energy utilization.

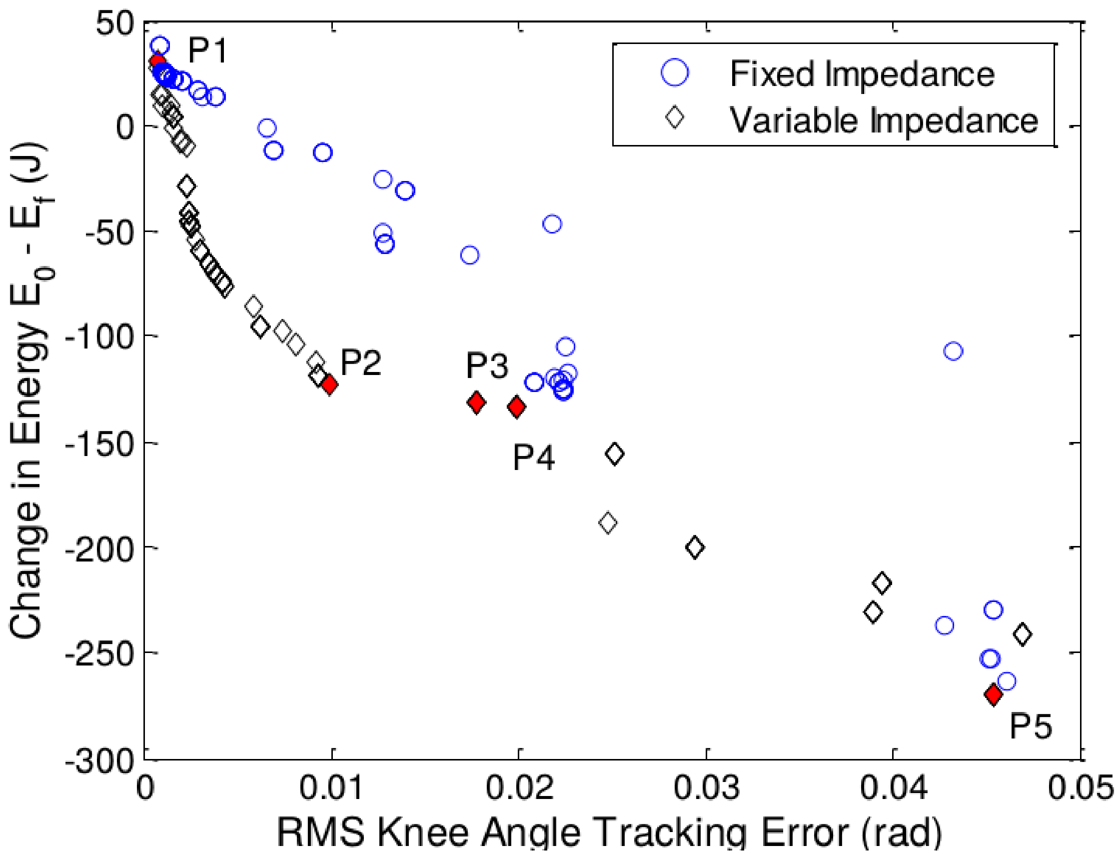
Example from Khalaf, P. and Richter, H., ASME DSCC 2015 and ASME JDSMC 2017.

Multi-Objective Optimization: Tracking vs. Regeneration

Suppose an impedance controller is used in a robot that will interact mechanically with an environment.
Intuitively, a very stiff impedance will result in accurate tracking regardless of environmental forces. But such “high impedance” reduces the back-drivability of the robot, so the regeneration effect is reduced.

Conversely, a very compliant impedance reduces tracking accuracy but can improve energy recovery. This represents a conflict, that can be managed with multi-objective optimization (MOO).

Multi-Objective Optimization...



Points on the graph are a *Pareto front*, the set of optimal solutions such that both objectives can't be improved upon simultaneously (non-dominated solutions).

Open Research Problems

The basic SVC has been verified in real-time experiments.
Ongoing/future research:

1. Optimal impedance control for regeneration, application to prosthetic legs (Poya Khalaf)
2. SVC with AC motors and drives (Amin Ghorbanpour)
3. Effect of interconnection topology (Rahul Cheppally)
4. Modeling and control for regenerative multi-robot systems (Amin Ghorbanpour)
5. Energy-consistent fractional-order modeling of ultracapacitors (Richter)